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A case study of flood dynamic wave simulation in natural waterways using numerical solution of unsteady flows

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Abstract

Flood routing has many applications in engineering projects and helps designers in understanding the flood flow characteristics in river flows. Floods are taken unsteady flows that vary by time and location. Equations governing unsteady flows in waterways are continuity and momentum equations which in case of one-dimensional flow the Saint-Venant hypothesis is considered. Dynamic wave model as one of the flood routing methods is used for flooding operations because of its high accuracy. Finding the best numerical methods is the main challenge for optimal modeling of dynamic waves and predicting the flood behavior. In this study, a 78-km reach of Ghare-Aghaj River (between two hydrometric stations) is investigated and in addition to the unsteady flow equations (nonlinear Hyperbolic partial differential equations), Preissman implicit scheme and Mac Cormak explicit scheme which are both based on the finite difference numerical methods have been used. Developed models have also been compared with one of the most reliable Mike series computer program. It is aimed to find the most suitable finite difference scheme by comparing the results of two above schemes with the results of Mike 11 program. The results confirmed the superiority of Preissmann implicit scheme in predicting flood wave characteristics for the studied area.

Keywords: Dynamical equations; Explicit and implicit numerical methods; Finite difference; Preissmann; Mc Cormak.

1. Introduction

Flood routing is an operational through which downstream flow hydrograph is determined by known upstream flow hydrograph. Graff was the first one who used flood routing method for translation of a flood hydrograph from one point to the other [1]. Mathematical methods of flood routing help designers in understanding the effects of flow on river stream and its surrounding area and they are important in designing the flood protection measures and proposing effective economical solutions to protect against flood waves behavior in waterways. Generally, computational methods for flood routing can be divided into two categories: Hydrological and Hydraulic. Hydrological routing is only based on one-dimensional continuity equation, during the routing operation and the flow hydrograph measured only at one section in downstream. Therefore, the flow discharge relative to time at

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a fixed distance is extracted from the upstream hydrograph. However, in the hydraulic method, one-dimensional continuity and momentum equations (Saint-Venant) are solved simultaneously by various numerical methods and the flow hydrograph can be calculated at any distance from the section which upstream hydrograph is known. The flow depth and other flood flow characteristics can also be determined based on a function of time and location [2].

Dynamic wave model which is one of flood routing methods is solved by different numerical schemes. Dynamic models use the full Saint-Venant equation solution and yield realistic results. Since the analytical solution of these equations is not possible, finding the best numerical method is one of the most challenging issues in flood routing operations. So what differentiates the stability, compatibility and accuracy of different schemes is selection of the best numerical method and the versatility of models by the observed flood. Finite difference schemes are among numerical methods with high accuracy, efficient consistency and good capabilities for computer programming in flood routing operations. Extended efforts have been made by various researchers which results in a wide variety of numerical methods and solutions. Stocker proposed his numerical method known as fixed mesh explicit method for solving unsteady flow equations [3]. To avoid sensitivity of explicit methods to the finite time interval, Fox used characteristic lines method proposed by Hartree named also as rectangular grid method [4]. The merit of this method is the easy handling of its simulation and computer programming. Preissmann used implicit schemes and Lip-Frog proposed explicit method from the second stage [5]. Abbott proposed an explicit method, in cooperation with the researchers at the Delft University of Netherlands [6]. Abbott's four-point method based on characteristic lines method ignored the energy line slope and the bed flow slope to solve the unsteady flow equations. However, the pioneers of dynamic wave method are confirmed by Preissmann, Blatzer and Lai, Dronkers, Amien and Fang [7]. The most effective effort was done by Amien and Fange for stable, quick and accurate solution of equations using Newton Raphson iterations. There are other studies performed in this field include; Mc Cormack, Anderson and Moretti [8].

In this study, two groups of finite difference numerical techniques i.e. Mc Cormack explicit scheme and Preissmann implicit scheme are investigated. MATLAB software is used with real river data. The results of this study are compared by Mike 11 computer numerical model designed at Danish Hydraulic Institute (DHI) [9]. The purpose of this comparison is to investigate the various finite difference schemes to determine the most convergent method in terms of waterway physical and hydraulic characteristics and the unsteady flow in flood routing operations. A river reach between two stations with a 78-kilometer length has been chosen in Ghare Aghaj River basin and cross sections of the river are assigned and the river bed is simulated. After hydraulic definition of the flow and analysis of the model, the output results are compared to Mike 11 results which are selected as references to examine the finite difference numerical schemes of Mc Cormak explicit scheme and Preissmann implicit scheme.

2. Saint-Venant equation (governing equations)

The Saint-Venant equations were first presented by Barre de saint-Venant in 1871 which explains unsteady and non-uniform one-dimensional flows in waterways. The study was published in the journal of French Academy of Sciences [2,4]. Saint-Venant equations include one-dimensional continuity and momentum equations used for flood flow hydraulic routing, regardless of side lateral flow, wind shear stress and eddy losses in the following conservation form:

Conservation form of one dimensional continuity equation:

$$\frac{\partial Q}{\partial x}+\frac{\partial A}{\partial t}=\frac{\partial Q}{\partial x}+\left(\frac{\partial A}{\partial Q}\right)\frac{\partial Q}{\partial t}=\frac{\partial Q}{\partial t}+\left(\frac{\partial Q}{\partial A}\right)\frac{\partial Q}{\partial x}=0 \tag{1}$$

Conservation form of momentum equation:

$$\frac{1}{A}\frac{\partial Q}{\partial t}+\frac{1}{A}\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right)+g\frac{\partial y}{\partial x}-g\left(S_0-S_f\right)=0 \tag{2}$$

where, Q is discharge flow, A is the mean cross-section area of waterway, G is acceleration of gravity, S_0 is the bottom slope and S_f *is* the energy line slope resulting from flow resistance. These equations are in the forms of non-linear partial differential equations and of hyperbolic type that can be solved based on different numerical methods. Equations (1) and (2) can be written in the form of a matrix as follows:

$$U_t+F_x+S=0 \tag{3}$$

$$U=\begin{pmatrix} A \\ VA \end{pmatrix} \quad ; \quad F=\begin{pmatrix} VA \\ V^2A+gA\bar{y} \end{pmatrix} \quad ; \quad S=\begin{pmatrix} 0 \\ -gA(S_0-S_f) \end{pmatrix}$$

where, A□ is flow area around the free surface.

3. Preissmann implicit model

This model is a finite difference model where the spatial partial derivatives are expressed in terms of the variables at the unknown time level and can be approximated as follows:

$$\frac{\partial f}{\partial x}=\frac{\beta(f_{i+1}^{k+1}-f_i^{k+1})+(1-\beta)(f_{i+1}^{k}-f_i^{k})}{\Delta x} \tag{4}$$

$$\frac{\partial f}{\partial t}=\frac{(f_i^{k+1}+f_{i+1}^{k+1})-(f_i^{k}-f_{i+1}^{k})}{2\Delta t} \tag{5}$$

$$f=\frac{1}{2}\beta(f_{i+1}^{k+1}+f_i^{k+1})+\frac{1}{2}(1-\beta)(f_{i+1}^{k}+f_i^{k}) \tag{6}$$

where, β is a weighting coefficient between zero and one in partial derivatives and f refers to functions of depth and flow velocity. In these equations, known and unknown time increments are shown by superscripts k and k +1 respectively. Also subscript *i* shows spatial location on *x* axis. By substitution of the finite difference approximations and coefficients mentioned in matrix form of governing equations, equation (3), finally equation (7) is obtained [5]:

$$\begin{aligned} &U_i^{k+1}+U_{i+1}^{k+1}+2\frac{\Delta t}{\Delta x}.(\beta(F_{i+1}^{k+1}-F_i^{k+1})+(1-\beta)(F_{i+1}^{k}-F_i^{k}))+ \\ &\Delta t(\beta(S_i^{k+1}+S_{i+1}^{k+1})+(1-\beta)(S_{i+1}^{k}+S_i^{k})=U_i^{k}+U_{i+1}^{k+1} \end{aligned} \tag{7}$$

4. Mac Cormak explicit model

Mac Cormak numerical model is an explicit finite difference models with two parts of prediction and correction. In prediction part, the following approximations are used [1]:

$$\frac{\partial u}{\partial t}=\frac{U_i^{*}-U_i^{k}}{\Delta t} \tag{8}$$

$$\frac{\partial f}{\partial x}=\frac{f_i^{k}-f_{i-1}^{k}}{\Delta x} \tag{9}$$

By substitution of the approximations mentioned in matrix form of governing equations, equation (3), equation (10) is obtained:

$$U_i^{*}=U_i^{k}-\frac{\Delta t}{\Delta x}(f_i^{k}-f_{i-1}^{k})-S_i^{k}\Delta t \tag{10}$$

Evaluated values of U_i^* provide the Q^*, and A^* values that represent the sectional area and the flow discharge that are used for estimation of velocity values and flow depth (V^* and y^*). The values are obtained for all grid points along the *x* axis in a time step and are used to calculate S^* and f^* in the correction part. Superscript * shows variables evaluated in the forecast part, which are used in the correction part. Also, in the correction part, the following approximations are used for the spatial and temporal derivatives:

$$\frac{\partial u}{\partial t}=\frac{U_i^{**}-U_i^{k}}{\Delta t} \tag{11}$$

$$\frac{\partial f}{\partial x}=\frac{f_{i+1}^{*}-f_i^{*}}{\Delta x} \tag{12}$$

By substitution of the approximations mentioned in matrix form of governing equations, equation (13) is obtained:

$$U_i^{**}=U_i^{k}-\frac{\Delta t}{\Delta x}(f_{i+1}^{*}-f_i^{*})-S_i^{*}\Delta t \tag{13}$$

where the superscript ** variables are evaluated variables in the correction part.

Finally, U_i^{k+1} value is calculated from the following equation:

$$U_i^{k+1}=\frac{1}{2}(U_{i1}^{*}+U_i^{**}) \tag{14}$$

5. Field evaluation model

Ghare-Aghaj River originates from mountainous region of north-west Shiraz and joins several other branches and finally, after crossing 685 km through Bushehr province enters Persian Gulf with the name of Mand river. It has steady river reaches over 20 meters width. Its annual average discharge has 20 cubic meters per second in Karzin canyon station. The image of Hydrometric stations (upstream and downstream) is shown in Figure 1.

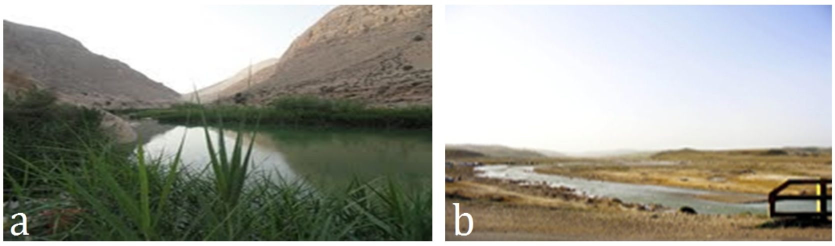

Figure 1. Images of river in selected hydrometric stations (a=upstream; b=downstream)

6. Initial and boundary conditions

Flow discharge and the initial flow condition for both numerical schemes of Mc Cormack and Preissmann is considered steady equal to 20 (m^3/ s). This is also adopted for the Mike11 numerical model. In other words, the basic discharge at the time of flood flows at all points of the reach. Discharge-depth relation and the inflow hydrograph are used as the downstream boundary condition and upstream boundary condition respectively. Upstream boundary condition (incoming flood hydrograph) is shown in Figure 2. Table 1 also shows the characteristics of the Ghare-Aghaj river in the studied area.

Table 1. Physical characteristics of the Ghare-Aghaj river in the studied area

Reach (KM)	B (m)	S_0	N
0 - 20	243	0.00027	0.033
20 - 40	217	0.00067	0.034
40 - 78	164	0.00051	0.037

7. Condition for stability of explicit numerical models

Saint-Venant equation is a set of hyperbolic equations in which the stability depends on the Courant number being kept smaller and more precisely near to 1. [10-12]:

$$C_n = \frac{|V| \pm \sqrt{gy}}{\Delta x / \Delta t} \tag{15}$$

To satisfy the criteria for Preissmann implicit model, weighting coefficient values of; 0.55 <β <1.00 is required [5, 12].

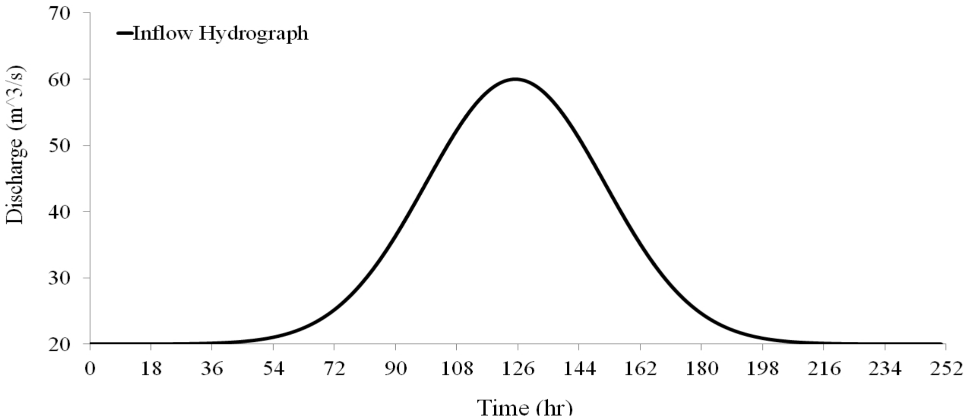

Figure 2. Upstream boundary condition (inflow flood hydrograph)

8. Method of solution

In the Mc Cormack model, equations (10), (13) and (14) are written for each time step and in all grid points. For all points at the end and beginning of the reach, characteristic equations are used. Upstream boundary condition is simultaneously solved by negative characteristic equation C^- at upstream boundary and the downstream boundary condition is simultaneously solved by positive characteristic equation C^+ at downstream boundary. Thus, the values of the flow variables at unknown time interval are calculated for all nodes. This is iterated and is followed for all time steps until the end of calculation time.

In Preissmann model, equation (7) is written for grid points at each unknown time step to calculate the values of discharge, velocity and depth of flow in all nodes of grid point at each time step. At this stage, Newton-Raphson iteration is used to solve the equations. For applying this method, we have:

$$X_1^p = X_0^p - W^{-1}(x_0^p) f(x_0^p) \tag{16}$$

where X_1^p is a columns matrix with $2n+2$ rows and in fact is the response matrix for depth values and flow velocity at each time step and reach length. X_0^p is a column matrix with $2n+2$ rows and in fact is the initial guess matrix for X_1^p, f is also a column matrix with $2n+2$ rows, W is Jacobin square matrix. These matrices are listed as follows:

$$f = \begin{pmatrix} f_1\left(V_1^{k+1}, A_1^{k+1}\right) \\ f_2\left(V_1^{k+1}, A_1^{k+1}, V_2^{k+1}, A_2^{k+1}\right) \\ f_3\left(V_1^{k+1}, A_1^{k+1}, V_2^{k+1}, A_2^{k+1}\right) \\ f_4\left(V_2^{k+1}, A_2^{k+1}, V_3^{k+1}, A_3^{k+1}\right) \\ f_5\left(V_2^{k+1}, A_2^{k+1}, V_3^{k+1}, A_3^{k+1}\right) \\ \vdots \\ \vdots \\ f_{2n}\left(V_n^{k+1}, A_n^{k+1}, V_{n+1}^{k+1}, A_{n+1}^{k+1}\right) \\ f_{2n+1}\left(V_n^{k+1}, A_n^{k+1}, V_{n+1}^{k+1}, A_{n+1}^{k+1}\right) \\ f_{2n+2}\left(V_{n+1}^{k+1}, A_{n+1}^{k+1}\right) \end{pmatrix} \; ; \; X_0^p = \begin{pmatrix} V_1^k \\ A_1^k \\ V_2^k \\ A_2^K \\ \vdots \\ \vdots \\ V_{n+1}^k \\ A_{n+1}^k \end{pmatrix} \; ; \; X_1^{p+1} = \begin{pmatrix} V_1^{k+1} \\ A_1^{k+1} \\ V_2^{k+1} \\ A_2^{K+1} \\ \vdots \\ \vdots \\ V_{n+1}^{k+1} \\ A_{n+1}^{k+1} \end{pmatrix} \tag{17}$$

$$
W=\begin{pmatrix}
\frac{\partial f_1}{\partial v_1} & \frac{\partial f_1}{\partial A_1} & \frac{\partial f_1}{\partial v_2} & \frac{\partial f_1}{\partial A_2} & \cdots & \frac{\partial f_1}{\partial V_{n+1}} & \frac{\partial f_1}{\partial A_{n+1}} \\
\frac{\partial f_2}{\partial v_1} & \frac{\partial f_2}{\partial A_1} & \frac{\partial f_2}{\partial v_2} & \frac{\partial f_2}{\partial A_2} & \cdots & \frac{\partial f_2}{\partial V_{n+1}} & \frac{\partial f_2}{\partial A_{n+1}} \\
\frac{\partial f_3}{\partial v_1} & \frac{\partial f_3}{\partial A_1} & \frac{\partial f_3}{\partial v_2} & \frac{\partial f_3}{\partial A_2} & \cdots & \frac{\partial f_3}{\partial V_{n+1}} & \frac{\partial f_3}{\partial A_{n+1}} \\
\vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\
\frac{\partial f_{2n+1}}{\partial v_1} & \frac{\partial f_{2n+1}}{\partial A_1} & \frac{\partial f_{2n+1}}{\partial v_2} & \frac{\partial f_{2n+1}}{\partial A_2} & \cdots & \frac{\partial f_{2n+1}}{\partial V_{n+1}} & \frac{\partial f_{2n+1}}{\partial A_{n+1}} \\
\frac{\partial f_{2n+2}}{\partial v_1} & \frac{\partial f_{2n+2}}{\partial A_1} & \frac{\partial f_{2n+2}}{\partial v_2} & \frac{\partial f_{2n+2}}{\partial A_2} & \cdots & \frac{\partial f_{2n+2}}{\partial V_{n+1}} & \frac{\partial f_{2n+2}}{\partial A_{n+1}}
\end{pmatrix} \tag{18}
$$

9. Results and discussion

To compare the results, Table 2 represents the maximum flood flow characteristics in 7 cross-sections and Table 3 represents the flood routing results at Km +70 upstream along the reach. Also, the graphs 3, 4, 5 and 6 compare the routed flood hydrographs, velocity variations, depth variations, discharge- depth curve by different numerical schemes. It may be noted that in data analysis by Mc Cormack explicit model, Preissmann implicit scheme and Mike 11 program, time step $\Delta t = 90s$ and spatial step $\Delta x == 1000m$ have been used for a rectangular grid of $x-t$ plane. Courant number values calculated varied between 0.43 and 0.98. In general, the results obtained are as follows:

1. Mc Cormack numerical model becomes unsteady for the large time step, but in Preissmann implicit model there is no limit on the choice of time step without reducing accuracy.

2. According to Table 2, hydrographs routed by Preissmann implicit scheme at all sections have the highest peak discharge. The highest difference is observed in the descending parts of hydrographs.

3. Figures (4) and (5) show that the flood routed by McCormack explicit method at +70 km has a lower discharge and peak velocity but its maximum depth is higher. An important point is that the peak times projected by both finite difference schemes are equal.

4. Hydrograph at the peak time to a distance of +45 km upstream is higher in Preissmann model than in Mc Cormack model. But in the next sections to +75 km the opposite is true.

5. The results show that an increase in the peak discharge time step slightly reduces the routed hydrograph.

6. The observations show that the flood routing results of Preissmann implicit scheme are close to those of predicted by Mike 11 which show the higher accuracy of the Preissmann scheme.

7. Results of discharge-depth at 70 km upstream along the reach highlighted by the three numerical models are presented in Figure (6). Although the predicted values of flow depth resulting from Preissmann model are less than the Mc Cormack model, however they are more close to the Mike 11 results.

Table 2. Evaluated characteristics of flood flows in the studied area

Distance Upstream (Km)	Preissmann implicit scheme				Mc Cormak diffusive			
	Q_p (m^3/s)	t_p (hr)	V_p (m/s)	y_p (m)	Q_p (m^3/s)	t_p (hr)	V_p (m/s)	y_p (m)
+12.50	59.756	129.83	0.707	3.058	59.752	129.75	0.713	3.044
+25.00	59.694	132.66	1.146	2.588	59.690	132.58	1.146	2.589
+32.50	59.686	133.91	1.002	2.078	59.680	133.83	1.001	2.077
+45.00	59.661	137.25	0.554	1.007	59.644	137.16	0.554	1.006
+57.50	59.586	141.42	0.528	1.114	59.476	141.50	0.475	1.176
+70.00	57.399	150.33	0.229	1.974	57.005	150.33	0.222	2.007
+78.00	57.078	155.16	0.355	1.303	56.658	155.083	0.354	1.30

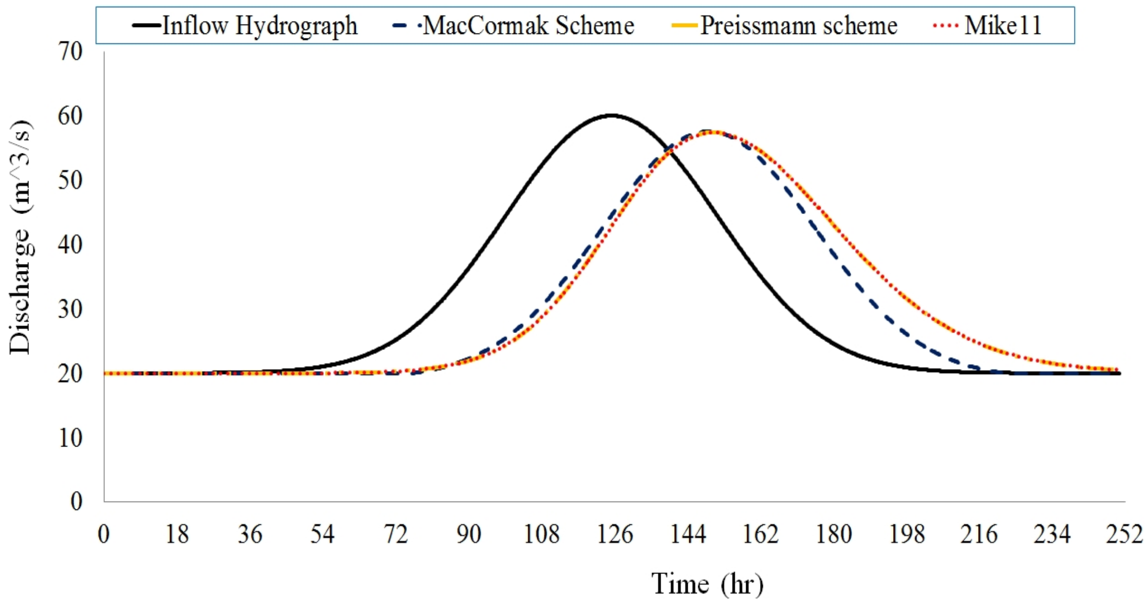

Figure 3. Flood hydrograph at +70 km upstream

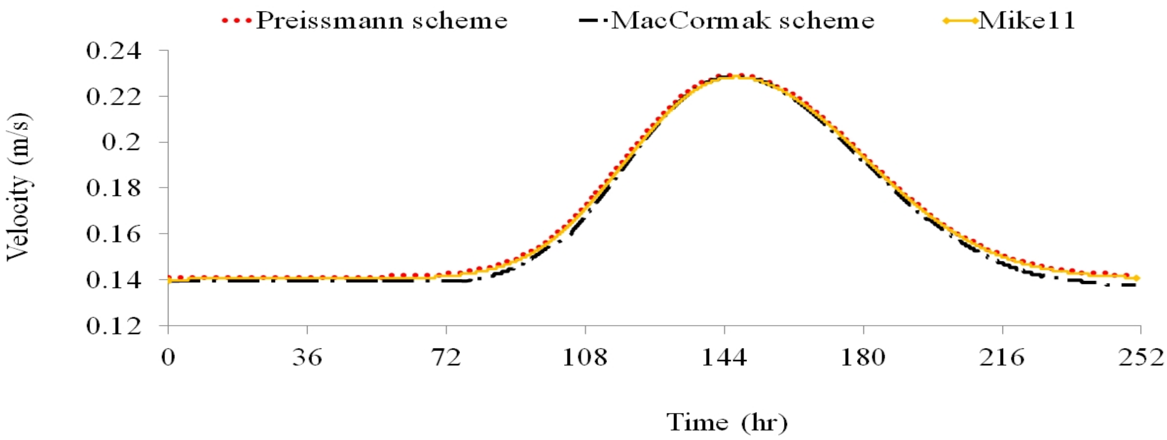

Figure 4. Velocity variations with respect to time at + 70 km upstream

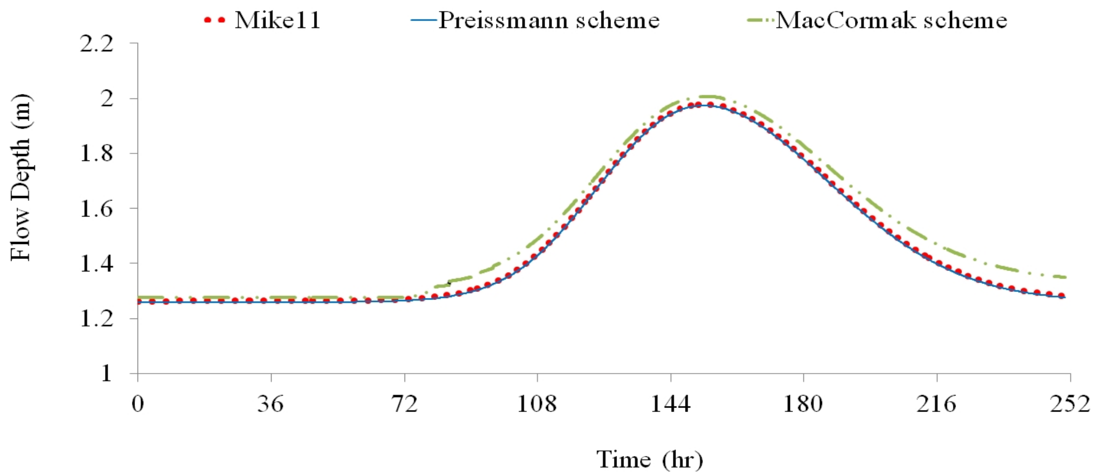

Figure 5. Flow depth variation with respect to time at section + 70 km upstream

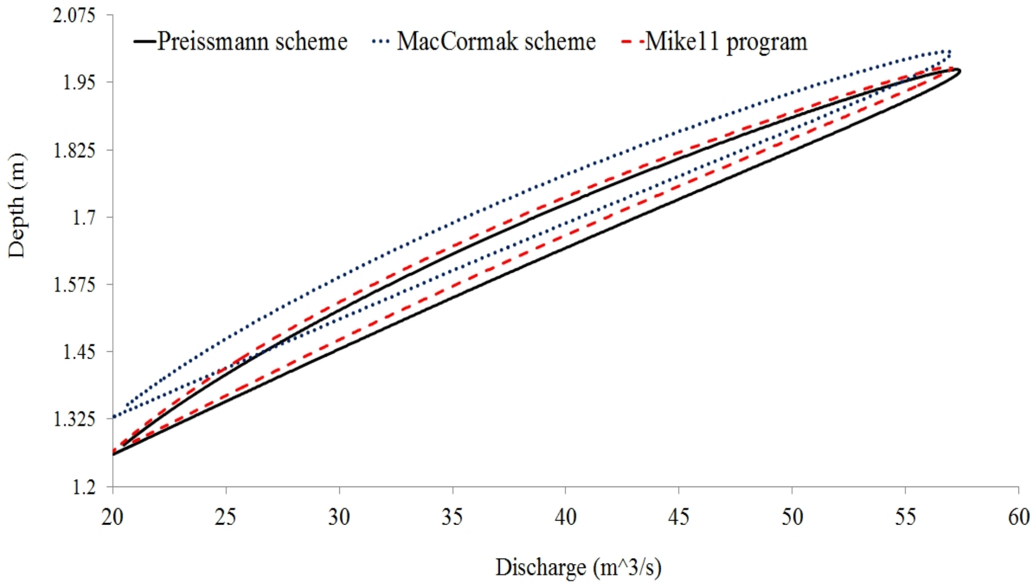

Figure 6. Discharge-Depth Curve at cross-section +70 km upstream

Table 3. Flood routing results by different numerical schemes at +70 km upstream

			70 Km from upstream end								
	Upstream-end		Preissmann			Mc Cormak			MIKE11		
Time (hour)	Q m^3/s)	Y (m)	Q (m^3/s)	Y (m)	V (m/s)	Q (m^3/s)	Y (m)	V (m)	Q (m^3/s)	Y (m)	V m/s
0	20	1.362	20	1.26	0.141	20	1.325	0.144	20	1.262	0.140
24	20.025	1.363	20.003	1.26	0.141	20	1.326	0.13	20	1.264	0.141
48	20.54	1.376	20.012	1.26	0.141	20	1.327	0.131	20.01	1.265	0.141
60	21.86	1.408	20.064	1.261	0.142	20.036	1.329	0.131	20.06	1.266	0.141
72	25.19	1.488	20.287	1.265	0.143	20.288	1.334	0.132	20.29	1.27	0.142
78	28.01	1.553	20.565	1.271	0.144	20.581	1.339	0.133	20.57	1.276	0.143
84	31.74	1.636	21.062	1.28	0.146	21.098	1.349	0.135	21.08	1.286	0.145
90	36.34	1.735	21.919	1.297	0.149	21.979	1.365	0.139	21.94	1.303	0.149
96	41.6	1.844	23.324	1.325	0.155	23.412	1.391	0.145	23.34	1.331	0.154
102	47.09	1.954	25.514	1.368	0.162	25.639	1.432	0.153	25.53	1.374	0.162
108	52.28	2.05	28.687	1.43	0.172	28.836	1.49	0.163	28.68	1.435	0.172
114	56.51	2.116	32.911	1.511	0.184	33.02	1.565	0.176	32.871	1.516	0.183
120	59.211	2.155	37.981	1.606	0.197	38.044	1.655	0.189	37.9	1.61	0.196
126	59.98	2.168	43.544	1.707	0.208	43.517	1.75	0.201	43.45	1.71	0.207
132	58.71	2.154	48.953	1.804	0.218	48.816	1.843	0.211	48.87	1.809	0.217
138	55.58	2.116	53.455	1.886	0.226	53.206	1.92	0.219	53.38	1.891	0.225
144	51.06	2.05	56.364	1.943	0.229	56.008	1.975	0.222	56.3	1.948	0.228
150	45.74	1.967	57.395	1.97	0.229	57.002	2.003	0.222	57.35	1.976	0.228
156	40.25	1.85	56.671	1.97	0.226	56.282	2.005	0.219	56.64	1.97	0.225
162	35.134	1.741	54.514	1.947	0.22	54.151	1.984	0.213	54.5	1.952	0.219
168	30.736	1.638	51.271	1.903	0.213	50.953	1.944	0.205	51.271	1.909	0.212
174	27.231	1.553	47.351	1.845	0.204	47.11	1.89	0.196	47.364	1.851	0.203
180	24.624	1.487	43.107	1.778	0.194	42.944	1.827	0.186	43.132	1.784	0.194
192	21.619	1.407	35.025	1.632	0.176	35.043	1.69	0.167	35.047	1.638	0.175
204	20.461	1.375	28.764	1.499	0.161	28.918	1.565	0.152	28.799	1.506	0.16
216	20.107	1.365	24.619	1.398	0.151	24.816	1.469	0.142	24.66	1.404	0.15
228	20.07	1.363	22.227	1.332	0.145	22.42	1.406	0.136	22.257	1.338	0.145
252	20.04	1.362	20.479	1.277	0.142	20.595	1.329	0.132	20.49	1.282	0.141

10. Conclusions

According to the results obtained, Preissmann scheme has the closest results to those of Mike 11 program and has an obvious advantage over other finite difference numerical method solutions. This method can be a reliable tool to help researchers in modeling the river flow variations. Although little differences have been found by Mc Cormack explicit model compared to Preissmann implicit model, the first showed high sensitivity to flood hydrograph characteristics in the upstream which is considered a weakness for this method. Also because Mc Cormack numerical model is a two-stage model for solving unsteady flows equations, its stability depends on choosing small time steps, which increased the time required to perform the calculations. However, based on stability, accuracy and low error in Preissmann implicit scheme, its application in river routing operations and water works related to natural waterways is strongly recommended.

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