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Assessment of the conventional control algorithms and proposing a modified displacement feedback control for performance-based design of structures

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Abstract

An enhanced displacement feedback structural control procedure is presented for performance-based design in this paper. At first, a comparative study is implemented assessing three most common active control algorithms including state, acceleration, and displacement feedback controls. The advantage of the displacement feedback algorithm for active control of structures against earthquakes is demonstrated through a number of representative examples. Second, the conventional displacement feedback control is modified to keep the lateral displacements under a preset value and to apply the control forces only when needed not continuously, hence to minimize the input energy. The lateral displacement at a certain level, e.g., the roof, is used as an on/off switch for the applied control forces. When the target displacement is going to be exceeded, the control forces are applied to reduce the displacements. As a result, it is shown that use of the proposed algorithm considerably reduces the energy demand of the control system compared to other procedures. Moreover, response of structure is controlled to any desired limit.

Keywords: Active control, State, Acceleration, Displacement, Feedback, Target.

1. Introduction

Advancement of science and technology in recent decades has also resulted in developing novel methods for seismic resistant systems in structures, out of which the active control is among the leading approaches. The most important applications of an active seismic control are: reduction of the response level of structure, strengthening of existing structures against earthquakes, retaining sensitive and expensive components in buildings when their post-earthquake serviceability is of prime importance, resilience of the system and the ability to conform to the nature of the earthquake loading when preparing for reaction. According to Soong [1], the first attempts for using active control as a method for lowering structural

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response dates back to early 60's when Freyssinet suggested use of prestressed tendons for stabilization of tall buildings. At the same time, Minai & Kobori [2] introduced the concept of dynamical smart structures being able to actively control the response in severe earthquakes. Zuk [3,4] and Zuk and Clark [5] introduced some basic actively controlled structures separating control systems designed for reduction of motion from motion-inducing systems. Nordell [6] proposed to include some reserved resistant members in structure to be activated in the event of an emergency loading to increase the structural strength. Yao [7] presented a comprehensive theory for the structural control.

In the years afterwards, parallel to the advent and development of digital computers and sensors, several control methods were proposed, including the optimum control method (LQR) [8,9,10], the sliding mode control approach (SMC) [11], and the predictive controlling procedure [12]. A comprehensive review on different active control methodologies can be found elsewhere [13,14]. A prime concern and disadvantage of the active seismic control of structures has been the severe energy demand of the control apparatus in a large earthquake. Therefore, optimizing the control algorithm has been a focus of research in recent years. The fundamental configuration of an active control system consists of a number of sensors installed at floor levels of a building to record response acceleration, velocity, and/or displacement at short time steps. These data are analyzed within a certain control algorithm to determine the control forces to be applied to structure immediately. Conversion of response to control force is generally implemented using the control efficacy matrix. In linear optimization, the control force vector is computed such that a performance index is minimized. This index is representative of the energy required for the operation of the control system and it is clearly preferred to be a minimum [13,14].

For structural applications, the linear optimized control is categorized to three different algorithms including the state, acceleration, and displacement feedback controls. In the state feedback control use is made of the velocity and displacement vectors of the structure’s DOF’s. Then, the control forces are calculated as the efficacy matrix multiplied by the state vector. While the state feedback control theoretically is the fundamental control algorithm, it requires measuring velocity and displacement at all degrees of freedom which is impractical. This major drawback of this method was the reason for development of other algorithms including the acceleration feedback and the displacement feedback controls [13,14].

At the same time, emerge of the performance and displacement based design of structures leads one to the idea that by modifying the displacement feedback algorithms and the corresponding control forces, it may be possible to retain the response within the limits ascertained by the current performance-based instructions for different performance levels. The above idea is followed and a performance-based displacement feedback algorithm is developed based on a target displacement. The control forces are to be applied only when the target displacement is to be exceeded, hence minimizing the required energy.

In the next sections first it is shown that the displacement feedback control (DFC) is the most efficient algorithm within the conventional control methods. Then, the new modified displacement feedback control (MDFC) algorithm is developed and implemented based on a selected target displacement and instantaneous application of control forces. The structural response is to be controlled to any desired limit.

2. Comparison of the conventional active control algorithms

2.1. The state feedback control (SFC)

The dynamic equilibrium equation for an MDOF structure under active control can be written as equation (1):

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = Du(t) + Ef(t) \tag{1}$$

In which M, C and K are the square mass, damping and stiffness matrices of the dimension n; x, u and f are the n×1 vector of structural displacements, m×1 vector of control forces and r×1 vector of structural loadings, respectively; and D and E are respectively the n×m and n×r position matrices of the control forces (CF's) and the structural loadings. Since solving the control equation (1) is easier in the state space, a 2n×1 collective vector of displacements and velocities is introduced as equation (2):

$$Z(t) = \begin{cases} x(t) \\ \dot{x}(t) \end{cases} \tag{2}$$

Replacing equation (2) in equation (1), results in equation (3):

$$\dot{Z}(t) = AZ(t) + Bu(t) + Hf(t) \tag{3}$$

The coefficient matrices of equation (3) are defined as:

$$A = \begin{bmatrix} 0_{n\times n} & I_{n\times n} \\ -(M^{-1}K)_{n\times n} & -(M^{-1}C)_{n\times n} \end{bmatrix}_{2n\times 2n} \tag{4}$$

$$B = \begin{bmatrix} 0_{n\times m} \\ (M^{-1}D)_{n\times m} \end{bmatrix}_{2n\times m} \tag{5}$$

$$H = \begin{bmatrix} 0_{n\times r} \\ (M^{-1}E)_{n\times r} \end{bmatrix}_{2n\times r} \tag{6}$$

In the above equations, I is the n×n unit matrix and 0 is a rectangular null matrix.
As of the optimized linear control theory, the target is finding a control vector u(t) minimizing the performance index J defined as follows:

$$J = \int_0^\infty J_1(t)dt \tag{6}$$

where:

$$J_1(t) = 0.5[Z^T(t)QZ(t) + u^T(t)Ru(t)] \tag{7}$$

in which Q is a 2n×2n weighting positive semi-definite matrix and R is an m×m weighting positive definite matrix calculated by trial and error.
In the linear control theory, the Hamiltonian of a dynamical system is defined by equation (8) [3]:

$$H = J_1(t) + \lambda^T \dot{Z}(t) \tag{8}$$

Substituting equations (3) and (7) in equation (8) in the absence of structural loadings results in equation (9):

$$H = 0.5(Z^T QZ + u^T Ru) + \lambda^T (AZ + Bu) \tag{9}$$

in which λ is a coefficient matrix defined as:

$$\dot{\lambda} = -\frac{\partial H}{\partial Z} = -QZ - A^T \lambda \tag{10}$$

But:

$$\frac{\partial H}{\partial Z} = Ru + B^T \lambda = 0 \tag{11}$$

Therefore:

$$u = -R^{-1} B^T \lambda \tag{12}$$

Equation (12) shows that the CF's are direct functions of λ; but calculating λ is not an easy task because the boundary condition λ(∞)=0 refers to the steady state response while value of Z is known only at t=0. Such problems are known as 2-point boundary value problems (TPBVP). To overcome the above difficulty, it is common to define λ as a multiple of the state vector as:

$$\lambda = S.Z(t) \tag{13}$$

Substituting equation (13) in equation (12) results in:

$$u(t) = -G.Z(t) \tag{14}$$

$$G = R^{-1} B^T S \tag{15}$$

S is a positive definite matrix known as the Riccati matrix and G is the control efficacy matrix relating the CF's to the measured system responses. Replacing equation (14) in equation (9), a Riccati equation is derived as:

$$SA + A^T S - SBR^{-1} B^T S + Q = 0 \tag{16}$$

There are quite a few methods for solving the Riccati equation (16) for S. Then the CF's are calculated using equations (14) and (15).

2.2. The acceleration feedback control (AFC)

The main goal in AFC is controlling the response only using the acceleration response values. Chung et al. [15] provided a solution for this problem which is summarized here. The main idea in this solution is again minimizing the performance index in the framework of the optimized control algorithm.

It is assumed that the output vector y(t) is a linear function of the state space vector, as:

$$\begin{aligned} y(t) &= DZ(t) \\ D &= \begin{bmatrix} -M^{-1}K & -M^{-1}C \end{bmatrix} \end{aligned} \tag{17}$$

in which D is a P×2n coefficient matrix where P is the number of acceleration sensors and y(t) is the vector of measured acceleration responses. Considering the definition of $y(t)$, the following simple change of equivalent notation is applied to equation (17):

$$\ddot{x}_a(t) = DZ(t) \tag{18}$$

in which $\ddot{x}_a(t)$ is the absolute acceleration response.

Next, the CF's are calculated with multiplying the response vector by the efficacy matrix *G* as:

$$u(t) = G.\ddot{x}_a(t) \tag{19}$$

Chung et al. [16] derived the following equations for calculating the efficacy matrix:

$$(A + BGD)^T H + H(A + BGD) + (Q + D^T G^T RGD) = 0 \tag{20}$$

$$Z_0 + L(A+BGD)^T + (A+BGD)L = 0 \tag{21}$$

$$B^T HLD^T + RGDLD^T = 0 \tag{22}$$

in which L is the Lagrangian coefficient matrix, $Z_0 = E[z_0 z_0^T]$, and z_0 is the initial value of the vector *z(t)*. An initial guess for G solves equations (20) and (21) for L and H matrices. These matrices are then substituted in equation (22) to derive a modified G. This process is repeated until convergence. Then using the final G matrix; the CF's are calculated by equation (18).

2.3. The displacement feedback control (DFC)

While in SFC knowing the state vector was a requirement for the optimized control, in DFC having the system displacements and a dynamic "observer" are enough for computing the CF's. The dynamic "observer" is one of the common algorithms in the modern control theory and analysis. On this basis, a mathematical model is developed in which only the measured displacements are used to prepare the dynamic system. Therefore, the system output vector can be shown as:

$$y(t) = C_0 Z(t) \tag{23}$$

The matrix C_0 defines the accessible entries of the state vector. Based on the control algorithm, the dynamic observer equation can be written as:

$$\dot{\hat{Z}}(t) = A\hat{Z}(t) + Bu(t) + L[y(t) - \hat{y}(t)] + Hf(t) \tag{24}$$

$$\hat{y}(t) = C_0 \hat{Z}(t) \tag{25}$$

in which L is the efficacy observer matrix to be computed, and a hat sign shows the output of the system. The first and second terms on the right of equation (24) correspond to the estimated system and the third term corrects the mathematical model of the system using a linear feedback of the difference between the measured output $y(t)$ and the calculated output $\hat{y}(t)$. Both of the later values are known and accessible.

Now, equation (24) is deduced from equation (3) resulting in:

$$\dot{Z}(t) - \dot{\hat{Z}}(t) = AZ(t) - A\hat{Z}(t) - L[y(t) - \hat{y}(t)] = (A - LC)(Z - \hat{Z}) \tag{26}$$

If the difference between the observer and the main systems is shown as the error function $er(t) = Z(t) - \dot{Z}(t)\hat{Z}(t)$ and is substituted in equation (25), the following linear first order equation emerges:

$$\begin{aligned} \dot{e}r &= (A - LC)er(t) \\ er(t) &= e^{(A-LC)t} er(0) \end{aligned} \tag{27}$$

It is seen that the error function is time dependent, thus it must tend to zero in the steady state response phase for a stable system. For this purpose, the stability condition of the closed loop observer can be considered as follows:

$$\lambda_i (A - LC) < 0 \tag{28}$$

in which λ_i are the eigenvalues of the $A-LC$ matrix. Since both matrices A and C are definite, it suffices to determine the efficacy matrix L. L should be selected such that the closed loop $A-LC$ system remains stable. Considering the property that the eigenvalues of a matrix and its transposed counterpart are identical, then:

$$\lambda_i(A-LC)=\lambda_i(A^T-C^TL^T) \tag{29}$$

Now efficacy of the observer can be computed using the closed loop $A^T-C^TL^T$. Then after calculating $Z(t)\hat{Z}(t)$ by equation (24) and substituting in equation (25), the CF's are calculated as:

$$u(t)=-G.\hat{Z}(t) \tag{30}$$

3. Numerical comparison of the conventional algorithms

3.1. Properties of the structural models

To assess and compare the mentioned control algorithms, three structures having 4, 8 and 12 stories are used in order to cover a wide range of periods. The structures are similar in plan having 8 frames in each of the perpendicular directions. The story height is 3 m. The structural system consists of special moment frames, located in a very high seismicity region on very firm soil. The buildings are residential and are regular both in plan and elevation. Table 1 shows the dynamic characteristics of the structures.

Table 1. Dynamic characteristics of the structures studied.

Structure		Stiffness (kN/m)	Story mass (ton)	First mode damping ratio (%)	Fundamental period (sec)
4-story	whole stories	680,000	600	2	0.537
8-story	lower 4 stories	871,000	600	2	0.920
	upper 4 stories	680,000	600		
12-story	lower 4 stories	1,463,000	600	2	1.167
	intermediate 4 stories	871,000	600		
	upper 4 stories	680,000	600		

3.2. Numerical results

The structural models described in the previous section are analyzed under the Elcentro (Imperial Valley station, 1940), Tabas (Tabas station, 1978), and Manjil (Abbar station, 1990) earthquakes, all scaled to a PGA of 0.35g. The response is controlled using the three methods explained earlier. The results are compared with those of the uncontrolled linear structures. It is assumed that the CF is applied only to the roof. The results are shown, as examples of similar trends, for the 12-story structure under Tabas (1978) in Figures 1-4. Also, the distributions of the responses along height, averaged between the earthquakes, are given for all of the buildings studied in Figures 5 and 6.

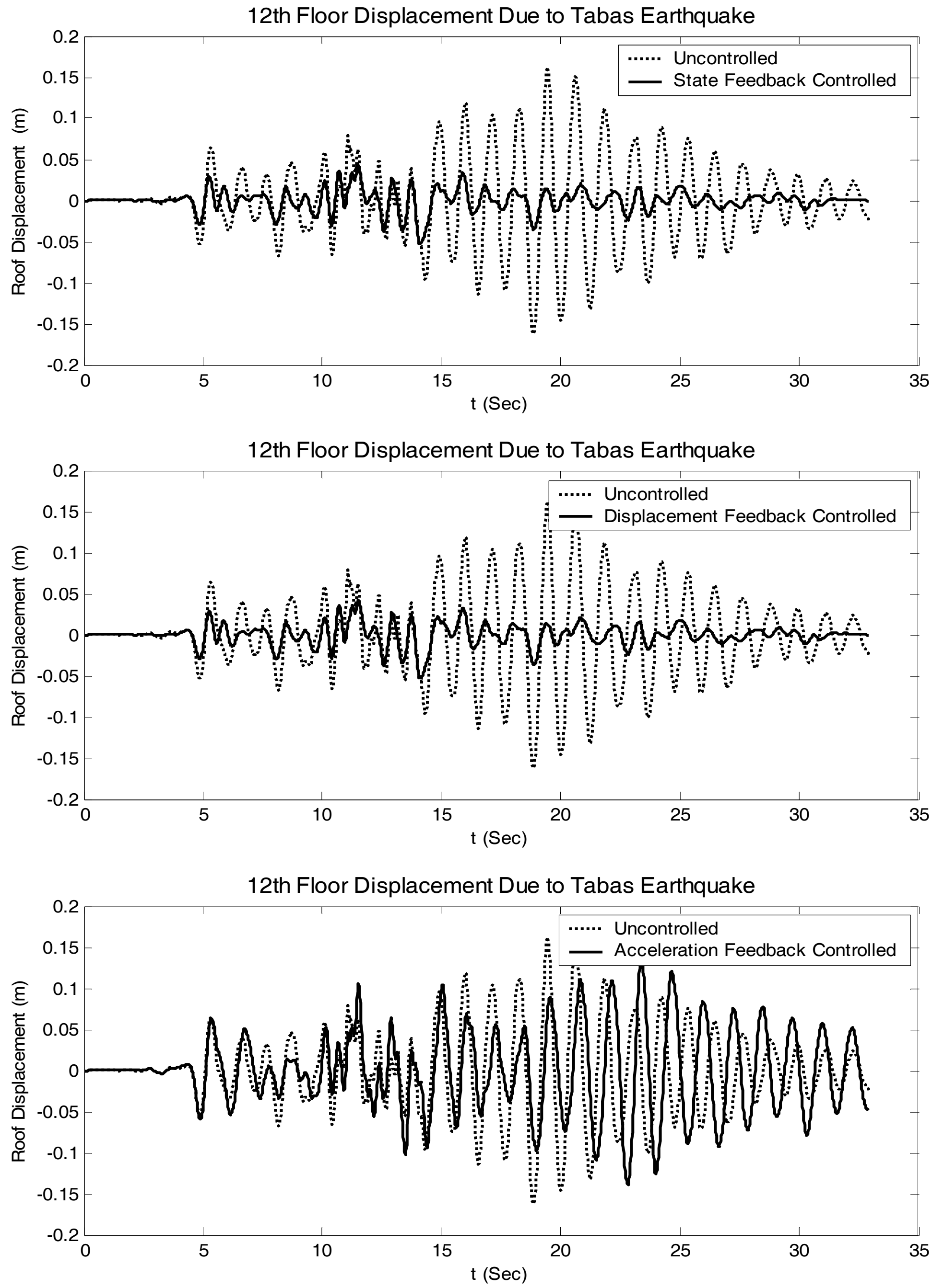


Figure 1. The displacement time history of the roof of the 12-story building under Tabas (1978)

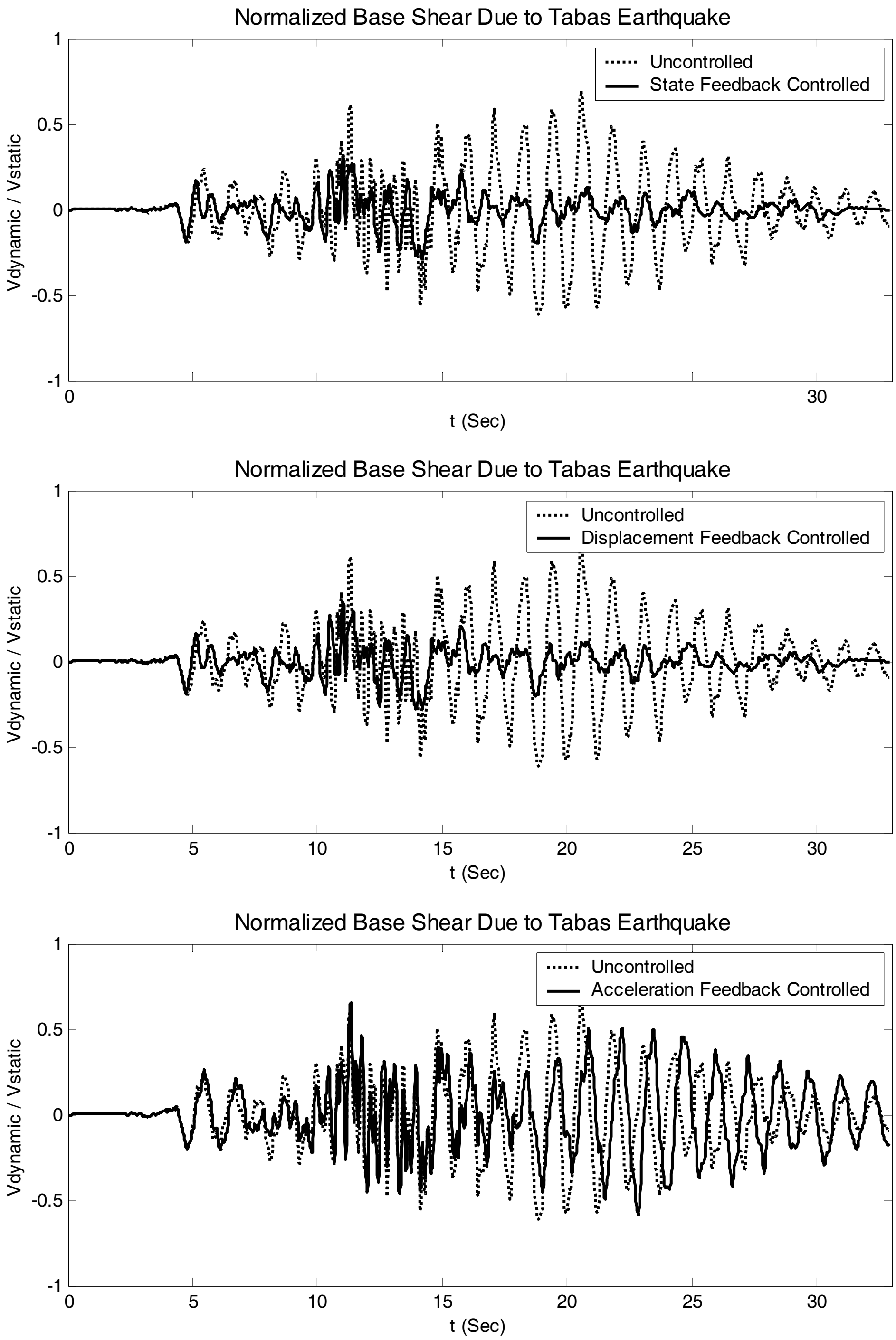


Figure 2. The normalized base shear time history, 12-story, Tabas (1978)

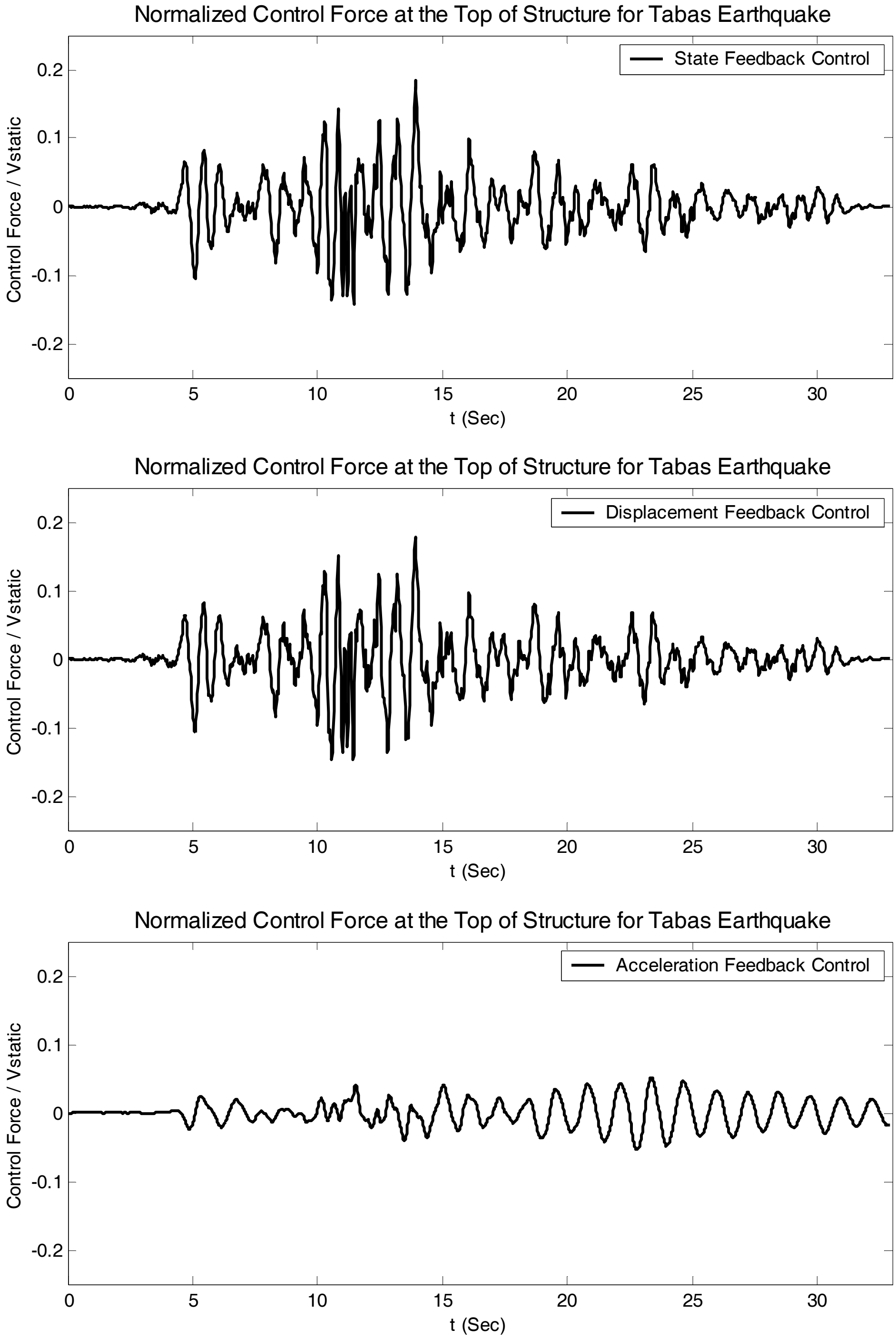


Figure 3. The control force time history (at roof), 12-story, Tabas (1978).

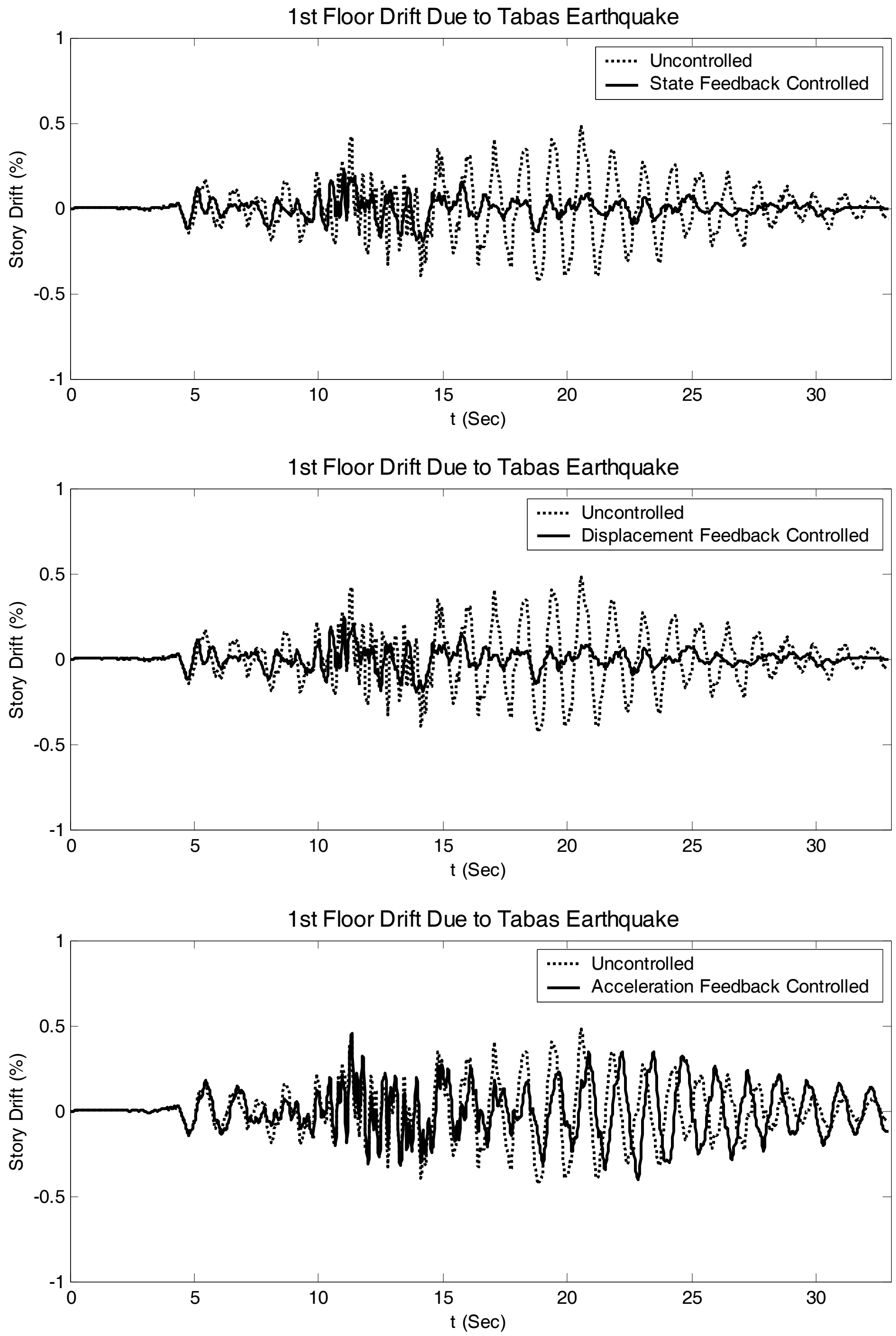


Figure 4. The first story drift time history, 12-story, Tabas (1978).

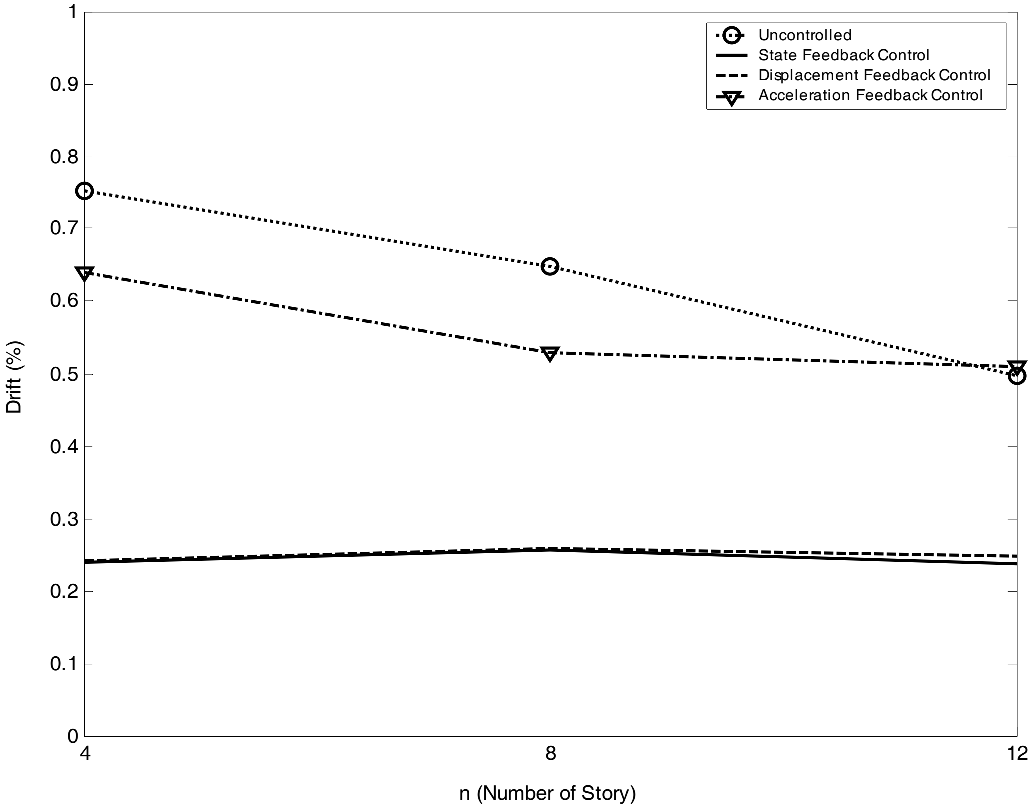


Figure 5. Distribution of mean story drift along the height of 4, 8, and 12-story buildings, Tabas (1978).

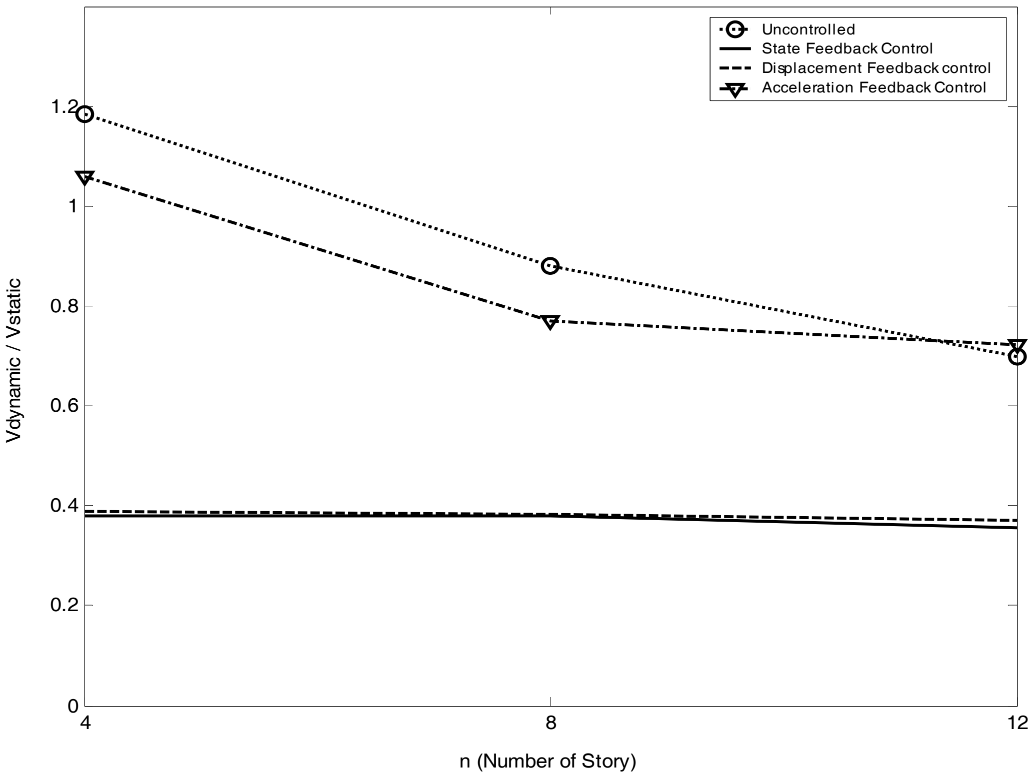


Figure 6. Distribution of mean normalized base shear for 4, 8, and 12-story buildings.

Studying the results demonstrated in the above figures, it is observed that the DFC has been as successful as SFC in controlling and lowering the structural response resulting in almost the same response values. These two algorithms both have reduced the displacements more than 50% in most cases. On the other hand, the AFC has not been successful in controlling the response as much, resulting in responses quite similar to the uncontrolled structure.

The reason can be attributed to much smaller variations of the displacement response compared to the acceleration response where due to severe fluctuations of the acceleration time history, determination of the appropriate direction and value of the CF at each time step cannot be appropriately accomplished. This is the case for the other response values too, e.g., the story drifts and the base shear.

Similarity of the responses provided by the SFC and DFC shows a very good estimation of the system's state in the DFC algorithm with the dynamic observer system. Therefore, noting that in a response control with the DFC algorithm only measurement of the structural displacements is required, the DFC can efficiently replace the SFC where in the later both of displacement and velocity responses must be at hand.

In the next section, the DFC that was proved as a most successful algorithm for structural response control in seismic events is further enhanced to be suitable for a performance-based design procedure.

4. The modified displacement feedback control (MDFC) for performance-based design

4.1. The proposed algorithm

The performance-based, especially the displacement based design, has been the prevalent tendency in earthquake engineering in recent years. The main philosophy in the displacement-based design is on the basis of a design beginning with deformations limited to certain values determined considering function and importance of structure. This is essential because it is now well known that seismic damage is a direct function of relative displacements. Therefore, paying attention to controlling displacements from the very first steps of design is imperative. The control algorithms studied in the previous sections do not consider displacement directly as the target of the control process. Actually, they replace matrix A in equation (4) with A+BG to change the stiffness and damping of structure. This requires calculation and application of the control force in all time steps to the system. Following such an algorithm, it is seen that there is not an explicit control on the values of the resulting displacements. In fact, the control forces act such that in SFC and DFC algorithms automatically a considerable decrease in displacement response is attained. It is not mandatory to set a certain level for the amount of response reduction in these algorithms. Moreover, and perhaps more important, is the fact that the control force must be applied continuously in time with large variations; see Figure 3.

The above discussion leads to the point that if the conventional control algorithms, especially DFC, are modified such that the displacement responses can be directly controllable to remain under certain limits, then the desired performance is maintained and the required control energy is minimized. This is clearly important when it is noticed that use of standby power sources like emergency generators and large batteries is inevitable for safe operation of control systems. On the other hand, at many moments structure's response is not as large as needing to be controlled. The control procedure can be idle in such times.

Now, it is decided not to apply the control force when the displacement is still under a predefined value, called the target displacement. This is implemented in this research in the framework of the displacement feedback control, as an algorithm adapted to the performance-based design. It is called the modified displacement feedback control, (MDFC) having an algorithm shown in Figure 7.

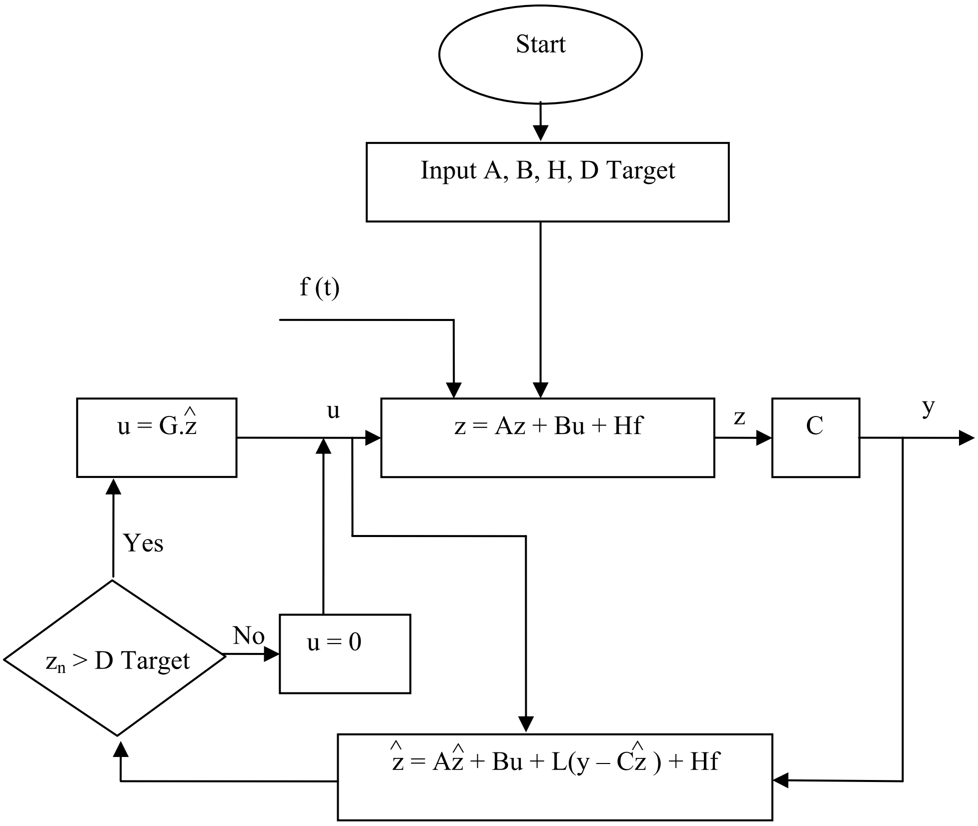
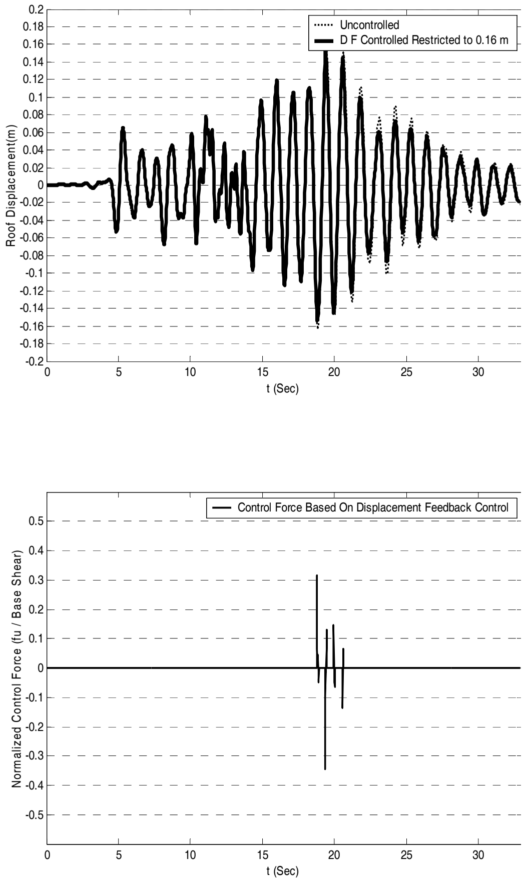


Figure 7. Algorithm for the modified displacement feedback control (MDFC).

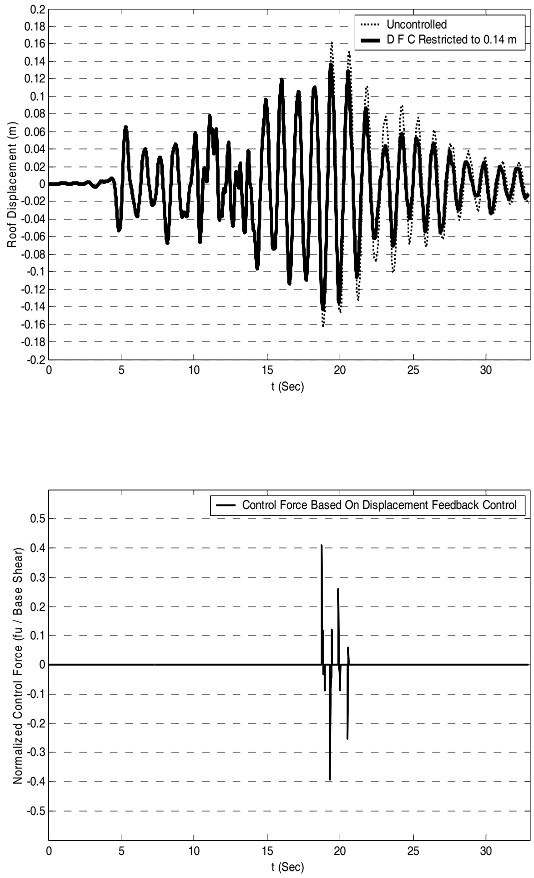
In the above algorithm first the structural characteristics are provided along with a target (maximum) displacement for the roof considering the function and importance of the building. At each time, the sensors record and relay the roof displacement to the control system enabling the observer system (lower part of the algorithm) to estimate the state vector. Calculation of the control force is disregarded until displacement of the roof exceeds the target value, when the control force is determined and applied to the roof. Since the control force is applied one time step later, to be on the safe side a reduction factor is considered for the target displacement and a decreased target displacement is used in the algorithm. The appropriate reduction factor was appeared to be 0.9 in the cases studied.

4.2. Numerical study

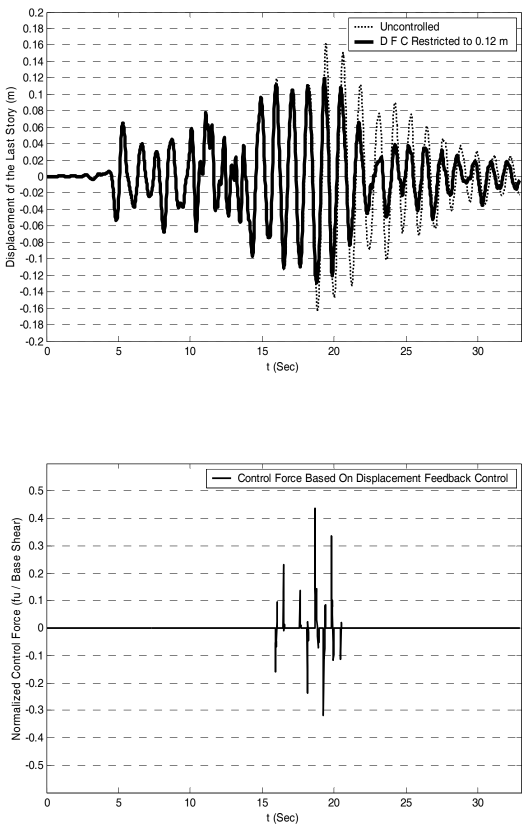
The same buildings introduced previously are studied again under the same earthquakes using the proposed MDFC algorithm (Figure 7). Because of similarity of the trends, only those of the 12-story structure under Tabas (1978) are presented in the following. As is seen in Figure 1, the maximum displacement of the roof of this building in the uncontrolled case under Tabas (1978) is about 17cm. Considering this value, several cases for the target displacement of the roof are evaluated selecting this quantity to be 16, 14, 12, 10, 8 and 6 cm. The controlled response of the structure is calculated for different target displacements. The roof displacement and control force time histories are shown in Figure 8. Also, Figure 9 shows the required input energy for the control system in each case.



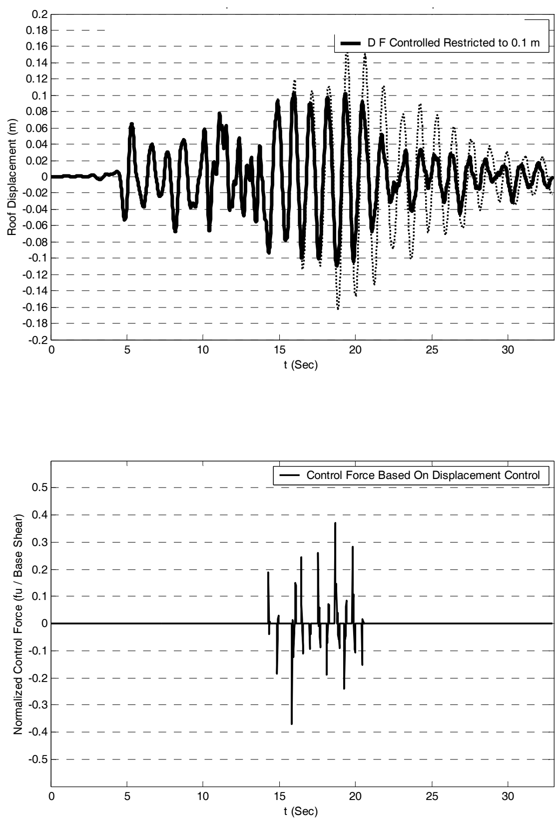
(a) TD=16 cm



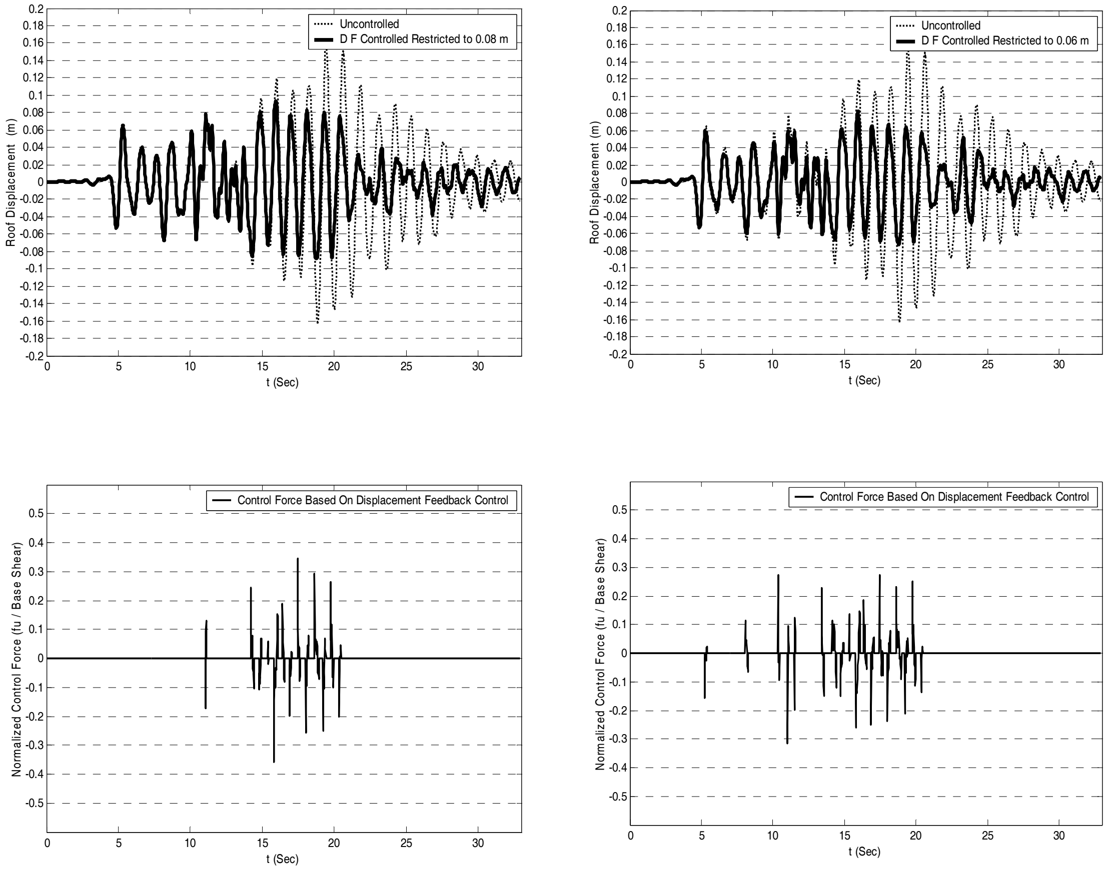
(b) TD=14 cm



(c) TD=12 cm



(d) TD=10 cm



(e) TD=8 cm (f) TD=6 cm

Figure 8. The roof displacement and control force for DFC, Tabas (1978)

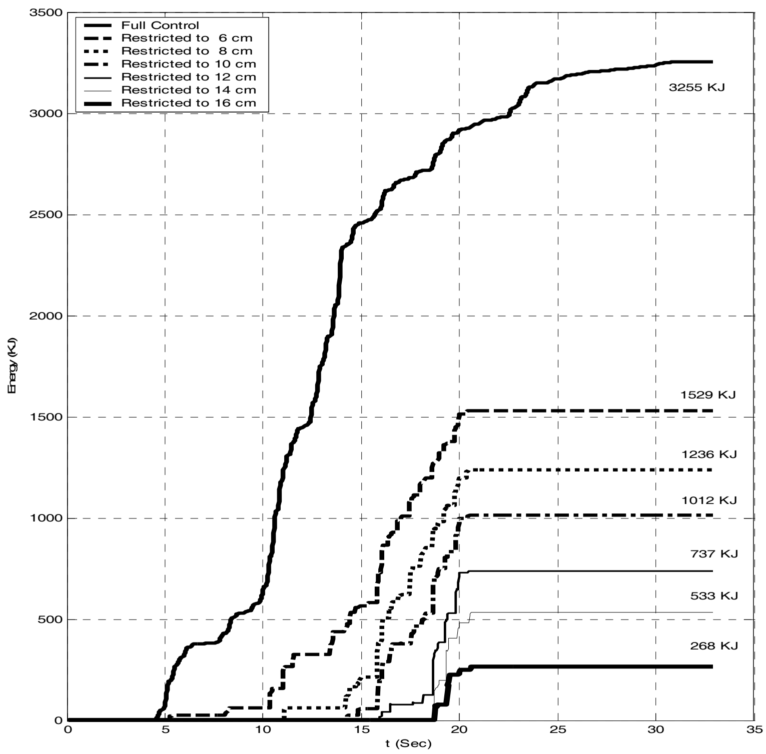


Figure 9. Time history of the required input energy of the control system for various target displacements, 12-story building, Tabas (1978).

As shown in Figure 8, the MDFC algorithm has been practically successful in limiting the roof displacement to the target values. Also, comparing Figure 8 with Figure 3, instantaneous application of the control force only when needed at certain moments in MDFC against its continuous application in the conventional DFC, has resulted in a drastic reduction of the total energy required for the active control system, even for a target displacement as small as 6 cm. Therefore, the MDFC control algorithm presented in Figure 7, has been successful in limiting the lateral displacements according to a performance-based design methodology and at the same time has needed a minimum energy compared to that of the conventional DFC.

5. Summary and conclusions

A performance-based modified displacement feedback control (MDFC) procedure was presented in this study. It was presented in the framework of the conventional displacement feedback control (DFC) adding the condition that the control force be applied if only the roof displacement exceeds a target maximum value. Therefore, in the proposed algorithm the response is controlled only when it is needed. The lateral displacement at any level can be used as the control criteria for the applied control forces.

Despite needing larger instantaneous control forces, it was shown that the MDFC procedure can considerably reduce the total energy required for the active control system. Moreover, the seismic response of structure can be contained to any desired limit based on existing energy resources.

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