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# Prediction of concrete mix ratios using modified regression theory

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**Abstract**

The strength of concrete is a function of the proportions of the constituent materials, namely, cement, water, fine and coarse aggregates. The conventional methods used to determine the mix proportions that will yield the desired strength, are laborious, time consuming and expensive. In this paper, a mathematical method based on modified regression theory is formulated for the prediction of concrete strength. The model can prescribe all the mixes that will produce a desired strength of concrete. It can also predict the strength of concrete if the mix proportions are specified. The adequacy of the mathematical model is tested using statistical tools.

***Keywords***: Concrete; Regression theory; Prediction; Mix ratios.

## 1. Introduction

Concrete is a composite construction material consisting of water, cement, fine aggregate (sand) and coarse aggregate. Mix design of concrete is a means of producing the most economical and durable concrete that meet with certain properties as consistency, strength and durability by properly and systematically combining the ingredients (concrete materials) of relative proportions [1]. The strength of concrete is very important because most of the desirable characteristic properties of concrete are qualitatively related to its compressive strength [2].

Various methods have been used to study and/or determine the strength of concrete [3-5]. All these methods are based on the conventional approach of selecting arbitrary mix proportions, subjecting concrete samples to laboratory and then adjusting the mix proportions in subsequent tests. Apparently, these methods are time consuming and expensive.

Studies on concrete mix design have been undertaken by some researchers using numerical method. Aggregate mix design with this method has been used with the aid of transformed Fuller's curve to calculate aggregate mixes for different types of concrete. The granulation number method of concrete mix design was also used to predict physical and mechanical properties of concrete [6].

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Various authors have used a standard multilayer feed forward artificial neural network to predict the compressive strength of concrete [7-10] where a back propagation algorithm is used to train the network existing datasets. The main advantage of using this method is their flexibility and ability to model nonlinear relationships. However, this method is limited because the models do not have the ability to incorporate additional expertise into the model [11].

Adaptive neuro-fuzzy inferencing system modelling for concrete strength estimation from concrete mix proportioning has been proposed by Tesfamariam [11]. It uses designer's intuitive experience as well as the numerical information included in the data sets. Sensitivity analysis is carried out to identify critical parameters that impact the concrete strength.

In this paper, a mathematical model based on modified regression theory, is formulated for the prediction of concrete mix ratios and strength.

## 2. Materials

The materials used in the production of the prototype concrete cubes are cement, fine aggregates, coarse aggregates and water. Eagle cement brand of Ordinary Portland cement with properties conforming to BS 12 was used in the preparation of the concrete cube specimens [12]. The fine aggregates were fine and medium graded river sand of zone 3 sourced from Otamiri River in Imo State .The coarse aggregates were crushed irregular shaped medium-graded coarse aggregates having a maximum size of 20mm and conforming to BS 882 [13]. They were free from clay lumps and organic materials. Potable water conforming to the specification of EN 1008, was used in the production of the prototype concrete cube specimens [14].

## 3. Methods

Two methods, namely analytical and experimental methods were used in this work.

### *3.1. Analytical methods*

Here, optimization method is used in formulating a mathematical model for predicting the modulus of rupture of concrete. The model is based on modified regression theory. A simplex lattice is described as a structural representation of lines joining the atoms of a mixture .The atoms are constituent components of the mixture. For a normal concrete mixture, the constituent elements are water, cement, fine and coarse aggregates. And so it gives a simplex of a mixture of four components. Hence the simplex lattice of this four-component mixture is a three-dimensional solid equilateral tetrahedron. Mixture components are subject to the constraint that the sum of all the components must be equal to one [15]. In order words:

$$X_1 + X_2 + X_3 + \ldots\ldots + X_q = 1 \tag{1}$$

$$\sum_{i}^{q} X_i = 1 \tag{2}$$

where $q$ is the number of components of a mixture and $X_i$ is the proportion of the $i^{\text{th}}$ component in the mixture. It is impossible to use the normal mix ratios such as 1:2:4 or 1:3:6 at a given water/cement ratio because of the requirement of the simplex that sum of the components must be one. Hence it is necessary to carry out a transformation from actual to pseudo components. The actual components represent the proportion of the ingredients while the pseudo components represent the proportion of the components of the $i^{\text{th}}$ component in the mixture i.e. $X_1$, $X_2$, $X_3$, $X_4$. Considering the four- component mixture tetrahedron simplex lattice, let the vertices of this tetrahedron (principal coordinates) be described by $A_1$, $A_2$, $A_3$, $A_4$.

The following arbitrary mix proportions which are based on past experiences and literature have been prescribed for the vertices of the tetrahedron and shown in Figure 1,

$A_1$ (0.55: 1: 2: 4)
$A_2$ (0.50: 1: 2.5: 6)
$A_3$ (0.45: 1: 3: 5.5)
$A_4$ (0.6: 1: 1.5: 3.5)
.

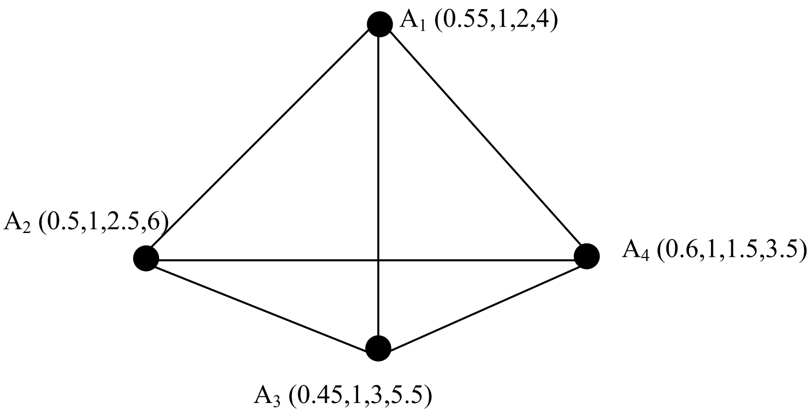


Figure 1. Vertices of a (4, 2) lattice (actual).

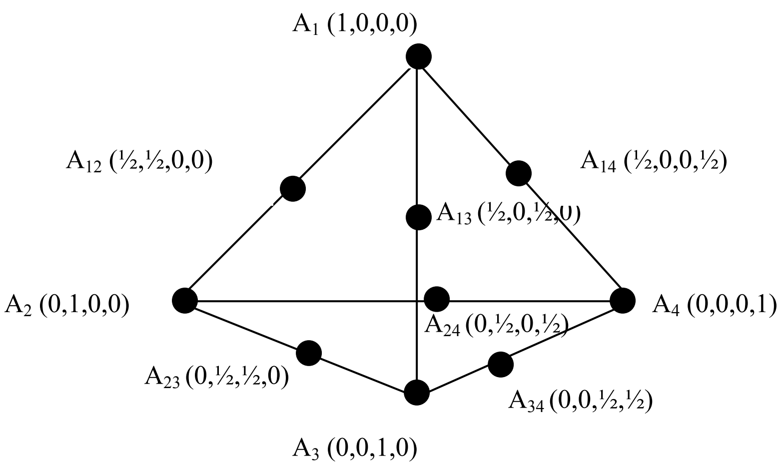


Figure 2. Vertices of a (4, 2) lattice (pseudo).

Let $X$ represent pseudo components and $Z$, actual components. For component transformation we use the following equations:

$$X = BZ \tag{3}$$

$$Z = AX \tag{4}$$

where $A$ is a matrix whose elements are from the arbitrary mix proportions chosen when equation (4) is opened and solved mathematically. $B$ is the inverse of matrix $A$ and $X$ is a matrix containing pseudo components. This is obtained from Figure 2.

Expanding and using equations (3) and (4), the actual components Z were determined and presented in Table 1 [15].

Table 1. Pseudo Components with their corresponding Actual Component Values.

| N | $X_1$ | $X_2$ | $X_3$ | $X_4$ | Response | $Z_1$ | $Z_2$ | $Z_3$ | $Z_4$ |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | $Y_1$ | 0.55 | 1 | 2 | 4 |
| 2 | 0 | 1 | 0 | 0 | $Y_2$ | 0.50 | 1 | 2.5 | 6 |
| 3 | 0 | 0 | 1 | 0 | $Y_3$ | 0.45 | 1 | 3 | 5.5 |
| 4 | 0 | 0 | 0 | 1 | $Y_4$ | 0.6 | 1 | 1.5 | 3.5 |
| 5 | 0.5 | 0.5 | 0 | 0 | $Y_{12}$ | 0.525 | 1 | 2.25 | 5 |
| 6 | 0.5 | 0 | 0.5 | 0 | $Y_{13}$ | 0.5 | 1 | 2.5 | 4.75 |
| 7 | 0.5 | 0 | 0 | 0.5 | $Y_{14}$ | 0.575 | 1 | 1.75 | 3.75 |
| 8 | 0 | 0.5 | 0.5 | 0 | $Y_{23}$ | 0.475 | 1 | 2.75 | 5.75 |
| 9 | 0 | 0.5 | 0 | 0.5 | $Y_{24}$ | 0.55 | 1 | 2 | 4.75 |
| 10 | 0 | 0 | 0.5 | 0.5 | $Y_{34}$ | 0.525 | 1 | 2.25 | 4.5 |
| Control points within the factor space | | | | | | | | | |
| 11 | 0.5 | 0.25 | 0.25 | 0 | $C_1$ | 0.5125 | 1 | 2.375 | 4.875 |
| 12 | 0.25 | 0.25 | 0.25 | 0.25 | $C_2$ | 0.525 | 1 | 2.25 | 4.75 |
| 13 | 0 | 0.25 | 0.25 | 0.5 | $C_3$ | 0.5375 | 1 | 2.125 | 4.625 |
| 14 | 0 | 0.25 | 0 | 0.75 | $C_4$ | 0.575 | 1 | 1.75 | 4.125 |
| 15 | 0.75 | 0 | 0.25 | 0 | $C_5$ | 0.525 | 1 | 2.25 | 4.375 |
| 16 | 0 | 0.5 | 0.25 | 0.25 | $C_6$ | 0.5125 | 1 | 2.375 | 5.25 |
| 17 | 0.25 | 0 | 0.5 | 0.25 | $C_7$ | 0.5125 | 1 | 2.375 | 4.625 |
| 18 | 0.75 | 0.25 | 0 | 0 | $C_8$ | 0.5375 | 1 | 2.125 | 4.5 |
| 19 | 0 | 0.75 | 0.25 | 0 | $C_9$ | 0.4875 | 1 | 2.625 | 5.875 |
| 20 | 0 | 0.4 | 0.4 | 0.2 | $C_{10}$ | 0.5 | 1 | 2.5 | 5.3 |
| Control points outside the factor space | | | | | | | | | |
| 21 | 0.5 | 0.5 | 0.5 | 0.5 | $C_{11}$ | 1.05 | 2 | 4.5 | 9.5 |
| 22 | 0.25 | 0 | 0.25 | 0 | $C_{12}$ | 0.35 | 0.5 | 1.375 | 2.875 |
| 23 | 0.5 | 0 | 0.5 | 0.5 | $C_{13}$ | 0.8 | 1.5 | 3.25 | 6.5 |
| 24 | 0.25 | 0.25 | 0.25 | 0 | $C_{14}$ | 0.375 | 0.75 | 1.875 | 3.875 |
| 25 | 0 | 0.5 | 0.5 | 0.25 | $C_{15}$ | 0.625 | 1.25 | 3.125 | 6.625 |

### *3.1.1. Formulation of the optimization model*

Modified regression model is used in the formulation of the mathematical model for the prediction of concrete strength. Osadebe assumed that the response function, F(z) given by equation (1) is continuous and differentiable with respect to its predictors, Zi [16].

$$F(z) = F(z^{(0)}) + \sum [\partial F(z^{(0)}) / \partial z_i ](z_i - z_i^{(0)}) + ½! \sum\sum [ \partial^2 F(z^{(0)}) / \partial z_i \partial z_j ](z_i - z_i^{(0)}) (z_j - z_j^{(0)}) + ½! \sum\sum [\partial^2 F(z^{(0)}) / \partial z_i^2] (z_i - z_i^{(0)})^2 + \ldots\ldots \tag{5}$$

where $1 \le i \le 4$ and $1 \le j \le 4$ respectively.
By making use of Taylor's series, the response function could be expanded in the neighborhood of a chosen point:

$$Z^{(0)} = Z_1^{(0)}, Z_2^{(0)}, Z_3^{(0)}, Z_4^{(0)}, Z_5^{(0)} \tag{6}$$

Without loss of generality of the formulation, the point $z^{(0)}$ will be chosen as the origin for convenience sake. It is worthy of note here that the predictor, $z_i$ is not the actual portion of the mixture component rather it is the ratio of the actual portions to the quantity of concrete. For convenience sake, let $z_i$ be the fractional portion and $s_i$ be the actual portions of the mixture components. If the total quantity of concrete is designated $s$, then

$$\sum s_i = s \tag{7}$$

For concrete of four components, $1 \le i \le 4$ and so equation (7) becomes:

$$s_1 + s_2 + s_3 + s_4 = s \tag{8}$$

If the total quantity of concrete required is a unit quantity, then equation (8) should be divided throughout by $s$. Hence

$$s_1/s + s_2/s + s_3/s + s_4/s = s/s \tag{9}$$

But fractional portions,

$$z_i = s_i/s \tag{10}$$

Substituting equation (10) into equation (9) gives equation (11)

$$z_1 + z_2 + z_3 + z_4 = 1 \tag{11}$$

In the formulation of the regression equation, the point, $z^{(0)}$ was chosen as the origin. This implies that $z^{(0)} = 0$ and so

$z_1^{(0)} = 0,\ z_2^{(0)} = 0,\ z_3^{(0)} = 0$ and $z_4^{(0)} = 0$

Let

$$b_0 = F(0) \tag{12}$$

$$b_i = \partial F(0) / \partial z_i \tag{13}$$

$$b_{ij} = \partial^2 F(0) / \partial z_i \partial z_j \tag{14}$$

$$b_{ii} = \partial^2 F(0) / \partial z_i^2 \tag{15}$$

Substituting equations (12 –15) into equation (5) gives:

$$F(z) = b_0 + \sum b_i z_i + \sum\sum b_{ij} z_i z_j + \sum b_{ii} z_i^2 + \ \dots \tag{16}$$

where $1 \le i \le 4$ and $1 \le j \le 4$
Multiplying equation (11) by $b_0$ gives the expression for $b_0$ i.e. equation (17)

$$b_0 = b_0 z_1 + b_0 z_2 + b_0 z_3 + b_0 z_4 \tag{17}$$

Multiplying equation (11) by $z_1$, $z_2$, $z_3$ and $z_4$, and rearranging the products, gives equation (18)-(21)

$$z_1^2 = z_1 - z_1z_2 - z_1z_3 - z_1z_4 \tag{18}$$

$$z_2^2 = z_2 - z_1z_2 - z_2z_3 - z_2z_4 \tag{19}$$

$$z_3^2 = z_3 - z_1z_3 - z_2z_3 - z_3z_4 \tag{20}$$

$$z_4^2 = z_4 - z_1z_4 - z_2z_4 - z_3z_4 \tag{21}$$

Substituting equations (17–21) into equation (16) and simplifying yields equation (22) as follows:

$$Y = \alpha_1z_1 + \alpha_2z_2 + \alpha_3z_3 + \alpha_4z_4 + \alpha_{12}z_1z_2 + \alpha_{13}z_1z_3 + \alpha_{14}z_1z_4 + \alpha_{23}z_2z_3 + \alpha_{24}z_2z_4 + \alpha_{34}z_3z_4 \tag{22}$$

where

$$\alpha_i = b_0 + b_i + b_{ii} \tag{23}$$

and

$$\alpha_{ij} = b_{ij} - b_{ii} - b_{jj} \tag{24}$$

In general, equation (22) is given as

$$Y = \sum\alpha_iz_i + \sum\alpha_{ij}z_iz_j \tag{25}$$

where $1 \leq i \leq j \leq 4$

Equations (22) and (25) are the optimization model equations. $Y$ is the response function at any point of observation, $z_i$ is the predictor and $\alpha_i$ is the coefficient of the optimization model equations.

Osadebe's regression model can be used which has been used successfully by some researchers for different responses. Ogah used that model to study the shear modulus of rice husk ash concrete, in which the concrete components were water, rice husk ash with 45% slaked lime mix, river sand and crushed rock [16].

In this paper, Osadebe's regression model was used to determine a new response which is compressive strength of concrete using normal concrete constituents which include cement, water, fine and coarse aggregates.

*3.1.2 . Determination of the coefficients of the optimization model equation*

Different points of observation will have different responses with different predictors at constant coefficients. At $n^{th}$ observation point, $Y^{(n)}$ will correspond with $Z_i^{(n)}$. That is,

$$Y^{(n)} = \sum \alpha_i z_i^{(n)} + \sum \alpha_{ij} z_i^{(n)} z_j^{(n)} \tag{26}$$

where $1 \leq i \leq j \leq 4$ and $n = 1,2,3, \ldots\ldots\ldots\ldots 10$.

Equation (26) can be put in matrix from as

$$[Y^{(n)}] = [Z^{(n)}]\,\{\alpha\} \tag{27}$$

Rearranging equation (27) gives:

$$\{\alpha\} = [Z^{(n)}]^{-1}\,[Y^{(n)}] \tag{28}$$

The actual mix proportions, $s_i^{(n)}$) and the corresponding fractional portions, $z_i^{(n)}$ are presented in Table 2. These values of the fractional portions $Z^{(n)}$ were used to develop $Z^{(n)}$ matrix and the inverse of $Z^{(n)}$ matrix. The values of $Y^{(n)}$ matrix are determined from laboratory tests and presented in Table 3. With the values of the matrices $Y^{(n)}$ and $Z^{(n)}$ known, it is easy to determine the values of the constant coefficients of equation (31).

Table 2. Values of actual mix proportions and the corresponding fractional portions.

| N | S1 | S2 | S3 | S4 | RESPONSE | Z1 | Z2 | Z3 | Z4 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.55 | 1 | 2 | 4 | $Y_1$ | 7.285 | 13.245 | 26.490 | 52.980 |
| 2 | 0.5 | 1 | 2.5 | 6 | $Y_2$ | 5.000 | 10.000 | 25.000 | 60.000 |
| 3 | 0.45 | 1 | 3 | 5.5 | $Y_3$ | 4.523 | 10.050 | 30.151 | 55.276 |
| 4 | 0.6 | 1 | 1.5 | 3.5 | $Y_4$ | 9.091 | 15.152 | 22.727 | 53.030 |
| 5 | 0.525 | 1 | 2.25 | 5 | $Y_{12}$ | 5.983 | 11.396 | 25.641 | 56.980 |
| 6 | 0.5 | 1 | 2.5 | 4.75 | $Y_{13}$ | 5.714 | 11.429 | 28.571 | 54.286 |
| 7 | 0.575 | 1 | 1.75 | 3.75 | $Y_{14}$ | 8.127 | 14.134 | 24.735 | 53.004 |
| 8 | 0.475 | 1 | 2.75 | 5.75 | $Y_{23}$ | 4.762 | 10.025 | 27.569 | 57.644 |
| 9 | 0.55 | 1 | 2 | 4.75 | $Y_{24}$ | 6.627 | 12.048 | 24.096 | 57.229 |
| 10 | 0.525 | 1 | 2.25 | 4.5 | $Y_{34}$ | 6.344 | 12.085 | 27.190 | 54.381 |

*3.2 Experimental method*

The actual components as transformed from equation (4) and Table 1 were used to measure out the quantities water ($Z_1$), cement ($Z_2$), sand ($Z_3$), and coarse aggregates ($Z_4$) in their respective ratios for the compressive strength test. For instance, the actual ratio for the test number 20 means that the concrete mix ratio is 1: 2.5: 5.3 at 0.5 free water/cement ratio. A total of 25 mix ratios were used to produce 50 prototype concrete cubes measuring 150mm x 150mm x 150mm that were cured and tested on the 28$^{th}$ day. Fifteen out of 20 mix ratios were used as control mix ratios to produce 30 cubes for the confirmation of the adequacy of the mixture design model given by equation (25). The cubes were then tested for compressive strength using the universal testing machine. The load under which the specimen failed was recorded and used to compute the compressive strength of the cubes.

## 4. Results and analysis

The test result of the compressive strength of concrete ($Y_i$) based on 28-day strength, is presented as part of Table 3.

The compressive strength of concrete was obtained from the following equation:

$$\alpha \quad = F/A \tag{29}$$

where α is the compressive strength in Kilo Newtons per meters squared (KNm$^{-2}$). $F$ is failure load (KN) and $A$ is nominal cross-sectional area (m$^2$).

Table 3. Test Results and Replication Variance.

| EXP NO | Replicates | Response $Y_i$ ($KNm^{-2}$) | Response Symbol | $Y^a$ | $\sum Y_i$ | $\sum Yi^2$ | $S_i^{2\ b}$ |
|---|---|---|---|---|---|---|---|
| 1 | 1A<br>IB | 27.10<br>25.34 | $Y_1$ | 26.22 | 52.44 | 1376.53 | 1.55 |
| 2 | 2B<br>2B | 31.12<br>29.32 | $Y_2$ | 30.22 | 60.44 | 1828.12 | 1.62 |
| 3 | 3A<br>3B | 25.20<br>22.80 | $Y_3$ | 24 | 48.00 | 1154.88 | 2.88 |
| 4 | 4A<br>4B | 27.90<br>27.20 | $Y_4$ | 27.55 | 55.10 | 1518.25 | 0.25 |
| 5 | 5A<br>5B | 27.58<br>30.20 | $Y_{12}$ | 28.89 | 57.78 | 1672.70 | 3.44 |
| 6 | 6A<br>6B | 23.31<br>25.57 | $Y_{13}$ | 24.44 | 48.88 | 1197.18 | 2.55 |
| 7 | 7A<br>7B | 20.13<br>23.41 | $Y_{14}$ | 21.77 | 43.54 | 953.25 | 5.38 |
| 8 | 8A<br>8B | 33.01<br>29.21 | $Y_{23}$ | 31.11 | 62.22 | 1942.88 | 7.22 |
| 9 | 9A<br>9B | 23.22<br>21.66 | $Y_{24}$ | 22.44 | 44.88 | 1008.32 | 1.21 |
| 10 | 10A<br>10B | 26.88<br>25.12 | $Y_{34}$ | 26.00 | 52.00 | 1353.55 | 1.55 |
| 11 | 11A<br>11B | 22.22<br>29.40 | $C_1$ | 25.81 | 51.62 | 1358.09 | 25.77 |
| 12 | 12A<br>12B | 22.22<br>30.56 | $C_2$ | 26.39 | 52.78 | 1427.64 | 34.78 |
| 13 | 13A<br>13B | 26.67<br>27.29 | $C_3$ | 26.98 | 53.96 | 1456.03 | 0.19 |
| 14 | 14A<br>14B | 23.78<br>22.86 | $C_4$ | 23.32 | 46.64 | 1088.07 | 0.43 |
| 15 | 15A<br>15B | 28.01<br>28.47 | $C_5$ | 28.24 | 56.48 | 1595.10 | 0.10 |
| 16 | 16A<br>16B | 29.33<br>26.17 | $C_6$ | 27.75 | 55.50 | 1545.12 | 4.99 |
| 17 | 17A<br>17B | 20.00<br>27.38 | $C_7$ | 23.69 | 47.38 | 1149.66 | 27.33 |
| 18 | 18A<br>18B | 26.44<br>21.60 | $C_8$ | 24.02 | 48.04 | 1165.63 | 11.71 |
| 19 | 19A<br>19B | 22.22<br>29.70 | $C_9$ | 25.96 | 51.92 | 1375.82 | 27.98 |
| 20 | 20A<br>20B | 24.66<br>28.48 | $C_{10}$ | 26.57 | 53.14 | 1419.23 | 7.30 |
| CONTROL OUTSIDE FACTOR SPACE | | | | | | | |
| 21 | 21A<br>21B | 25.00<br>25.22 | $C_{11}$ | 25.11 | 50.22 | 1261.05 | 0.03 |
| 22 | 22A<br>22B | 25.78<br>29.98 | $C_{12}$ | 27.88 | 55.76 | 1563.41 | 8.82 |
| 23 | 23A<br>23B | 19.11<br>26.77 | $C_{13}$ | 22.94 | 45.88 | 1081.83 | 29.34 |

*Cont. Table 3.*

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 24 | 24A<br>24B | 29.55<br>23.91 | $C_{14}$ | 26.73 | 53.46 | 1444.89 | 15.90 |
| 25 | 25A<br>25B | 30.67<br>23.77 | $C_{15}$ | 27.22 | 54.44 | 1505.66 | 23.80 |
| | | | | | | ∑∑ | 246.02 |

[a] Y is calculated using the following

$$Y = \sum_{i=1}^{n} Y_i/n$$

[b] $S_i^2$ is calculated as follows:

$$S_i^2 = [1/(n\text{-}1)]\{\sum Y_i^2 - [1/n(\sum Y_i)^2]\}$$

The values of the mean of responses, $Y$ and the variances of replicates $S_i^2$ presented in columns 5 and 8 of Table 3 are gotten from the following equations (30) and (31):

$$Y = \sum_{i=1}^{n} Y_i/n \tag{30}$$

$$S^2_i = [1/(n\text{-}1)]\{\sum Y_i^2 - [1/n(\sum Y_i)^2]\} \tag{31}$$

where 1≤ i ≤n and the following equation is an expanded form of equation (31)

$$S^2_i = [1/(n\text{-}1)][\sum_{i=1}^{n} (\mathrm{Y_i} - \mathrm{Y})^2] \tag{32}$$

where $Y_i$ = responses, $Y$ is the mean of the responses for each control point, $n$ is number of parallel observations at every point, $n-1$ indicates the degree of freedom and $S^2_i$ is the variance at each design point

Considering all the design points, the number of degrees of freedom, $V_e$ is given as

$$V_e = \sum N\text{-}1 = 25 - 1 = 24 \tag{33}$$

where $N$ is the number of points. Replication variance can be found as follows

$$S^2_y = (1/V_e) \sum_{i=1}^{N} S_i^2 = 246.02/24 = 10.25 \tag{34}$$

where $S_i^2$ is the variance at each point

Using equations (33) and (34), the replication error, $S_y$ can be determined as follows:

$$S_y = \sqrt{S^2_y} = 3.2 \tag{35}$$

This replication error value was used below to determine the t-statistics values for the model.

*4.1. Determination of the optimization model based on the modified theory*

Substituting the values of $Y^{(n)}$ from test results presented in Table 3 into equation (28) gives the following values of the coefficients of the model developed i.e. equation (22).

$\alpha_1$ = -394790933.1 $\alpha_2$ = – 220057975.6 $\alpha_3$ = - 4093499.945
$\alpha_4$ = -1283.021096 $\alpha_5$ = 1204352313 $\alpha_6$ = 318501118.4
$\alpha_7$ = 395949693.6 $\alpha_8$ = 284162641.2 $\alpha_9$ = 219194875.1
$\alpha_{10}$ = 4214942.072

Substituting the values of these coefficients into equation (31) yields:

$$\begin{aligned} Y = & -394790933.1Z_1 - 220057975.6Z_2 - 4093499.945Z_3 - 1283.021096Z_4 \\ & +1204352313Z_1Z_2 + 318501118.4 Z_1Z_3 + 395949693.6Z_1Z_4 \\ & +284162641.2 Z_2Z_3 + 219194875.1Z_2Z_4 + 4214942.072Z_3Z_4 \end{aligned} \quad (36)$$

Equation (36) is the modified mathematical model of compressive strength concrete based on the 28-day strength.

## 4.2 . Test of the adequacy of the model

The model equation was tested for adequacy against the controlled experimental results. It will be recalled that the hypothesis for this mathematical model are as follows:
Null Hypothesis ($H_o$): There is no significant difference between the experimental and the theoretically expected results at an α- level of 0.05
Alternative Hypothesis ($H_1$): There is a significant difference between the experimental and theoretically expected results at an α –level of 0.05.

The student's t-test and fisher test statistics were used for this test. The expected values ($Y_{predicted}$) for the test control points were obtained by substituting the values of $Z_i$ from $Z^n$ matrix into the model equation i.e. equation (36). These values were compared with the experimental result ($Y_{observed}$) from Table 4.

### 4.2.1 . Student's test

For this test, the parameters $\Delta_Y$, Є and t are evaluated using the following equations respectively

$$\Delta_Y = Y_{(observed)} - Y_{(predicted)} \quad (37)$$

$$\epsilon = (\sum a_i^2 + \sum a_{ij}^2) \quad (38)$$

$$t = \Delta y\sqrt{n} / (Sy\sqrt{1+ \epsilon}) \quad (39)$$

where Є is the estimated standard deviation or error, $t$ is the t-statistics, $n$ is the number of parallel observations at every point, $S_y$ is the replication error, $a_i$ and $a_{ij}$ are coefficients while $i$ and $j$ are pure components. Also $a_i = X_i(2X_i-1)$ , $a_{ij} = 4X_iX_j$, $Y_{obs}$ is the experimental results and $Y_{pre}$ is the Predicted results.

Table 4. T –Statistics for test control points.

| N | CN | i | j | $a_i$ | $a_{ij}$ | $a^2_i$ | $A^2_{ij}$ | Є | $Y_{obs}$ | $Y_{pre}$ | $\Delta_Y$ | T |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | $C_1$ | 1 | 2 | 0 | 0.5 | 0 | 0.25 | | 25.81 | 26.16 | -0.35 | -0.12 |
| | | 1 | 3 | 0 | 0.5 | 0 | 0.25 | | | | | |
| | | 1 | 4 | 0 | 0 | 0 | 0 | | | | | |
| | | 2 | 3 | -0.125 | 0.25 | 0.0156 | 0.0625 | | | | | |
| | | 2 | 4 | -0.125 | 0 | 0.0156 | 0 | | | | | |
| | | 3 | 4 | -0.125 | 0 | 0.0156 | 0 | | | | | |
| | | 4 | - | 0 | - | 0 | 0 | | | | | |
| | | | | | ∑ | 0.0468 | 0.5625 | 0.6093 | | | | |
| Similarly | | | | | | | | | | | | |
| 2 | | - | - | - | - | - | - | 0.4842 | 26.39 | 26.69 | -0.3 | -0.10 |
| 3 | | - | - | - | - | - | - | 0.6093 | 26.98 | 27.26 | -0.28 | -0.097 |
| 4 | | - | - | - | - | - | - | 0.7343 | 23.32 | 28.69 | -5.37 | -1.80 |
| 5 | | - | - | - | - | - | - | 0.9999 | 28.24 | 28.23 | 0.01 | 0.00 |
| 6 | | - | - | - | - | - | - | 0.5937 | 27.75 | 29.49 | -1.74 | -0.61 |
| 7 | | - | - | - | - | - | - | 0.6249 | 23.69 | 25.77 | -2.08 | -0.72 |
| 8 | | - | - | - | - | - | - | 1.0155 | 24.02 | 25.49 | -1.47 | -0.46 |
| 9 | | - | - | - | - | - | - | 0.8593 | 25.96 | 30.52 | -4.56 | -1.48 |
| 10 | | - | - | - | - | - | - | 0.648 | 26.57 | 29.37 | 2.8 | 0.96 |
| 11 | | - | - | - | - | - | - | 7.0 | 25.11 | 26.69 | -1.58 | -0.25 |
| 12 | | - | - | - | - | - | - | 0.1249 | 27.88 | 29.19 | -1.13 | -0.47 |
| 13 | | - | - | - | - | - | - | 3 | 22.94 | 24.51 | -1.57 | -0.09 |
| 14 | | - | - | - | - | - | - | 0.2811 | 26.73 | 28.13 | -1.4 | -0.55 |
| 15 | | - | - | - | - | - | - | 1.5156 | 27.22 | 29.37 | -2.15 | 0.60 |

At significant level, $\alpha = 0.05$, $t_{\alpha/l}(Ve) = t_{0.05/10} = t_{0.005(14)} = 2.977$. The *t*- value is obtained from standard *t*-statistics table. Since this is greater than any of the *t*- values calculated in Table 4, we accept the Null hypothesis. Hence the model is adequate.

*4.2.2 . Fisher test*

For this test, the parameter y, is evaluated using the following equation:

$$y = \sum Y/n \tag{40}$$

where *Y* is the response and n the number of responses.
Using variance,

$$S^2 = [1/(n-1)][\sum (Y\text{-}y)^2]\ ,\ \ y = \sum Y/n \quad for \quad 1 \le i \le n \tag{41}$$

The fisher test statistics is presented in Table 5.

Table 5. F-Statistics for the controlled points.

| Response Symbol | $Y_{(observed)}$ | $Y_{(predicted)}$ | $Y_{(obs)}-y_{(obs)}$ | $Y_{(pre)}-y_{(pre)}$ | $Y_{(obs)}-y_{(obs)}^2$ | $(Y_{(pre)}-y_{(pre)})^2$ |
|---|---|---|---|---|---|---|
| $C_1$ | 25.81 | 26.16 | -0.09733 | -0.936 | 0.009474 | 0.876096 |
| $C_2$ | 26.39 | 26.69 | 0.482667 | -0.406 | 0.232967 | 0.164836 |
| $C_3$ | 26.98 | 27.26 | 1.072667 | 0.164 | 1.150614 | 0.026896 |
| $C_4$ | 23.32 | 28.69 | -2.58733 | 1.594 | 6.694294 | 2.540836 |
| $C_5$ | 28.24 | 28.23 | 2.332667 | 1.134 | 5.441334 | 1.285956 |
| $C_6$ | 27.75 | 29.49 | 1.842667 | 2.394 | 3.39542 | 5.731236 |
| $C_7$ | 23.69 | 25.77 | -2.21733 | -1.934 | 4.916567 | 3.740356 |
| $C_8$ | 24.02 | 25.49 | -1.88733 | -1.606 | 3.562027 | 2.579236 |
| $C_9$ | 25.96 | 30.52 | 0.052667 | 3.424 | 0.002774 | 11.72378 |
| $C_{10}$ | 26.57 | 29.37 | 0.662667 | 2.274 | 0.439127 | 5.171076 |
| $C_{11}$ | 25.11 | 26.69 | -0.79733 | -0.406 | 0.63574 | 0.164836 |
| $C_{12}$ | 27.88 | 29.19 | 1.972667 | 2.094 | 3.891414 | 4.384836 |
| $C_{13}$ | 22.94 | 24.51 | -2.96733 | -6.706 | 8.805067 | 10.20164 |
| $C_{14}$ | 26.73 | 28.13 | 0.822667 | 1.034 | 0.67678 | 1.069156 |
| $C_{15}$ | 27.22 | 29.37 | 1.312667 | 2.274 | 1.723094 | 5.171076 |
| Sum | 388.61 | 415.56 | | | 41.57669 | 43.79056 |
| Mean | *y(obs) =25.90733* | *y(pre)= 27.704* | | | | |

Therefore from Table 5,

$S^2_{(obs)} = 41.57669/14 = 2.97$ and $S^2_{(pre)} = 43.79056/14 = 3.127$

But the fisher test statistics is given by:

$$F = S^2_1 / S^2_2 \tag{42}$$

where $S^2_1$ is the larger variance. Hence $S^2_1 = 3.127$ and $S^2_2 = 2.97$, therefore $F = 3.127 / 2.97 = 1.05$

From standard Fisher table, $F_{0.95}(14, 14) = 2.41$, hence the regression equation is adequate.

## *4.3 . Comparison of results*

The compressive strength test results obtained from the model were compared with those obtained from the experiment, as presented in Table 6.

Table 6. Comparison of some Predicted Result with Experimental Results.

| Mix No. | Experimental Compressive Strength ($KNm^{-2}$) | Predicted Compressive Strength(($KNm^{-2}$) | Percentage Difference |
|---|---|---|---|
| 1 | 25.81 | 26.16 | 1.36 |
| 2 | 26.39 | 26.69 | 1.14 |
| 3 | 26.98 | 27.26 | 1.04 |
| 4 | 25.11 | 26.69 | 6.29 |
| 5 | 28.24 | 28.23 | 0.04 |
| 6 | 27.75 | 29.49 | 6.27 |

A comparison of the predicted results with the experimental results shows that the percentage difference ranges from a minimum of 0.04% to a maximum of 6.29%, which is insignificant.

## 5. Conclusion

The following conclusions can be drawn from this study:

- Modified regression model using Taylor's series has been applied and used successfully to develop mathematical models for prediction of compressive strength of concrete.
- The student's t-test and the fisher test used in the statistical hypothesis showed that the model developed is adequate.
- Since the maximum percentage difference between the experimental result and the predicted result is insignificant (i.e.6.29), the optimization model will yield accurate values of compressive strength of concrete if given the mix proportions and vice versa.

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