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Non-Linear New Product ABPi Derivations on Rings

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ABSTRACT

Let R be a unital prime ring with unit I which contains a non-trivial idempotent P_1. We will show that any arbitrary not necessarily linear map Φ:R→R satisfies the condition

$$\Phi(ABP_i) = \Phi(A)BP_i + A\Phi(B)P_i + AB\Phi(P_i).\ \ i = 1.2 \qquad (1)$$

for all A.B∈R, where P_2=I-P_1, is an additive derivation.

1. Introduction

Let $\mathcal{R}$ be a ring. For $A.B \in \mathcal{R}$, the Jordan product of the two elements A and B is defined explicitly by $A \circ B = AB + BA$ and their Lie product is described by $[A.B] = AB - BA$. Let $\mathcal{A}$ be a $*-$algebra over complex field $\mathbb{C}$. For $A.B \in \mathcal{A}$, $A \bullet B = AB + BA^*$ and $[A.B]_* = AB - BA^*$ characterize the $*-$Jordan product and the skew Lie product of A and B respectively. These products are fairly meaningful and important in some research topics. Recall that an additive map $\Phi: \mathcal{A} \to \mathcal{A}$ is said to be an additive derivation if $\Phi(AB) = \Phi(A)B + A\Phi(B)$ for all $A.B \in \mathcal{A}$. Furthermore, Φ is said to be an additive $*-$derivation if it is an additive derivation and satisfied $\Phi(A^*) = \Phi(A)^*$ for all $A \in \mathcal{A}$. Studying the features of these multiplications and derivations has recently attracted the attention of many authors [1,2,3,4,5,6,7,8,9,10]. For example, Yaoxian et al. [11] studied the possible structure of nonlinear mixed Lie triple derivation on factor von Neumann algebras. Indeed, they have shown that every nonlinear mixed Lie triple derivation on factor von Neumann algebra is an additive $*-$derivation. Changjin Li, Dongfon Zhang [12] proved that Φ is a nonlinear mixed Jordan triple derivation on factor von Neumann algebras if and only if Φ is an additive $*-$derivation. Thagavi et al. [13] and Zhang [14] independently investigated nonlinear Jordan $*-$derivations on factor von Neumann algebras. In general case, in [15], it has shown such

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facts hold for n-tuple preserving maps over factor von Neumann algebras. The other types of the $*$- additive derivations with respect to the different kind of multiplication have been considered in [16] and [11] for algebras which are factors. As a motivation for conducting this study, we need to mention that, recently many mathematicians have devoted their studies to analyze the new kind of triple products, ABA and AB^*A, which are called Jordan triple product and $*-$Jordan triple product, respectively, over different algebraic structures. The main reason for studying such maps is their close relationship with operator theoretical approach of Quantum physics. Actually, these kind of derivatives may provide the tools that we need to compute some properties.

A map $\Phi: \mathcal{A} \to \mathcal{A}$ is said to be a nonlinear Jordan triple $*-$derivation triple derivation if

$$\Phi((A \bullet B) \bullet C) = (\Phi(A) \bullet B) \bullet C + (A \bullet \Phi(B)) \bullet C + (A \bullet B) \bullet \Phi(C). \quad (2)$$

for all $A.B.C \in \mathcal{A}$, where $A \bullet B = AB + BA^*$. Zhao and Li, [9], have proven that every nonlinear Jordan triple $*-$derivation between von Neumann algebras with no central summands of type I_1 is an additive $*-$derivation. In [17], Taghavi showed that the map $\Phi: \mathcal{A} \to \mathcal{A}$ satisfies (2) for every $A.B \in \mathcal{A}$ and $C \in \mathcal{A}_{p_1}$ if and only if Φ is an additive $*-$derivation, whenever $C \in \mathcal{A}_{p_1} = \left\{P_1.I - P_1.I - 2P_1.\frac{I}{2}.\frac{iI}{2}\right\}$.

In this study, we apply triple product ABP_i for $i = 1.2$, and then we check the mentioned result for our interesting case over algebras which have fewer features. Consequently, it is possible to say that this work results in the previous works.

2. Main Results

Our main theorem is characterized as following:

Theorem 2.1. Let $\mathcal{R}$ be a unital prime ring with I and a nontrivial idempotent P_1. Then the map $\Phi: \mathcal{R} \to \mathcal{R}$ satisfies the following condition:

$$\Phi(ABP_i) = \Phi(A)BP_i + A\Phi(B)P_i + AB\Phi(P_i). \quad i = 1.2 \quad (3)$$

for all $A.B \in \mathcal{R}$, where $P_2 = I - P_1$, is additive derivation.

Proof. Let P_1 be a nontrivial idempotent in $\mathcal{R}$ and $P_2 = I - P_1$. Denote $\mathcal{R}_{ij} = P_i\mathcal{R}P_j$, $i.j = 1.2$,then $\mathcal{R} = \sum_{i.j=1}^{2} \mathcal{R}_{ij}$. For every A$\in \mathcal{R}$ we can write $A = A_{11} + A_{12} + A_{21} + A_{22}$. In all that follow, when we write A_{ij}, it indicates that $A_{ij} \in \mathcal{R}_{ij}$.

For showing additivity of Φ on $\mathcal{R}$, we use above partition of $\mathcal{R}$ and give some claims that prove Φ is additive on each $\mathcal{R}_{ij}$. $i.j = 1.2$.

Claim 1. $\Phi(0) = 0$.

Proof. It is easily proved**.**

Claim 2. For every $A_{ij} \in \mathcal{R}_{ij}. B_{ji} \in \mathcal{R}_{ji}$, for $1 \leq i \neq j \leq 2$, we have

$$\Phi(A_{ij} + B_{ji}) = \Phi(A_{ij}) + \Phi(B_{ji}).$$

Proof. Let $T = \Phi(A_{ij} + B_{ji}) - \Phi(A_{ij}) - \Phi(B_{ji})$, we should prove that $T = 0$.

Using Claim 1 we have

$$\Phi\big(P_i(A_{ij}+B_{ji})P_i\big)=\Phi\big(P_i(A_{ij})P_i\big)+\Phi(P_i(B_{ji})P_i).$$

From this, using the relation (3) we write

$$\Phi(P_i)(A_{ij}+B_{ji})P_i+P_i\Phi(A_{ij}+B_{ji})P_i+P_i(A_{ij}+B_{ji})\Phi(P_i)$$
$$=\Phi\big(P_i(A_{ij}+B_{ji})P_i\big)$$
$$=\Phi\big(P_i(A_{ij})P_i\big)+\Phi(P_i(B_{ji})P_i)$$
$$=\Phi(P_i)(A_{ij})P_i+P_i\Phi(A_{ij})P_i+P_i(A_{ij})\Phi(P_i)$$
$$+\Phi(P_i)(B_{ji})P_i+P_i\Phi(B_{ji})P_i+P_i(B_{ji})\Phi(P_i).$$

In this case, $P_i\left(\Phi(A_{ij}+B_{ji})-\Phi(A_{ij})-\Phi(B_{ji})\right)P_i=0$. So $T_{ii}=0$ and it can be shown in the same way $T_{jj}=0$. By using Claim 1 we have

$$\Phi\big(P_j(A_{ij}+B_{ji})P_i\big)=\Phi\big(P_j(A_{ij})P_i\big)+\Phi(P_j(B_{ji})P_i).$$

And by using relation (3) we get

$$\Phi\big(P_j(A_{ij}+B_{ji})P_i\big)+P_j\Phi(A_{ij}+B_{ji})\mathrm{P_i}+P_j(A_{ij}+B_{ji})\Phi(\mathrm{P_i})$$
$$=\Phi\big(P_j(A_{ij}+B_{ji})P_i\big)$$
$$=\Phi\big(P_j(A_{ij})P_i\big)+\Phi(P_j(B_{ji})P_i)$$
$$=\Phi(P_j)(A_{ij})P_i+P_j\Phi(A_{ij})P_i+P_j(A_{ij})\Phi(P_i)$$
$$+\Phi(P_j)(B_{ji})P_i+P_j\Phi(B_{ji})P_i-P_j(B_{ji})\Phi(P_i).$$

We get $P_j\left(\Phi(A_{ij}+B_{ji})-\Phi(A_{ij})-\Phi(B_{ji})\right)P_i=0$, so $T_{ji}=0$. It can be shown in the same way $T_{ij}=0$, so $T=0$.

Claim 3. For every $A_{ii}\in\mathcal{R}_{ii}.B_{ij}\in\mathcal{R}_{ij}.C_{ji}\in\mathcal{R}_{ji}$, , for every $1\le i\ne j\le 2$, we have

$$\Phi(A_{ii}+B_{ij}+C_{ji})=\Phi(A_{ii})+\Phi(B_{ij})+\Phi(C_{ji}).$$

Proof. Let $T=\Phi(A_{ii}+B_{ij}+C_{ji})-\Phi(A_{ii})-\Phi(B_{ij})-\Phi(C_{ji})$,

we should prove that $T=0$.

Using Claim 1 we have

$$\Phi\big(P_i(A_{ii}+B_{ij}+C_{ji})P_i\big)=\Phi(P_i(A_{ii})P_i)+\Phi\big(P_i(B_{ij})P_i\big)+\Phi(P_i(C_{ji})P_i).$$

By using relation (3),

$$\Phi(P_i)(A_{ii}+B_{ij}+C_{ji})P_i+P_i\Phi(A_{ii}+B_{ij}+C_{ji})P_i+P_i(A_{ii}+B_{ij}+C_{ji})\Phi(P_i)$$
$$=\Phi\big(P_i(A_{ii}+B_{ij}+C_{ji})P_i\big)$$

$$= \Phi(P_i(A_{ii})P_i) + \Phi(P_i(B_{ij})P_i) + \Phi(P_i(C_{ji})P_i)$$

$$= \Phi(P_i)(A_{ii})P_i + P_i\Phi(A_{ii})P_i + P_i(A_{ii})\Phi(P_i)$$

$$+\Phi(P_i)(B_{ij})P_i + P_i\Phi(B_{ij})P_i + P_i(B_{ij})\Phi(P_i)$$

$$+\Phi(P_i)(C_{ji})P_i + P_i\Phi(C_{ji})P_i + P_i(C_{ji})\Phi(P_i).$$

We get $P_i(\Phi(A_{ii} + B_{ij} + C_{ji}) - \Phi(A_{ii}) - \Phi(B_{ij}) - \Phi(C_{ji}))P_i = 0$, so $T_{ii} = 0$. It can be shown in the same way $T_{jj} = 0$.

By using Claim 1 we have

$$\Phi(P_j(A_{ii} + B_{ij} + C_{ji})P_i) = \Phi(P_j(A_{ii})P_i) + \Phi(P_j(B_{ij})P_i) + \Phi(P_j(C_{ji})P_i).$$

By using relation (3),

$$\Phi(P_j)(A_{ii} + B_{ij} + C_{ji})P_i + P_j\Phi(A_{ii} + B_{ij} + C_{ji})P_i + P_j(A_{ii} + B_{ij} + C_{ji})\Phi(P_i)$$

$$= \Phi(P_j(A_{ii} + B_{ij} + C_{ji})P_i)$$

$$= \Phi(P_j(A_{ii})\mathrm{P_i}) + \Phi(P_j(B_{ij})\mathrm{P_i}) + \Phi(P_j(C_{ji})\mathrm{P_i})$$

$$= \Phi(P_j)(A_{ii})\mathrm{P_i} + P_j\Phi(A_{ii})\mathrm{P_i} + P_j(A_{ii})\Phi(P_i)$$

$$+\Phi(P_j)(B_{ij})P_i + P_j\Phi(B_{ij})P_i + P_j(B_{ij})\Phi(P_i)$$

$$+\Phi(P_j)(C_{ji})P_i + P_j\Phi(C_{ji})P_i + P_j(C_{ji})\Phi(P_i).$$

We get $P_j(\Phi(A_{ii} + B_{ij} + C_{ji}) - \Phi(A_{ii}) - \Phi(B_{ij}) - \Phi(C_{ji}))\mathrm{P_i} = 0$, then $T_{ji} = 0$. It can be shown in the same way $T_{ij} = 0$, so $T = 0$.

Claim 4. For every $A_{ii} \in \mathcal{R}_{ii}.B_{ij} \in \mathcal{R}_{ij}.C_{ji} \in \mathcal{R}_{ji}$ and $D_{jj} \in \mathcal{R}_{jj}$, for every $1 \leq i \neq j \leq 2$ we have

$$\Phi(A_{ii} + B_{ij} + C_{ji} + D_{jj}) = \Phi(A_{ii}) + \Phi(B_{ij}) + \Phi(C_{ji}) + \Phi(D_{jj}).$$

Proof. Let $T = \Phi(A_{ii} + B_{ij} + C_{ji} + D_{jj}) - \Phi(A_{ii}) - \Phi(B_{ij}) - \Phi(C_{ji}) - \Phi(D_{jj})$.

By Claim 1 we have

$$\Phi(\mathrm{P_i}(A_{ii} + B_{ij} + C_{ji} + D_{jj})\mathrm{P_i}) = \Phi(\mathrm{P_i}(A_{ii})\mathrm{P_i}) + \Phi(\mathrm{P_i}(B_{ij})\mathrm{P_i})$$

$$+\Phi(\mathrm{P_i}(C_{ji})\mathrm{P_i}) + \Phi(\mathrm{P_i}(D_{jj})\mathrm{P_i}).$$

By relation (3),

$$\Phi(\mathrm{P_i})(A_{ii} + B_{ij} + C_{ji} + D_{jj})\mathrm{P_i} + \mathrm{P_i}\Phi(A_{ii} + B_{ij} + C_{ji} + D_{jj})\mathrm{P_i}$$

$$+\mathrm{P_i}(A_{ii} + B_{ij} + C_{ji} + D_{jj})\Phi(\mathrm{P_i})$$

$$= \Phi(P_i(A_{ii} + B_{ij} + C_{ji} + D_{jj})P_i)$$

$$= \Phi(P_i(A_{ii})P_i) + \Phi(P_i(B_{ij})P_i)$$

$$+\Phi\big(P_i(C_{ji})P_i\big)+\Phi\big(P_i(D_{jj})P_i\big)$$

$$=\Phi(P_i)(A_{ii})P_i+P_i\Phi(A_{ii})P_i+P_i(A_{ii})\Phi(P_i)$$

$$+\Phi(P_i)(B_{ij})P_i+P_i\Phi(B_{ij})P_i+P_i(B_{ij})\Phi(P_i)$$

$$+\Phi(P_i)(C_{ji})P_i+P_i\Phi(C_{ji})P_i+P_i(C_{ji})\Phi(P_i)$$

$$+\Phi(P_i)(D_{jj})P_i+P_i\Phi(D_{jj})P_i+P_i(D_{jj})\Phi(P_i).$$

So we get $T_{ii}=0$. As the same way $T_{jj}=0$. By Claim 1,

$$\Phi\big(P_j(A_{ii}+B_{ij}+C_{ji}+D_{jj})P_i\big)=\Phi\big(P_j(A_{ii})P_i\big)+\Phi\big(P_j(B_{ij})P_i\big)$$

$$+\Phi\big(P_j(C_{ji})P_i\big)+\Phi\big(P_j(D_{jj})P_i\big).$$

By relation (3),

$$P_j(\Phi(A_{ii}+B_{ij}+C_{ji}+D_{jj})-\Phi(A_{ii})-\Phi(B_{ij})-\Phi(C_{ji})-\Phi(D_{jj}))P_i=0.$$

We get $T_{ji}=0$. As the same way $T_{ij}=0$, so $T=0$.

Claim 5. For every $A_{ij}.B_{ij}\in\mathcal{R}_{ij}$, for $1\le i\ne j\le 2$ we have

$$\Phi\big(A_{ij}+B_{ij}\big)=\Phi\big(A_{ij}\big)+\Phi\big(B_{ij}\big).$$

Proof. Since the $\big(I+A_{ij}\big)\big(P_j+B_{ij}\big)P_j=P_j+A_{ij}+B_{ij}$.

By using relation (3) and Claim 3, we write

$$\Phi\big(P_j+A_{ij}+B_{ij}\big)=\Phi\Big(\big(I+A_{ij}\big)\big(P_j+B_{ij}\big)P_j\Big)$$

$$=\Phi\big(I+A_{ij}\big)\big(P_j+B_{ij}\big)P_j$$

$$+\big(I+A_{ij}\big)\Phi\big(P_j+B_{ij}\big)P_j$$

$$+\big(I+A_{ij}\big)\big(P_j+B_{ij}\big)\Phi\big(P_j\big)$$

$$=\Phi\big(\mathrm{I}+A_{ij}\big)P_j\big(P_j\big)+\Phi\big(\mathrm{I}+A_{ij}\big)B_{ij}\big(P_j\big)$$

$$+\big(\mathrm{I}+A_{ij}\big)\Phi\big(P_j\big)P_j+\big(\mathrm{I}+A_{ij}\big)\Phi\big(B_{ij}\big)\big(P_j\big)$$

$$+\big(\mathrm{I}+A_{ij}\big)P_j\Phi\big(P_j\big)+\big(\mathrm{I}+A_{ij}\big)\big(B_{ij}\big)\Phi\big(P_j\big)$$

$$=\Phi\Big(\big(\mathrm{I}+A_{ij}\big)(P_j)P_j\Big)+\Phi\Big(\big(\mathrm{I}+A_{ij}\big)\big(B_{ij}\big)P_j\Big)$$

$$=\Phi\big(P_j+A_{ij}\big)+\Phi\big(B_{ij}\big).$$

So $\Phi\big(P_j+A_{ij}+B_{ij}\big)=\Phi\big(P_j+A_{ij}\big)+\Phi\big(B_{ij}\big)$, by using Claim 3, we get

$$\Phi\big(A_{ij}+B_{ij}\big)=\Phi\big(A_{ij}\big)+\Phi\big(B_{ij}\big).$$

Claim 6. For every $A_{ii}.B_{ii}\in\mathcal{R}_{ii}$, for $1\le i\ne j\le 2$, we have

$$\Phi(A_{ii}+B_{ii})=\Phi(A_{ii})+\Phi(B_{ii}).$$

Proof. Let $T=\Phi(A_{ii}+B_{ii})-\Phi(A_{ii})-\Phi(B_{ii})$, we should prove that $T=0$.

By using relation (3) and Claim 1 we have

$$\begin{aligned}\Phi(P_j)(A_{ii}+B_{ii})P_j+P_j\Phi(A_{ii}+B_{ii})P_j+P_j(A_{ii}+B_{ii})\Phi(P_j)\\=\Phi\big(P_j(A_{ii}+B_{ii})P_j\big)\\=\Phi\big(P_j(A_{ii})P_j\big)+\Phi\big(P_j(B_{ii})P_j\big)\\=\Phi(P_j)A_{ii}P_j+P_j\Phi(A_{ii})P_j+P_jA_{ii}\Phi(P_j)\\+\Phi(P_j)B_{ii}P_j+P_j\Phi(B_{ii})P_j+P_jB_{ii}\Phi(P_j).\end{aligned}$$

So $T_{jj}=0$.

By using relation (3) and Claim 1 we have

$$\begin{aligned}\Phi(P_j)(A_{ii}+B_{ii})P_i+P_j\Phi(A_{ii}+B_{ii})P_i+P_j(A_{ii}+B_{ii})\Phi(P_i)\\=\Phi(P_j(A_{ii}+B_{ii})P_i)\\=\Phi(P_jA_{ii}P_i)+\Phi(P_jB_{ii}P_i)\\=\Phi(P_j)A_{ii}P_i+P_j\Phi(A_{ii})P_i+P_jA_{ii}\Phi(P_i)\\+\Phi(P_j)B_{ii}P_i+P_j\Phi(B_{ii})P_i+P_jB_{ii}\Phi(P_i).\end{aligned}$$

We get $P_j\big(\Phi(A_{ii}+B_{ii})-\Phi(A_{ii})-\Phi(B_{ii})\big)P_i=0$, then $T_{ji}=0$. It can be shown in the same way $T_{ij}=0$.

For every $X_{ji}\in R_{ji}$, by relation (3) and Claim 1 we have

$$\begin{aligned}\Phi(X_{ji})(A_{ii}+B_{ii})P_i+X_{ji}\Phi(A_{ii}+B_{ii})P_i+X_{ji}(A_{ii}+B_{ii})\Phi(P_i)\\=\Phi\big(X_{ji}(A_{ii}+B_{ii})P_i\big)\\=\Phi\big(X_{ji}A_{ii}P_i\big)+\Phi\big(X_{ji}B_{ii}P_i\big)\\=\Phi(X_{ji})\,A_{ii}P_i+X_{ji}\Phi(A_{ii})P_i+X_{ji}A_{ii}\Phi(P_i)\\+\Phi(X_{ji})\,B_{ii}P_i+X_{ji}\Phi(B_{ii})P_i+X_{ji}B_{ii}\Phi(P_i).\end{aligned}$$

We get $X_{ji}\big(\Phi(A_{ii}+B_{ii})-\Phi(A_{ii})-\Phi(B_{ii})\big)P_i=0$, then $X_{ji}T_{ii}=0$.

By primness, we obtain $T_{ii}=0$, so $T=0$.

Claim 7. Φ is additive.

Proof. Using from Claim 1-6 we conclude that Φ is additive.

Claim 8. $\Phi(\mathrm{I})=0$.

Proof. By using relation (3), we write

$$\Phi(\mathrm{II}P_i) = \Phi(\mathrm{I})\mathrm{I}P_i + \mathrm{I}\Phi(\mathrm{I})P_i + \mathrm{II}\Phi(P_i)$$

and

$$\Phi\left(\mathrm{II}P_j\right) = \Phi(\mathrm{I})\mathrm{I}P_j + \mathrm{I}\Phi(\mathrm{I})P_j + \mathrm{II}\Phi(P_j).$$

Adding the two above relation, we get $\Phi(\mathrm{I}) = 0$.

Claim 9. Φ is a derivation.

Proof. By using the relation (3), we write

$$\Phi(ABP_i) = \Phi(\mathrm{A})\mathrm{B}P_i + \mathrm{A}\Phi(\mathrm{B})P_i + \mathrm{AB}\Phi(P_i)$$

and

$$\Phi\left(ABP_j\right) = \Phi(\mathrm{A})\mathrm{B}P_j + \mathrm{A}\Phi(\mathrm{B})P_j + \mathrm{AB}\Phi(P_j).$$

Adding the two above relation, we get $\Phi(AB) = \Phi(A)B + A\Phi(B)$. So Φ is a derivation.

Theorem 2.2. Let $\mathcal{A}$ be a unital prime $*-$algebra with I and a non-trivial projection P_1. Then the map $\Phi\colon \mathcal{A} \to \mathcal{A}$ satisfies in the following conditions:

$$\Phi(\mathrm{A}B^*P_i) = \Phi(\mathrm{A})B^*P_i + A\Phi(B)^*P_i + AB^*\Phi(P_i). \quad \mathrm{i} = 1.2 \qquad (4)$$

for all $\mathrm{A}.\mathrm{B} \in \mathcal{A}$, where $P_2 = I - P_1$, is additive.

Moreover, if $\Phi(I)$ is self-adjoint, then Φ is a $*-$derivation.

Proof. The proof of this theorem is the same as theorem 2.1, we only need to add proof

$\Phi(A^*) = \Phi(A)^*$.

By using relation (4), we write

$$\Phi(IA^*P_i) = \Phi(\mathrm{I})A^*P_i + I\Phi(A)^*P_i + IA^*\Phi(P_i)$$

And

$$\Phi\left(IA^*P_j\right) = \Phi(\mathrm{I})A^*P_j + I\Phi(A)^*P_j + IA^*\Phi(P_j).$$

Adding the two above relations and $\Phi(\mathrm{I}) = 0$, we get $\Phi(A^*) = \Phi(A)^*$.

The following example shows that the self-adjoint condition of $\Phi(\mathrm{I})$ in the above theorem is necessary.

Example 1. Let $\mathcal{A}$ be a prime $*-$algebra with unit I and nontrivial projection. Define a map $\Phi\colon \mathcal{A} \to \mathcal{A}$ where $\Phi(\mathrm{A}) = \mathrm{iA}$ for all $\mathrm{A} \in \mathcal{A}$. In this mapping $\Phi(\mathrm{I})$ is not self-adjoint. It can be easily shown that the mapping Φ in (4) applies, but is not a derivation.

3. CONCLUSION

The current work investigated non-linear maps on rings satisfies in (3) are additive derivation. Also we studied map Φ on prime $*-$algebrasby applying triple product $\mathrm{A}B^*\mathrm{P}_\mathrm{i}$, i=1.2, provided that $\Phi(I)$ is self-adjoint. We showed by providing an example that the self-adjoint condition of $\Phi(I)$ in the above theorem is necessary.

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