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Theory of constraints driven approach to tackle product mix problem with joint material

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ABSTRACT

Determining the type and quantity of products to produce holds critical significance in multi-product manufacturing systems. This problem has been named the product mix problem. Several heuristics have been frequently applied to solve the product mix problems. The previous heuristics lead to ineffective decisions when joint material costs are allocated to single products. This paper seeks to establish a new constructive heuristic derived from the theory of constraints (TOC) to tackle problem of product mix with joint material. A comparison is done between the traditional TOC-based approach, modified TOC-based approach, integer linear programming, and proposed constructive heuristic. The provided numerical example illustrates the reasonableness and applicability of the proposed method.

1. Introduction

Product mix problem entails the task of determining the volume and mix of products to be manufactured. The primary structure of this challenge is to optimize the profit of the enterprise through the combination of manufactured products, while adhering to constraints related to demand and production resources [1]. The product mix decision plays a key role in multi-product companies. So, this problem has been discussed by many researchers. There are three major approaches to solve product mix problem [2, 3]:

1. The exact approaches, such as the integer linear programming (ILP).
2. The heuristics methods, such as the traditional theory of constraints (TOC)-based algorithm, the revised TOC (RTOC) and the improved algorithm.

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3. The meta-heuristics techniques, such as genetic algorithm (GA), simulated annealing (SA), hybrid tabu-simulated annealing (TS-SA), immune algorithm (IA) and imperialist competitive algorithm (ICA).

Although the ILP technique can find the optimum solution, its application needs a high-level expertise and it is also time consuming [4]. Moreover, due to the exponential growth in the number of possible product mix with the number of product types, the problem of product mix is an NP-complete problem [5-7]. As a result, extensive studies have been carried out to develop heuristics algorithms. Traditional TOC-based algorithm was proposed by Goldratt [8] for solving product mix problem. This approach was inefficient or could lead to non-optimal solutions in handling several types of problems, such as:

(i) The problems related to adding new product to the current manufacturing process.
(ii) The problems dealing with multi-bottleneck.
(iii) The problems considering joint material. In other words, the problems under the conditions where two or more products are extracted from one raw material [9].
(iv) The problems associated with non-linear objective function.

Previous research in the literature has predominantly concentrated on addressing the second type of problems mentioned above. However, this paper shifts its focus towards the third type, specifically developing a novel heuristic approach to effectively tackle product mix problems involving joint materials.

This paper is organized as follows: Section 2 presents a concise review of the literature on the theory of constraints (TOC)-based product mix problem. Section 3 outlines the problem formulation. Section 4 introduces the proposed heuristic. Section 5 provides a numerical example illustrating the procedure of the suggested heuristic. Additionally, this section includes the solutions obtained by three other approaches (traditional TOC-based, modified TOC-based, and Integer Linear Programming). Finally, the study concludes in Section 6.

2. Literature review

In 1984, Goldratt [10] introduced the theory of constraints as a groundbreaking management philosophy. He proposed a systematic process known as the Five Focusing Steps (5FS) to manage constraints and drive continuous improvement. These steps are as follows:

1. Identify the constraint(s).
2. Exploit the constraint(s).
3. Subordinate everything else to the constraint(s).
4. Elevate the constraint(s).
5. If a constraint is broken, return to Step 1 to prevent inertia from becoming the constraint.

The detailed explanation of TOC's 5FS can be found in various literature sources, such as Aryanezhad et al. [11] and Badri & Aryanezhad [12]. One significant application of the TOC's 5FS is in product mix decision, as highlighted by Hsu and Chung [13]. Goldratt also proposed an algorithm, based on TOC principles to determine the optimal product mix. Researchers like Luebbe and Finch [14] and Patterson [15] have validated the traditional TOC-based approach. Additionally, Luebbe and Finch [14], Balakrishnan and Cheng [16], and Finch and Luebbe [17] have compared the traditional TOC-based approach with the linear programming approach. In a study by Lee and Plenert [18], it was shown that the TOC-based product mix may result in lower profits compared to the integer linear programming when new product alternatives are introduced. Plenert [19] further noted that the TOC-based algorithm may not be efficient in finding the optimal solution when multiple constrained resources are involved.

Fredendall and Lea [20] introduced a revised heuristic, known as the RTOC, to optimize product mix in environments with multiple constrained resources. This was necessary as the traditional TOC heuristic was unable to do so effectively. In a similar vein, Hsu and Chung [13] introduced a dominance rule-based algorithm that categorized non-critically constrained resources into three levels. This approach was designed to address the problem of product mix in situations where multiple bottlenecks were present.

Onwubolu [21] demonstrated the effectiveness of utilizing the tabu search (TS) approach to identify the product mix in scenarios with multiple bottlenecks. While Onwubolu's solution outperformed the traditional TOC heuristic, it underperformed when compared to the RTOC and the ILP solutions. Onwubolu and Mutingi [22,23] introduced GA to tackle the challenges posed by multiple bottlenecks. Aryanezhad and Komijan [24] highlighted a flaw in the RTOC, showing that it led to suboptimal solutions. They also pointed out various drawbacks of the RTOC method and put forth an improved algorithm to achieve optimal results.

Mishra et al. [25] attempted a TS-SA approach, but their solution proved infeasible due to an overloaded constraint (resource 40 in their study). Souren et al. [9] delved into the fundamental principles underlying product mix decisions, distinguishing cases where the traditional TOC-based heuristic yields optimal outcomes from those where it falls short. Tsai et al. [26] expanded on the improved algorithm to encompass systems with joint products. Bhattacharya et al. [27] presented findings from their fuzzy linear programming approach to the problem, while Wang et al. [28] employed an intelligent search approach based on immune algorithms and TOC. Komijan et al. [29] introduced a novel heuristic algorithm to address product mix challenges, showcasing in an example how the RTOC method and the improved algorithm by Aryanezhad and Komijan led to suboptimal solutions. Linhares [5] provided an illustrative example demonstrating the limitations of the traditional TOC-based approach in cases with a single bottleneck.

Sobreiro and Nagano [6] introduced a novel constructive heuristic that combines Theory of Constraints (TOC) principles with the knapsack problem. Their research revealed that this new heuristic outperformed the RTOC and other existing algorithms. Tanhaei and Nahavandi [30] proposed a heuristic algorithm designed to optimize product mix in environments with two constraints. Furthermore, De Souza et al. [3] utilized a numerical example, known as the P&Q problem, to evaluate the feasibility of achieving an optimal production mix in a non-deterministic situation. They also devised a heuristic approach that integrates TOC and Banared's factor to ensure a predetermined minimum level of protective capacity. Sobreiro et al. [7] introduced a novel constructive heuristic aimed at optimizing the product mix based on throughput per day. Their innovative approach yielded high-quality solutions and efficient CPU time utilization, outperforming traditional enumeration methods. Rajesh [31] proposed a mixed integer linear goal programming (MILGP) model to address the challenges of managing multiple constrained resources in product-mix optimization within the Theory of Constraints (TOC) framework. Furthermore, Badri et al. [32] tackled the product mix problem with interval parameters by introducing a multi-criteria decision-making approach aligned with TOC principles. Golmohammadi and Mansouri [33] developed the COLOMAPS algorithm to streamline master production scheduling (MPS) under the TOC methodology. Hadidi and Moawad [34] leveraged an ILP model to address the product-mix probkem in a steel plant in Saudi Arabia. Mansouri et al. [35] devised a mixed-effects model to predict job shop system throughput based on six key problem characteristics and four MPS methods, including an ILP model and algorithms proposed by Fredendall and Lea [20], Sobreiro and Nagano [6], and Golmohammadi and Mansouri [33].

In light of the literature review provided above, in the following the product mix problem with joint material is discussed.

3. Product mix problem with joint material

The product mix decision discussed in this study differs from the classical version. In the classical version of product mix decision problem, each product utilizes only separable raw materials. However, this paper explores a scenario where two distinct products are derived from a single raw material source.

3.1. Notation

To model the product mix problem considering joint material, the following notation will be used:

i	product index,
j	resource index,
h	joint material index,
n	total number of products,
m	total number of resources,
l	total number of joint material types,
S_h	set of products requiring joint material type *h*,
t_{ij}	processing time for product *i* on resource *j*,
D_i	product *i* demand,
p_i	selling price of the product *i*,
m_i	cost of separable materials for product *i*,
M_h	unit cost of joint material type *h*,
AC_j	available capacity of resource *j*,
OE	operational expenses,
Q_i	decision variable indicating the quantity of product *i* produced,

3.2. The ILP model

The problem of determining the best product mix can be stated as follows:

Maximize Net profit (NP) = Total revenue – Total separable material cost – Total joint material cost – Operating expenses.

i.e.

$$Maximize\ NP = \sum_{i=1}^{n} p_i Q_i - \sum_{i=1}^{n} m_i Q_i - \sum_{h=1}^{l} \left[\max_{i \in S_h} \{Q_i\}\, M_h \right] - OE \qquad \textbf{(1)}$$

Subject to:

(Resource capacity constraints):

$$\sum_{i=1}^{n} t_{ij} Q_i \leq AC_j \quad j = 1,2,\dots,m \qquad \textbf{(2)}$$

(Market demand constraints):

$$Q_i \leq D_i \quad i = 1,2,\dots,n \qquad \textbf{(3)}$$

(Non-negative constraints):

$$Q_i \geq 0 \text{ and } integer \quad i = 1,2,\dots,n \qquad \textbf{(4)}$$

This problem can be reformulated equivalently by introducing a parameter q_h as follows:

$$Maximize\ NP = \sum_{i=1}^{n} p_i Q_i - \sum_{i=1}^{n} m_i Q_i - \sum_{h=1}^{l} q_h M_h - OE \qquad \textbf{(5)}$$

Subject to:

$$q_h \geq Q_i \quad h = 1,2,\dots,l; \quad i \in S_h \qquad \textbf{(6)}$$

$$\sum_{i=1}^{n} t_{ij} Q_i \leq AC_j \quad j = 1,2,\dots,m \qquad \textbf{(7)}$$

$$Q_i \leq D_i \quad i = 1,2,\dots,n \qquad \textbf{(8)}$$

$$Q_i \geq 0 \text{ and } integer \quad i = 1,2,\dots,n \qquad \textbf{(9)}$$

4. Proposed heuristic

In this section, a constructive heuristic based on the TOC is proposed for solving product mix problem with joint material. In proposed heuristic, both individual products and products set which need joint material are considered in order to determine the priority sequences. Then, the product mix is determined with regard to the priorities. The steps of the proposed algorithm are presented as follows:

Step1. Identify the dominant bottleneck of the system:

(a) Calculate the difference between the required capacity and the available capacity for each resource.

Overload of each resource (O_j) = required capacity – available capacity.

i.e.

$$O_j = \sum_{i=1}^{n} D_i t_{ij} - AC_j \tag{10}$$

(b) Determine the most overload resource and call it dominant bottleneck (BN).

Step2. Calculate the contribution margin for each product:

The contribution margin for individual products, as well as for product sets requiring joint materials, is calculated as follows:

Contribution margin of product *i* (CM_i) = selling price of the product *i* – separable material cost of product *i* – joint material cost of product *i*.

i.e.

$$CM_i = p_i - m_i - \sum_{h \in Set_i} M_h \tag{11}$$

where $Set_i = \{h | i \in S_h\}$.

Contribution margin of products set which need joint material type *h* (CM_{S_h}) = total selling price of the products – total separable material cost of products – unit cost of joint material type *h*.

i.e.

$$CM_{S_h} = \sum_{i \in S_h} (p_i - m_i) - M_h \tag{12}$$

Step3. Determine the production priority:

(a) For dominant bottleneck calculate $R_{i,BN}$ and $R_{ik,BN}$ as follows:

Priority of product i in view of *BN* ($R_{i,BN}$) = Contribution margin of product i ÷ processing time for product *i* on *BN*.

i.e.

$$R_{i,BN} = \frac{CM_i}{t_{i,BN}} \tag{13}$$

Priority of products set which need joint material type h in view of *BN* ($R_{S_h,BN}$) = Contribution margin of products set which need joint material type h ÷ total processing time of products set on *BN*.

i.e.

$$R_{S_h,BN} = \frac{CM_{S_h}}{\sum_{i \in S_h} t_{i,BN}} \tag{14}$$

(b) Arrange the products processed on *BN* in non-ascending order based on $R_{i,BN}$ and $R_{S_h,BN}$. Arrange the product with same $R_{i,BN}$ or $R_{S_h,BN}$ in non-ascending order based on CM_ior CM_{S_h}.

(c) Arrange the products not processed on *BN* in non-ascending order based on CM_i and CM_{ik}.

Step4. Determine the product mix:

In this step, the product mix is established according to prioritization determined in step 3. For each product i, schedule the maximum quantity considering market demand and available capacity of *BN*. If $Q_{i'} > Q_i$, increase the production priority for $(Q_{i'} - Q_i)$ unit(s) of product i by $\frac{\sum_{h \in Set_{i,i'}} M_h}{t_{i,BN}}$. Where $i \& i' \in S_h$ and $Set_{i,i'} = \{h | i, i' \in S_h\}$.

Step5. Determine the net profit

The net profit of the product mix is calculated by ***Eq. (1)***.

5. Numerical example

To exemplify the steps outlined in the preceding section, consider the following problem adapted from Souren et al., [9]. A manufacturing facility produces three products: A, B, and C, utilizing four resources (machines) labeled I, II, III, and IV. The weekly demand for products A, B, and C is 100, 80, and 50 units, respectively. The available capacity of all resources per week is 2400 minutes, except for resource III, which is 2800 minutes. The separable raw material costs for products A, B, and C are $100, $80, and $50, respectively. Additionally, products A and B require joint raw material. The unit cost of the joint material is $30, allocated based on the weight of the products: 30% for product A ($9) and 70% for product B ($21). The operating expense is $3000 per week. The processing time for each product on each resource and the selling price of each product are depicted in ***Figure 1***.

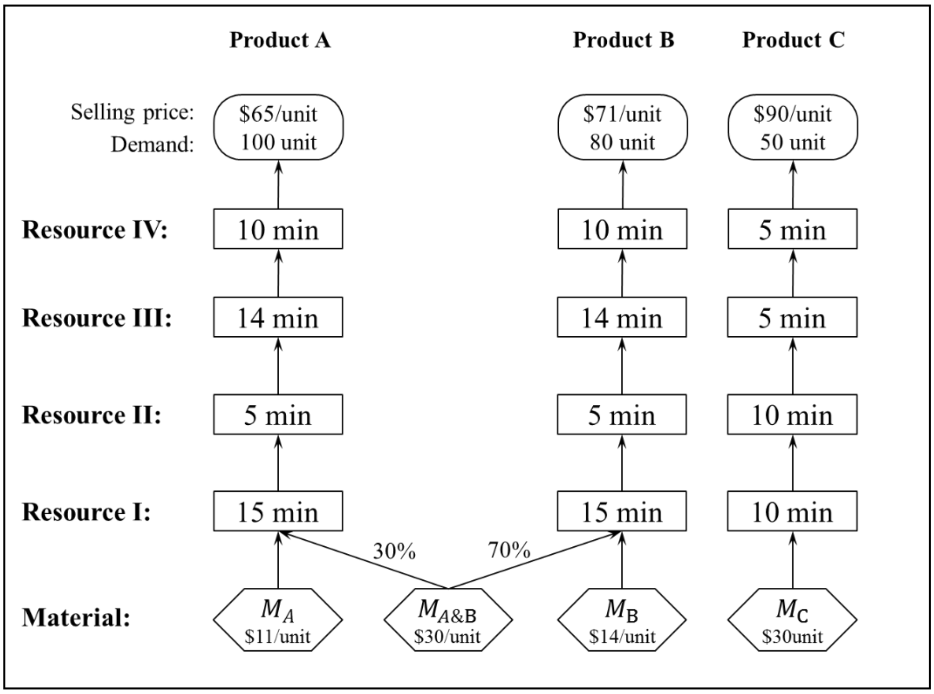

Figure. 1. Product structure and process times taken from Souren et al., [9]

5.1. Determination of the product mix according to the traditional TOC-based approach

Following the TOC, production priority is determined based on the R_i ratio, representing throughput to processing time on the bottleneck. As depicted in ***Table 1***, resource I emerges as the most overloaded bottleneck (BN). Notably, product C exhibits the highest R_i ratio, R_C= (90-30)/10=$6/min. Given the demand for product C stands at 50 units, prioritizing its production, 50 units of product C are initially manufactured. This decision consumes 50×10=500 minutes of the BN. Subsequently, product A, boasting a ratio of $3/min, is produced to fulfill its total demand of 100 units, consuming 1500 minutes of the BN. Consequently, the remaining time available for resource I is 800 minutes, allowing the production of only 26 units of product B.

As shown in ***Table 2***, the traditional TOC-based solution is 50C, 100A and 26B. Taking into account the operating expense of $3000, the net profit can be determined in the following manner:

Net profit = 50×60 + 100×54 + 26×57 – (100×30) – 3000 = $3882

5.2. Determination of the product mix according to the modified TOC-based approach

As per Souren *et al.*, [9], when only individual material costs are accounted for, the modified R_i ratio values are as follows: R_A= (65-11)/15=3.6, R_B= (71-14)/15=3.8 and R_C= (90-30)/15=6. Thus, the priority sequence is C, B and A. As shown in ***Table 3***, the modified TOC-based solution is 50C, 80B and 46A with the total net profit of $4542.

Net profit = 50×60 + 80×54 + 46×57 – (80×30) – 3000 = $4542

Table 1. Time required for processing in minutes, available capacity, and required capacity

Product	Resource			
	I	II	III	IV
A	15	5	14	10
B	15	5	14	10
C	10	10	5	5
Available Capacity	2400	2400	2800	2400
Required Capacity	3200	1400	2770	2050
Difference (O_j)	800	-1000	-30	-350

Table 2. Product mix of the traditional TOC-based approach

Product	Demand	Product mix	Resource I		
			Processing time	Used	Left
C	50	50	10	500	1900
A	100	100	15	1500	400
B	80	26	15	390	10

Table 3. Product mix of the modified TOC-based approach

Product	Demand	Product mix	Resource I		
			Processing time	Used	Left
C	50	50	10	500	1900
B	80	80	15	1200	700
A	100	46	15	690	10

5.3. Determination of the product mix according to the proposed heuristic

The proposed heuristic solves the product mix problem with joint material in the following manner:

Step1. Identify the system's dominant bottleneck:

(a) ***Table 1*** illustrates the variance between available capacity and required capacity for each resource.

(b) According to the last row of ***Table 1***, resource I is dominant bottleneck (BN).

Step2. Calculate the contribution margin of each product:

Using the material cost and selling price of each product shown in ***Figure 1***, CM_i and CM_{S_h} are calculated as follows:

$CM_A = 65 - 11 - 30 = 24$
$CM_B = 71 - 14 - 30 = 27$
$CM_C = 90 - 30 = 60$
$CM_{\{A,B\}} = 65 + 71 - 11 - 14 - 30 = 81$

Step3. Determine the production priority:

The production priority ratio with respect to resource I is determined as follows:

$$R_{A,I} = \frac{24}{15} = 1.6$$
$$R_{B,I} = \frac{27}{15} = 1.8$$
$$R_{C,I} = \frac{60}{10} = 6$$
$$R_{\{A,B\},I} = \frac{81}{15 + 15} = 2.7$$

Hence, the priority sequences regarding resource I are C, {A,B}, B and A.

Step4. Determine the product mix:

The product mix regarding the priority obtained in Step 4, is shown in ***Table 4***.

Table 4. Product mix of the proposed approach

Product	Demand	Product mix	Resource I		
			Processing time	Used	Left
C	50	50	10	500	1900
{A,B}	Min(100,80)	63	15+15	1890	10
B	80-63	0	15	0	10
A	100-63	0	15	0	10

As product C has the highest priority, it is produced first. After meeting its demand (which is 50 units), product A and product B (i.e. {A,B}) are scheduled simultaneously. The left time for resource I is only enough to produce 63 units of {A,B}. So the product mix is 50C, 63A and 63B.

Step5: Determine the net profit:

The net profit is calculated as follows:

Net profit = 50×60 + 63×54 + 63×57 – (63×30) – 3000 = $5103

5.4. Determination of the optimal product mix according to the ILP model

To compare the solution of proposed heuristic with the optimum solution, the example is formulated as follows:

$$maximize\ NP = (65-11)Q_A + (71-14)Q_B + (90-30)Q_C - 30q_{\{A,B\}} - 3000$$

Subject to:

$$\begin{aligned}
&15Q_A + 15Q_B + 10Q_C \leq 2400\\
&5Q_A + 5Q_B + 10Q_C \leq 2400\\
&14Q_A + 14Q_B + 5Q_C \leq 2800\\
&10Q_A + 10Q_B + 5Q_C \leq 2400\\
&q_{\{A,B\}} \geq Q_A\\
&q_{\{A,B\}} \geq Q_B\\
&Q_A \leq 100\\
&Q_B \leq 80\\
&Q_C \leq 50\\
&Q_A, Q_B \text{ and } Q_C \text{ are integer and non negative}
\end{aligned}$$

The optimum solution obtained through the ILP is 63A, 63B and 50C and the net profit is 5103 dollars which is the same solution resulted from the proposed heuristic.

The summary of results obtained by proposed heuristic and other approaches is presented in ***Table 5***.

Table 5. Comparison of the product mix and the resulting net profits achieved through various approaches

The solution method	Traditional TOC-based	Modified TOC-based	Proposed heuristic	Optimal solution
Product mix	100A, 26B, 50C	46A, 80B, 50C	63A, 63B , 50C	63A, 63B , 50C
Net profit	3882	4542	5103	5103

The comparison between the proposed algorithm and alternative approaches highlights that:

- The proposed algorithm performs better than both the traditional TOC-based and the modified TOC-based approaches.
- The traditional TOC-based and the modified TOC-based approaches fail to obtain the optimum product mix in problems considering joint material even with a single bottleneck.
- The proposed algorithm is suitable in solving product mix problem considering joint material and can find the optimal solutions.

6. Conclusion and further research

Product mix decision plays a key role in multi-product companies. Several algorithms have been devised to find the optimum product mix based on theory of constraint. Because of the inefficiency of the previous approaches in handling the problems considering joint material, this paper proposed a new approach for product mix problem with joint material. The problem addressed in this paper deviates from the classical version of product mix problem. In the classical version, each product uses only separable raw material. This paper considers the case that two products are extracted from one raw material. In proposed heuristic, after bottleneck identification, both individual products and products set which need joint material are considered in order to determine the priority sequences. The comparison between the proposed algorithm and the traditional TOC-based and modified TOC-based approaches reveals that the proposed algorithm achieves an optimal product mix, outperforming the other approaches. Future research directions include the development of meta-heuristic algorithms for solving large-scale problems and incorporating uncertainty into the models.

References

[1] Karakas, E., Koyuncu, M., Erol, R., & Kokangul, A. (2010). Fuzzy programming for optimal product mix decisions based on expanded ABC approach. *International Journal of Production Research, 48*(3), 729-744.

[2] Chaharsooghi, S. K., & Jafari, N. (2007). A simulated annealing approach for product mix decisions. *Scientia Iranica*, *14*(3), 230-235.

[3] De Souza, F. B., Sobreiro, V. A., Nagano, M. S., & de Souza Manfrinato, J. W. (2013). When less is better: Insights from the product mix dilemma from the Theory of Constraints perspective. *International Journal of Production Research*, *51*(19), 5839-5852.

[4] Ray, A., Sarkar, B., & Sanyal, S. (2010). The TOC-based algorithm for solving multiple constraint resources. *Engineering Management, IEEE Transactions on*, *57*(2), 301-309.

[5] Linhares, A. (2009). Theory of constraints and the combinatorial complexity of the product-mix decision. *International Journal of Production Economics, 121*(1), 121-129.

[6] Sobreiro, V. A., & Nagano, M. S. (2012). A review and evaluation on constructive heuristics to optimise product mix based on the Theory of Constraints. *International Journal of Production Research*, *50*(20), 5936-5948.

[7] Sobreiro, V. A., Mariano, E. B., & Nagano, M. S. (2014). Product mix: the approach of throughput per day. *Production Planning & Control*, 25(12), 1015-1027.

[8] Goldratt, E. M. (1990). The haystack syndrome: Sifting information from the data ocean. *New York, North River*.

[9] Souren, R., Ahn, H., & Schmitz, C. (2005). Optimal product mix decisions based on the theory of constraints? Exposing rarely emphasized premises of throughput accounting. *International Journal of Production Research*, *43*(2), 361-374.

[10] Goldratt, E. M. (1990b). Theory of Constraints: What is this thing called the Theory of Constraints and how should it be implemented. *Croton-on-Hudson, North River, New York*.

[11] Aryanezhad, M. B., Badri, S. A., & Rashidi Komijan, A. (2010). Threshold-based method for elevating the system's constraint under theory of constraints. *International Journal of Production Research*, *48*(17), 5075-5087.

[12] Badri, S. A., & Aryanezhad, M. B. (2011). A mathematical method for managing the system constraint. *International Journal of Engineering-Transactions A: Basics*, *24*(1), 37-47.

[13] Hsu, T. C., & Chung, S. H. (1998). The TOC-based algorithm for solving product mix problems. *Production planning & control*, *9*(1), 36-46.

[14] Luebbe, R., & Finch, B. (1992). Theory of constraints and linear programming: a comparison. *THE International Journal of Production Research*, *30*(6), 1471-1478.

[15] Patterson, M. C. (1992). The product-mix decision: a comparison of theory of constraints and labor-based management accounting. *Production and Inventory Management Journal*, *33,* 80-80.

[16] Balakrishnan, J., & Cheng, C. H. (2000). Discussion: Theory of constraints and linear programming: A re-examination. *International Journal of Production Research*, *38*(6), 1459-1463.

[17] Finch, B. J., & Luebbe, R. L. (2000). Response to 'Theory of constraints and linear programming: a re-examination'. *International Journal of Production Research*, *38*(6), 1465-1466.

[18] Lee, T. N., & Plenert, G. (1993). Optimizing theory of constraints when new product alternatives exist. *Production and Inventory Management Journal*, *34*, 51-51.

[19] Plenert, G. (1993). Optimizing theory of constraints when multiple constrained resources exist. *European Journal of Operational Research*, *70*(1), 126-133.

[20] Fredendall, L. D., & Lea, B. R. (1997). Improving the product mix heuristic in the theory of constraints. *International Journal of Production Research*, *35*(6), 1535-1544.

[21] Onwubolu, G. C. (2001). Tabu search-based algorithm for the TOC product mix decision. *International Journal of Production Research*, *39*(10), 2065-2076.

[22] Onwubolu, G. C., & Mutingi, M. (2001a). Optimizing the multiple constrained resources product mix problem using genetic algorithms. *International Journal of Production Research*, *39*(9), 1897-1910.

[23] Onwubolu, G. C., & Mutingi, M. (2001b). A genetic algorithm approach to the theory of constraints product mix problems. *Production Planning & Control*, *12*(1), 21-27.

[24] Aryanezhad, M. B., & Komijan, A. R. (2004). An improved algorithm for optimizing product mix under the theory of constraints. *International Journal of Production Research*, *42*(20), 4221-4233.

[25] Mishra, N., Tiwari, M. K., Shankar, R., & Chan, F. T. (2005). Hybrid tabu-simulated annealing based approach to solve multi-constraint product mix decision problem. *Expert systems with applications*, *29*(2), 446-454.

[26] Tsai, W. H., Lai, C. W., & Chang, J. C. (2007). An algorithm for optimizing joint products decision based on the Theory of Constraints. *International journal of production research*, *45*(15), 3421-3437.

[27] Bhattacharya, A., Vasant, P., Sarkar, B., & Mukherjee, S. K. (2008). A fully fuzzified, intelligent theory-of-constraints product-mix decision. *International Journal of Production Research*, *46*(3), 789-815.

[28] Wang, J. Q., Sun, S. D., Si, S. B., & Yang, H. A. (2009). Theory of constraints product mix optimisation based on immune algorithm. *International Journal of Production Research*, *47*(16), 4521-4543.

[29] Komijan, A. R., Aryanezhad, M. B., & Makui, A. (2009). A new heuristic approach to solve product mix problems in a multi-bottleneck system. *Journal of Industrial Engineering International*, *5*(9), 46-57.

[30] Tanhaei, F., & Nahavandi, N. (2013). Algorithm for solving product mix problem in two-constraint resources environment. *The International Journal of Advanced Manufacturing Technology*, *64*(5-8), 1161-1167.

[31] Rajesh, M. (2014). A Mixed Integer Linear Goal Programming Model for Optimizing Multiple Constrained Resources Product-Mix Problem Under the Theory of Constraints. *IUP Journal of Operations Management*, *13*(1), 7-19.

[32] Badri, S. A., Ghazanfari, M., & Shahanaghi, K. (2014). A multi-criteria decision-making approach to solve the product mix problem with interval parameters based on the theory of constraints. *The International Journal of Advanced Manufacturing Technology*, *70*(5-8), 1073-1080.

[33] Golmohammadi, D., & Mansouri, S. A. (2015). Complexity and workload considerations in product mix decisions under the theory of constraints. *Naval Research Logistics (NRL)*, *62*(5), 357-369.

[34] Hadidi, L. A., & Moawad, O. A. (2017). The product-mix problem for multiple production lines in sequenced stages: a case study in the steel industry. *The International Journal of Advanced Manufacturing Technology*, *88*(5-8), 1495-1504.

[35] Mansouri, S. A., Golmohammadi, D., & Miller, J. (2019). The moderating role of master production scheduling method on throughput in job shop systems. *International Journal of Production Economics*, *216*, 67-80.