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Obtaining Solitary Solutions of two well-known systems using a coupled modified simplest Method

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ABSTRACT

One of the topics of curiosity in recent years in miscellaneous sciences such as physics and engineering is to attain analytical solutions to evolution equations. In this work, a novel modification of the simplest equation method has been suggested to solve the nonlinear system of the partial differential equation. As an example, the proposed methods have been applied to achieve the exact solutions of the coupled generalized Schrödinger–Boussinesq system and Boussinesq-type coupled system. By giving specific values to the parameters, private answers are obtained and the plots of solutions are drawn. The newly-used method is highly accurate, flexible, effective, and programmable to solve systems of differential equations.

1. Introduction

Nonlinear coupled partial differential equations play an important role in various scientific fields, especially in mathematics and physics. Most problems and scientific phenomena occur nonlinearly. Therefore, researchers have long been interested in numerical and analytical solutions. In the last decade, various methods have been introduced and used for this purpose [1-15]. One of the most important and simplest of these methods is the simplest equation method (MSE method) [13-15]. The primary merits of procedure used in this research over other analytical methods is that $\varphi(\xi)$ is not satisfy in any special equation.

In this method, to solve a system of equations, first, it will be converted into an equation and then the desired method will be applied. The present paper's motivation is to develop the above-mentioned method to directly solve a system of differential equations. The structure and steps of

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proposed method with the help of mathematical software is quite simple. To demonstrate the ability and capability of the method exact solutions and especially soliton solutions of two famous and widely used coupled system called the coupled generalized Schrödinger–Boussinesq system and the Boussinesq-type coupled system equation have been attained.

John Scott Russell first described soliton phenomenon in 1834. The "translation wave" was the name he gave to the phenomenon that was reproduced in a wave reservoir. A wave propagating at a constant rate and shape in a nonlinear dispersive system is called soliton.

We can say that a solitary wave called Soliton satisfies to the following three conditions:

1. They are constant shape.
2. They are localized within a region.
3. In interaction with other solitons they come out unchanged (except phase shift).

The continuation process of the article is as follows. Section 2 describes the coupled modified simple equation (CMSE) method. In Section 3, the CMSE method is used to obtain the exact answer of the coupled generalized Schrödinger–Boussinesq system. The Boussinesq-type coupled system equation is discussed in Section 4. Finally, a conclusion is presented in Section 5.

2. The CMSE method

To explain, Coupled modified simplest equation method, the following nonlinear system of PDE should be noted:

$$\begin{cases} F(u, v, u_x, u_t, v_x, v_t, \dots) = 0, \\ G(u, v, u_x, u_t, v_x, v_t, \dots) = 0. \end{cases} \quad (1)$$

By using the nonlinear complex transformation

$$\xi = \delta x - \rho t, \quad (2)$$

Where ρ and δ are nonzero parameters, Eq. (2) turns to a system of ODE

$$\begin{cases} f(u, v, u', v', \dots) = 0, \\ g(u, v, u', v', \dots) = 0. \end{cases} \quad (3)$$

In this method, we get the following form for solution of (3)

$$\begin{cases} u(\xi) = \sum_{i=0}^N a_i \left(\frac{\varphi'(\xi)}{\varphi(\xi)} \right)^i, \\ v(\xi) = \sum_{i=0}^M b_i \left(\frac{\varphi'(\xi)}{\varphi(\xi)} \right)^i. \end{cases} \quad (4)$$

Where a_i 's and b_i 's are unknown constants and $\varphi(\xi)$ is an unknown function.

To obtain the number M , and N we strike a balance between the sentences with the highest derivative order and the highest nonlinear order in EQ. (3). In this step, we place Eq. (4) in to the ODE obtained. Therefore, a polynomial of $\frac{\varphi'(\xi)}{\varphi(\xi)}$ and its derivatives will be obtained. By considering the coefficients of $\varphi^{-i}(\xi)$ as zero, a system of equations that can be obtained to determine unknown constants, $\varphi(\xi)$ and $\varphi'(\xi)$. Finally, solutions of Eq. (1) will be obtained by placing obtained results into Eq. (4).

3- Application MSE method to coupled generalized Schrödinger–Boussinesq system

In this part we use the above method for the coupled generalized Schrödinger–Boussinesq system (CGSBs), as the following form

$$\begin{cases} iu_t + u_{xx} + \kappa u = uv, \\ 3v_{tt} - v_{xxxx} + 3(v^2)_{xx} + \omega v_{xx} = (|u|^2)_{xx} \end{cases} \tag{5}$$

For this goal, we use the transformation $u(x,t) = p(x,t)e^{i(\lambda x + \mu t)}$, So, Eq. (1) convert to

$$\begin{cases} (\alpha - \mu - \lambda^2)p + p_{xx} - pv + i(p_t + 2\lambda p_x) = 0, \\ 3v_{tt} - v_{xxxx} + 3(v^2)_{xx} + \beta v_{xx} = (p^2)_{xx} \end{cases} \tag{6}$$

So we derive

$$\begin{cases} (\alpha - \mu - \lambda^2)p + p_{xx} - pv = 0, \\ p_t + 2\lambda p_x = 0, \\ 3v_{tt} - v_{xxxx} + 3(v^2)_{xx} + \beta v_{xx} = (p^2)_{xx}. \end{cases} \tag{7}$$

we introduce a complex variable ξ , defined as

$$\xi = \delta x - \rho t,$$

So, above equation turns to the following system of ODE,

$$\begin{cases} (\alpha - \mu - \lambda^2)p + \delta^2 p'' - pv = 0, \\ -\rho p' + 2\delta\lambda p' = 0, \\ (3\rho^2 + \beta\delta^2)v'' - \delta^4 v^{(4)} + 3\delta^2(3v^2 - p^2)'' = 0. \end{cases} \tag{8}$$

From the second equation of (8), we get

$$\rho = 2\delta\lambda$$

So

$$\begin{cases} (\alpha - \mu - \lambda^2)p + \delta^2 p'' - pv = 0, \\ \delta^2(12\lambda^2 + \beta)v'' - \delta^4 v^{(4)} + 3\delta^2(3v^2 - p^2)'' = 0. \end{cases} \tag{9}$$

By twice integration from second equation (9), we derive

$$\begin{cases} (\kappa - \mu - \lambda^2)p + \delta^2 p'' - pv = 0, \\ (12\lambda^2 + \omega)v - \delta^2 v'' + 3v^2 - p^2 = 0. \end{cases} \quad (10)$$

Homogeneous balance between linear and nonlinear terms in each equation of (10) leads to

$$\begin{cases} M = 2, \\ N = 2. \end{cases} \quad (11)$$

So (4) convert to

$$\begin{cases} p(\xi) = a_2 \left(\frac{\varphi'}{\varphi} \right)^2 + a_1 \left(\frac{\varphi'}{\varphi} \right) + a_0, \quad a_2 \neq 0, \\ v(\xi) = b_2 \left(\frac{\varphi'}{\varphi} \right)^2 + b_1 \left(\frac{\varphi'}{\varphi} \right) + b_0, \quad b_2 \neq 0, \end{cases} \quad (12)$$

Putting (12) in the first equation of (10) and equalizing coefficient $\varphi^{-i}(\xi)$ to zero, leads to

$$-a_0\lambda^2 + a_0\kappa - a_0\mu - a_0b_0 = 0, \quad (13)$$

$$\delta^2 a_1 \varphi''' + (-a_0 b_1 - a_1 b_0 - a_1(\lambda^2 - \kappa + \mu))\varphi' = 0, \quad (14)$$

$$2\delta^2 a_2 \varphi''' \varphi' - 3\delta^2 a_1 \varphi'' \varphi' + 2\delta^2 a_2 \varphi''^2 - (a_0 b_2 + a_1 b_1 + a_2 b_0 + a_2(\lambda^2 - \kappa + \mu))\varphi'^2 = 0, \quad (15)$$

$$-10\delta^2 a_2 \varphi'' \varphi'^2 + (2\delta^2 a_1 - a_1 b_2 - a_2 b_1)\varphi'^3 = 0, \quad (16)$$

$$(6\delta^2 a_2 - a_2 b_2)\varphi'^4 = 0. \quad (17)$$

And by placing (12) in the second equation of (10), we derive

$$3b_0^2 - a_0^2 + 12b_0\lambda^2 + \omega b_0 = 0, \quad (18)$$

$$-\delta^2 b_1 \varphi''' + (12b_1\lambda^2 + \omega b_1 + 6b_1 b_0 - 2a_0 a_1)\varphi' = 0, \quad (19)$$

$$-2\delta^2 b_2 \varphi''' \varphi' + 3\delta^2 b_1 \varphi'' \varphi' - 2\delta^2 b_2 \varphi''^2 + (12b_2\lambda^2 + \omega b_2 + 6b_2 b_0 - 2a_0 a_2 - a_1^2 + 3b_1^2)\varphi'^2 = 0, \quad (20)$$

$$10\delta^2 b_2 \varphi'' \varphi'^2 + (-2\delta^2 b_1 + 6b_1 b_2 - 2a_1 a_2)\varphi'^3 = 0, \quad (21)$$

$$(-6\delta^2 b_2 - a_2^2 + 3b_3^2)\varphi'^4 = 0, \quad (22)$$

From Eq (17) and (22), we have

$$b_2 = 6\delta^2, a_2 = 6\delta^2\sqrt{2}. \quad (23)$$

Substituting (23) into (16) and (21) leads to

$$-60\delta^2\sqrt{2}\varphi'' - (4a_1 + 6\sqrt{2}b_1)\varphi' = 0, \quad (24)$$

$$60\delta^2\varphi'' + (34b_1 - 12a_1\sqrt{2})\varphi' = 0, \quad (25)$$

From (24), we have

$$\frac{\varphi''}{\varphi'} = -\frac{\sqrt{2}a_1 + 3b_1}{30\delta^2}$$

So,

$$\varphi' = Ae^{-\frac{\sqrt{2a_1+3b_1}}{30\delta^2}\xi} \tag{26}$$

By replacing (26) into (25), It can be result

$$a_1 = \sqrt{2} b_1, \tag{27}$$

so, (26) convert to $\varphi' = Ae^{-\frac{b_1}{6\delta^2}\xi}$. Solving Eqs (13) and (18), leads to

Case 1: $a_0 = 0, b_0 = 0$

Case 2: $a_0 = 0, b_0 = \frac{-1}{3}(12\lambda^2 + \omega)$

Case 3: $b_0 = -\lambda^2 + k - \mu$, and $3b_0^2 - a_0^2 + 12b_0\lambda^2 + \omega b_0 = 0$,

Case 1: If $a_0 = 0, b_0 = 0$, By placing (23), (26) and (27), into (14),(15),(19) and (20), we derive

$$\begin{cases} 36\delta^2(-\lambda^2 + \kappa - \mu) + b_1^2 = 0, \\ 36\delta^2(12\lambda^2 + \omega) - b_1^2 = 0, \end{cases} \tag{28}$$

Solving above system leads to

$$\mu = 11\lambda^2 + \kappa + \omega, \quad b_1 = \pm 6\delta\sqrt{12\lambda^2 + \omega} \tag{29}$$

Therefore,

$$\varphi' = Ae^{\mp\frac{\sqrt{12\lambda^2+\omega}}{\delta}\xi} \quad \text{and} \quad \varphi = \mp\frac{A\delta}{\sqrt{12\lambda^2 + \omega}} e^{\mp\frac{\sqrt{12\lambda^2+\omega}}{\delta}\xi} + B$$

Placement φ and φ' into Eq. (12), the general solution of Eq. (5) has been resulted

$$\begin{cases} u(x,t) = \left(\frac{6\sqrt{2}\delta^2 A^2 e^{\mp 2\sqrt{12\lambda^2+\omega}(x-2\lambda)t}}{\left(\mp\frac{A\delta}{\sqrt{12\lambda^2+\omega}} e^{\mp\sqrt{12\lambda^2+\omega}(x-2\lambda)t} + B\right)^2} \pm \frac{6\sqrt{2}\delta\sqrt{12\lambda^2+\omega} A e^{\mp\sqrt{12\lambda^2+\omega}(x-2\lambda)t}}{\mp\frac{A\delta}{\sqrt{12\lambda^2+\omega}} e^{\mp\sqrt{12\lambda^2+\omega}(x-2\lambda)t} + B} \right) e^{i(\lambda x + (11\lambda^2 + \kappa + \omega)t)} \\ v(x,t) = \frac{6\delta^2 A^2 e^{\mp 2\sqrt{12\lambda^2+\omega}(x-2\lambda)t}}{\left(\mp\frac{A\delta}{\sqrt{12\lambda^2+\omega}} e^{\mp\sqrt{12\lambda^2+\omega}(x-2\lambda)t} + B\right)^2} \pm \frac{6\delta\sqrt{12\lambda^2+\omega} A e^{\mp\sqrt{12\lambda^2+\omega}(x-2\lambda)t}}{\mp\frac{A\delta}{\sqrt{12\lambda^2+\omega}} e^{\mp\sqrt{12\lambda^2+\omega}(x-2\lambda)t} + B}, \end{cases} \tag{30}$$

The plots of above solution for some value of parameters illustrated in Fig. 1.

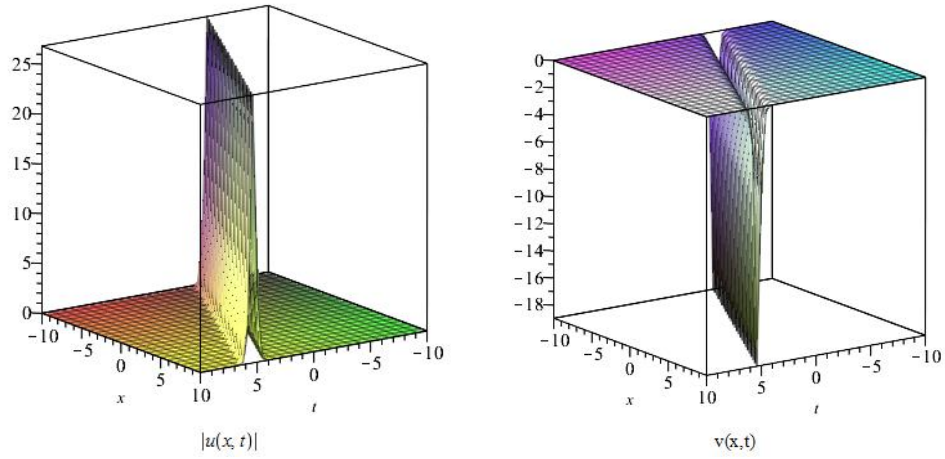


Fig.1: plots of solutions (30) for $A = 1, \delta = 1, B = -4, \lambda = 1, \omega = 1$ and $\kappa = 1$.

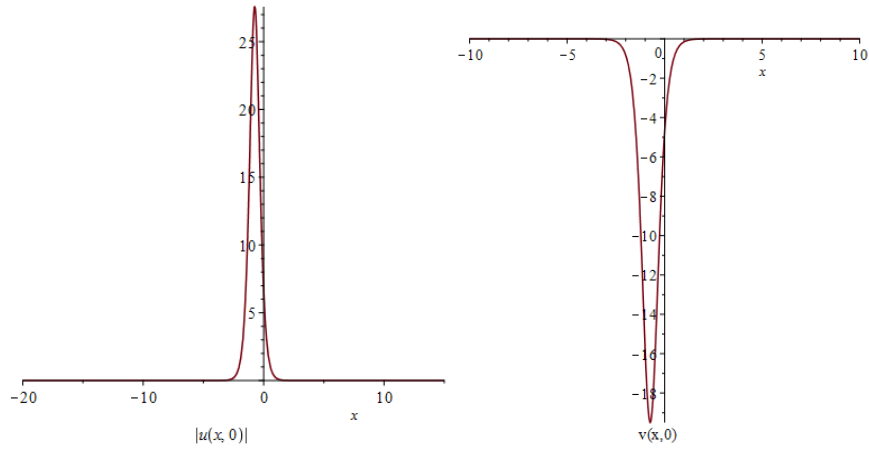


Fig.2: plots of solutions (30) for $A = 1, \delta = 1, B = -4, \lambda = 1, \omega = 1, t = 0$ and $\kappa = 1$.

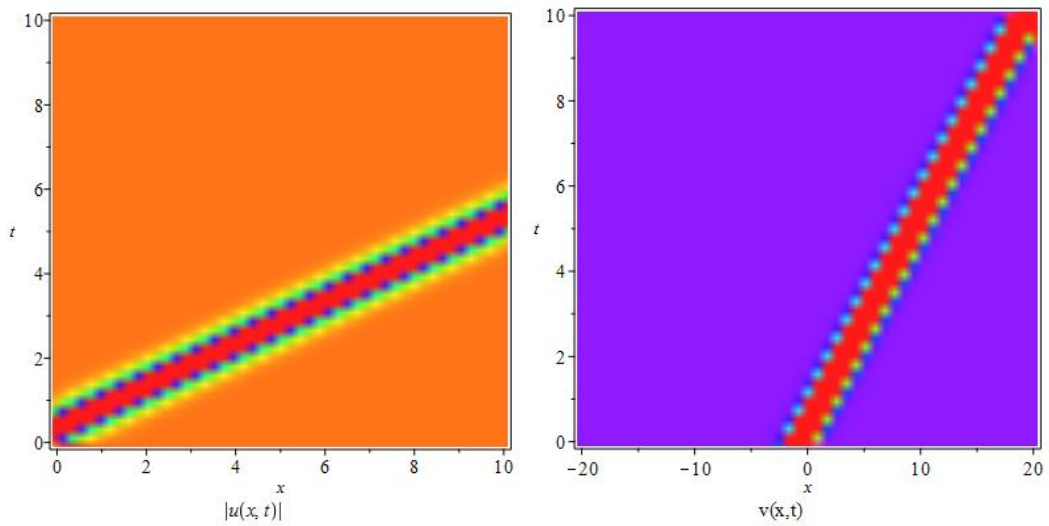


Fig.3: densityplots of solutions (30) for $A = 1, \delta = 1, B = -4, \lambda = 1, \omega = 1, t = 0$ and $\kappa = 1$.

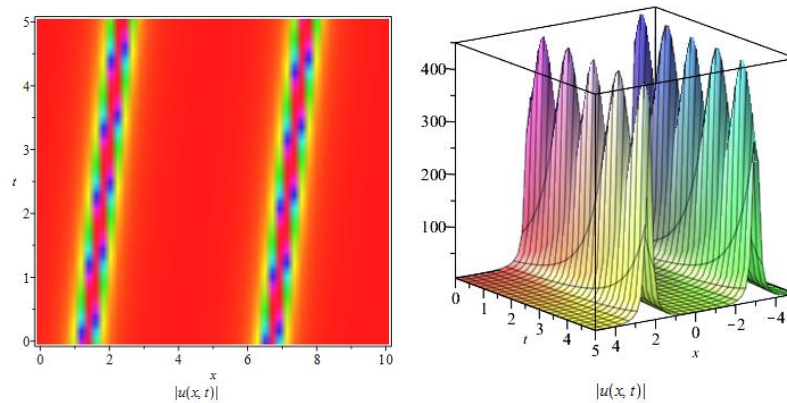


Fig.4 : densityplots and plots of $|u(x,t)|$ in (30) for $A=1, \delta=1, B=-1, \lambda=0.1, \omega=-1.5$, and $\kappa=1$.

Case 2: If $a_0=0, b_0 = \frac{-1}{3}(12\lambda^2 + \omega)$, By placing (23), (26) and (27), into (14),(15),(19) and (20),

we derive

$$\begin{cases} 36\delta^2(-\lambda^2 + \kappa - \mu) + 12\delta^2(12\lambda^2 + \omega) + b_1^2 = 0, \\ 36\delta^2(12\lambda^2 + \omega) + b_1^2 = 0, \end{cases} \tag{31}$$

Solving above system leads to

$$\mu = -9\lambda^2 + \kappa - \frac{2}{3}\omega, \quad b_1 = \pm 6\delta\sqrt{-12\lambda^2 - \omega} \tag{32}$$

Therefore,

$$\varphi' = Ae^{\mp \frac{\sqrt{-12\lambda^2 - \omega}}{\delta} \xi} \quad \text{and} \quad \varphi = \mp \frac{A\delta}{\sqrt{12\lambda^2 + \omega}} e^{\mp \frac{\sqrt{-12\lambda^2 - \omega}}{\delta} \xi} + B$$

Placement φ and φ' into Eq. (12), the general solution of Eq. (5) has been resulted

$$\begin{cases} u(x,t) = \left[\frac{6\sqrt{2}\delta^2 A^2 e^{\mp 2\sqrt{-12\lambda^2 - \omega}(x-2\lambda)t}}{\left(\mp \frac{A\delta}{\sqrt{-12\lambda^2 - \omega}} e^{\mp \sqrt{-12\lambda^2 - \omega}(x-2\lambda)t} + B\right)^2} \pm \frac{6\sqrt{2}\delta\sqrt{-12\lambda^2 - \omega} A e^{\mp \sqrt{-12\lambda^2 - \omega}(x-2\lambda)t}}{\mp \frac{A\delta}{\sqrt{-12\lambda^2 - \omega}} e^{\mp \sqrt{-12\lambda^2 - \omega}(x-2\lambda)t} + B} \right] e^{i(\lambda x + (-9\lambda^2 + \kappa - \frac{2}{3}\omega)t)} \\ v(x,t) = \frac{6\delta^2 A^2 e^{\mp 2\sqrt{-12\lambda^2 - \omega}(x-2\lambda)t}}{\left(\mp \frac{A\delta}{\sqrt{-12\lambda^2 - \omega}} e^{\mp \sqrt{-12\lambda^2 - \omega}(x-2\lambda)t} + B\right)^2} \pm \frac{6\delta\sqrt{-12\lambda^2 - \omega} A e^{\mp \sqrt{-12\lambda^2 - \omega}(x-2\lambda)t}}{\mp \frac{A\delta}{\sqrt{-12\lambda^2 - \omega}} e^{\mp \sqrt{-12\lambda^2 - \omega}(x-2\lambda)t} + B} - \frac{1}{3}(12\lambda^2 + \omega), \end{cases} \tag{33}$$

The plots of above solution for some value of parameters illustrated in Fig. 2.

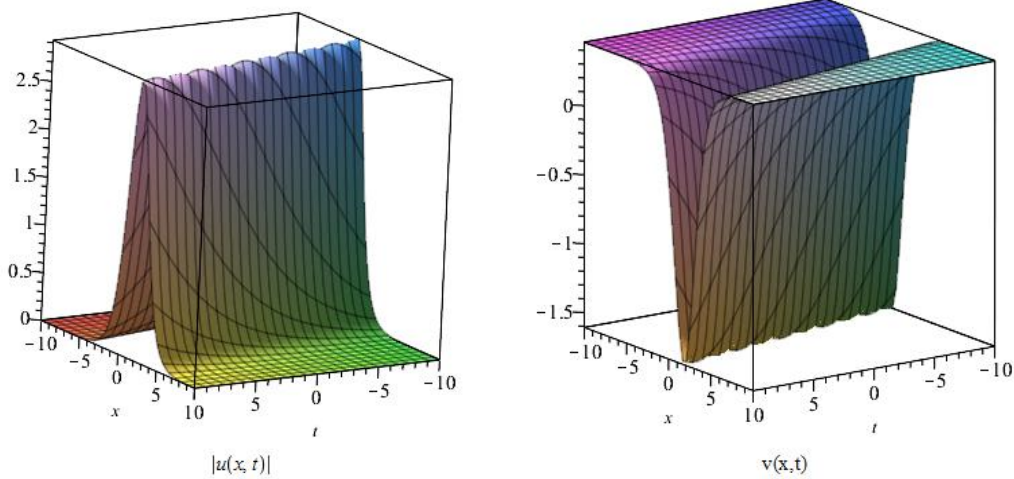


Fig.5 : plots of solutions (33) for $A = 1, \delta = 1, B = -1, \lambda = 0.1, \omega = -1.5$ and $\kappa = 1$.

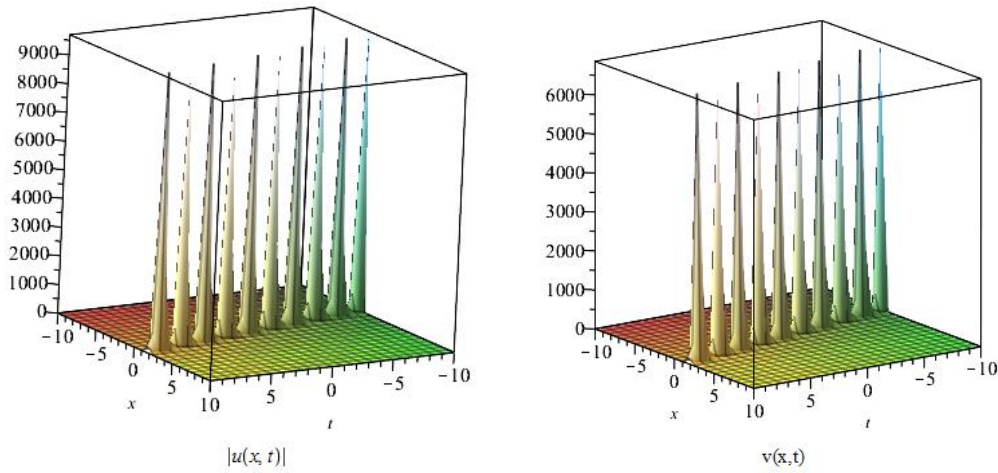


Fig.6 : plots of solutions (33) for $A = 1, \delta = 1, B = 1, \lambda = 0.1, \omega = -1.5$ and $\kappa = 1$.

Case 3: lets assume that

$$\begin{cases} b_0 = -\lambda^2 + k - \mu, \\ 3b_0^2 - a_0^2 + 12b_0\lambda^2 + \omega b_0 = 0, \end{cases} \tag{34}$$

Placing (23), (26) and (27), into (14),(15),(19) and (20) leads to

$$\begin{cases} 36\delta^2(-a_0 - \sqrt{2}b_0 - \sqrt{2}(\lambda^2 - \kappa + \mu)) + \sqrt{2}b_1^2 = 0, \\ 36\delta^2(12\lambda^2 + \omega - 2a_0\sqrt{2} + 6b_0) - b_1^2 = 0, \end{cases} \tag{35}$$

Solving (34) and (35) result in

case 3.1:

$$\mu = 11\lambda^2 + \kappa + \omega, \quad b_1 = \pm 6\delta\sqrt{-12\lambda^2 - \omega}, \quad b_0 = -12\lambda^2 - \omega, \quad a_0 = -\sqrt{2}(12\lambda^2 + \omega)$$

Therefore,

$$\varphi' = Ae^{\mp \frac{\sqrt{-12\lambda^2 - \omega}}{\delta} \xi} \text{ and } \varphi = \mp \frac{A\delta}{\sqrt{12\lambda^2 + \omega}} e^{\mp \frac{\sqrt{-12\lambda^2 - \omega}}{\delta} \xi} + B$$

Placement φ and φ' into Eq. (12), the general solution of Eq. (5) has been resulted

$$\left\{ \begin{aligned} u(x,t) &= \left(\frac{6\sqrt{2}\delta^2 A^2 e^{\mp 2\sqrt{-12\lambda^2 - \omega}(x-2\lambda)t}}{\left(\mp \frac{A\delta}{\sqrt{-12\lambda^2 - \omega}} e^{\mp \sqrt{-12\lambda^2 - \omega}(x-2\lambda)t} + B \right)^2} \pm \left(\frac{6\sqrt{2}\delta \sqrt{-12\lambda^2 - \omega} A e^{\mp \sqrt{-12\lambda^2 - \omega}(x-2\lambda)t}}{\mp \frac{A\delta}{\sqrt{-12\lambda^2 - \omega}} e^{\mp \sqrt{-12\lambda^2 - \omega}(x-2\lambda)t} + B} \right) - \sqrt{2}(12\lambda^2 + \omega) \right) e^{i(\lambda x + (11\lambda^2 + \kappa + \omega)t)} \\ v(x,t) &= \frac{6\delta^2 A^2 e^{\mp 2\sqrt{-12\lambda^2 - \omega}(x-2\lambda)t}}{\left(\mp \frac{A\delta}{\sqrt{-12\lambda^2 - \omega}} e^{\mp \sqrt{-12\lambda^2 - \omega}(x-2\lambda)t} + B \right)^2} \pm \left(\frac{6\delta \sqrt{-12\lambda^2 - \omega} A e^{\mp \sqrt{-12\lambda^2 - \omega}(x-2\lambda)t}}{\mp \frac{A\delta}{\sqrt{-12\lambda^2 - \omega}} e^{\mp \sqrt{-12\lambda^2 - \omega}(x-2\lambda)t} + B} \right) - 12\lambda^2 - \omega, \end{aligned} \right. \quad (36)$$

case 3.2:

$$\mu = -9\lambda^2 + \kappa - \frac{2}{3}\omega, \quad b_1 = \pm 6\delta\sqrt{12\lambda^2 + \omega}, \quad b_0 = 8\lambda^2 + \frac{2}{3}\omega, \quad a_0 = \sqrt{2}(12\lambda^2 + \omega)$$

Therefore,

$$\varphi' = Ae^{\pm \frac{\sqrt{12\lambda^2 + \omega}}{\delta} \xi} \text{ and } \varphi = \pm \frac{A\delta}{\sqrt{12\lambda^2 + \omega}} e^{\pm \frac{\sqrt{12\lambda^2 + \omega}}{\delta} \xi} + B$$

Placement φ and φ' into Eq. (12), the general solution of Eq. (5) has been resulted

$$\left\{ \begin{aligned} u(x,t) &= \left(\frac{6\sqrt{2}\delta^2 A^2 e^{\mp 2\sqrt{12\lambda^2 + \omega}(x-2\lambda)t}}{\left(\mp \frac{A\delta}{\sqrt{12\lambda^2 + \omega}} e^{\mp \sqrt{12\lambda^2 + \omega}(x-2\lambda)t} + B \right)^2} \pm \left(\frac{6\sqrt{2}\delta \sqrt{12\lambda^2 + \omega} A e^{\mp \sqrt{12\lambda^2 + \omega}(x-2\lambda)t}}{\mp \frac{A\delta}{\sqrt{12\lambda^2 + \omega}} e^{\mp \sqrt{12\lambda^2 + \omega}(x-2\lambda)t} + B} \right) + \sqrt{2}(12\lambda^2 + \omega) \right) e^{i(\lambda x + (-9\lambda^2 + \kappa - \frac{2}{3}\omega)t)} \\ v(x,t) &= \frac{6\delta^2 A^2 e^{\mp 2\sqrt{12\lambda^2 + \omega}(x-2\lambda)t}}{\left(\mp \frac{A\delta}{\sqrt{12\lambda^2 + \omega}} e^{\mp \sqrt{12\lambda^2 + \omega}(x-2\lambda)t} + B \right)^2} \pm \left(\frac{6\delta \sqrt{12\lambda^2 + \omega} A e^{\mp \sqrt{12\lambda^2 + \omega}(x-2\lambda)t}}{\mp \frac{A\delta}{\sqrt{12\lambda^2 + \omega}} e^{\mp \sqrt{12\lambda^2 + \omega}(x-2\lambda)t} + B} \right) + 8\lambda^2 + \frac{2}{3}\omega. \end{aligned} \right. \quad (37)$$

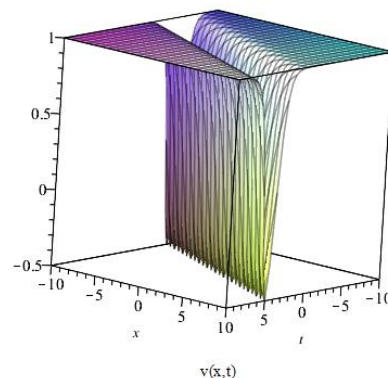
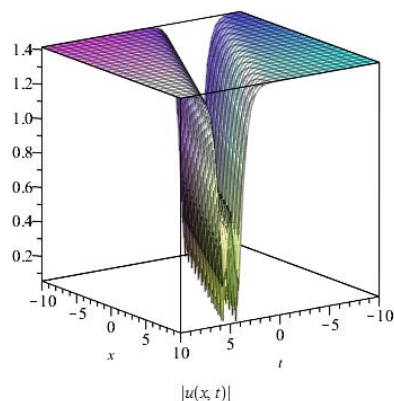


Fig.7 : plots of solutions (36) for $A = 1, \delta = 1, B = -1, \lambda = 1, \omega = -13$ and $\kappa = 1$.

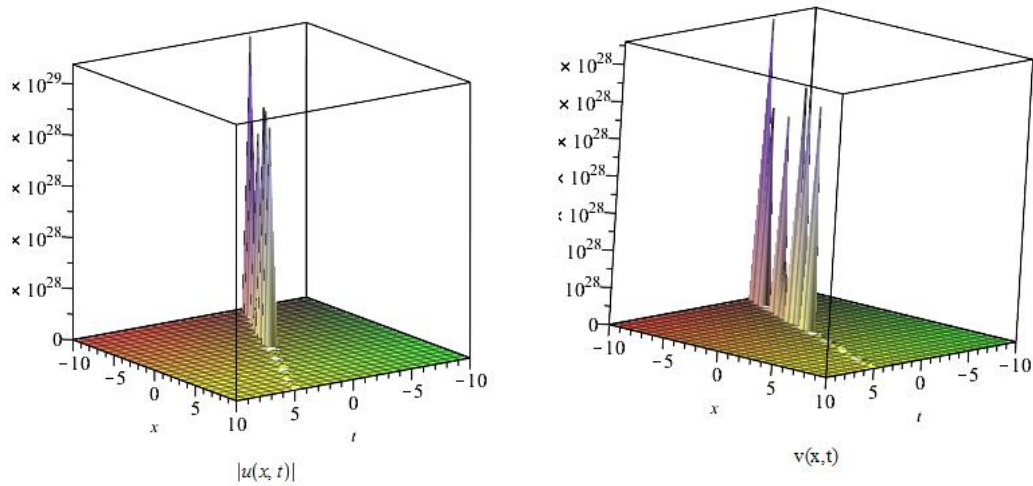


Fig.8 : plots of solutions (36) for $A = 1, \delta = 1, B = 1, \lambda = 1, \omega = -13$ and $\kappa = 1$.

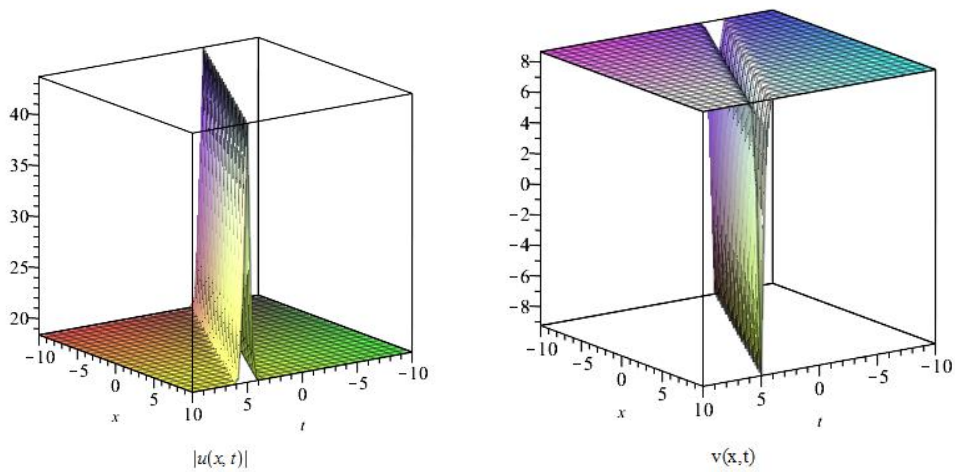


Fig.9 : plots of solutions (37) for $A = -2, \delta = 1, B = 1, \lambda = 1, \omega = 1$ and $\kappa = 1$.

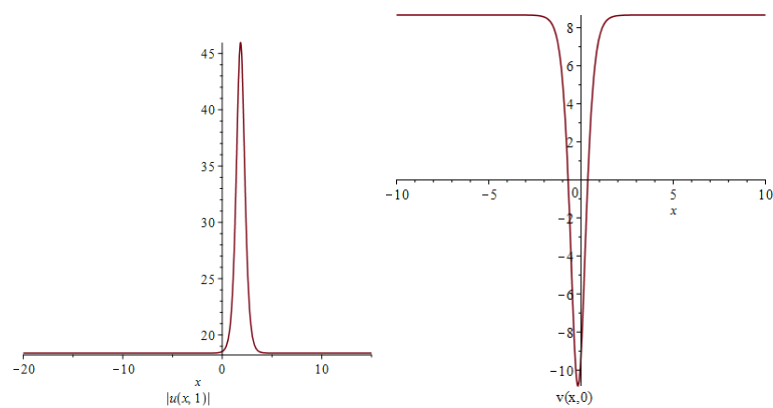


Fig.10 : plots of solutions (37) for $A = -2, \delta = 1, B = 1, \lambda = 1, \omega = 1, t = 1$ and $\kappa = 1$.

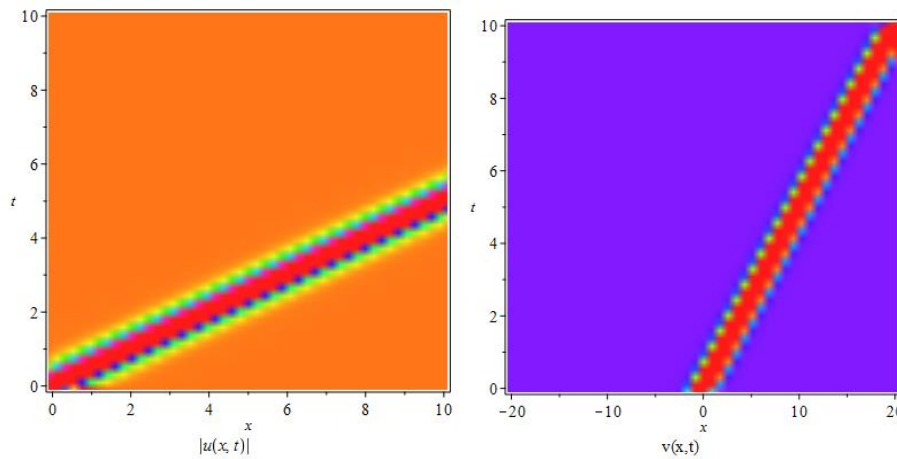


Fig.11 : densityplots of solutions (37) for $A = -2, \delta = 1, B=1, \lambda=1, \omega=1,$ and $\kappa=1.$

4. Application CMSE method to a Boussinesq-type coupled system

Let's consider the following Boussinesq-type coupled system,

$$\begin{cases} u_t + v_{xx} + v v_x - u u_x = 0, \\ v_t - u_{xx} + u v_x + v u_x = 0. \end{cases} \tag{38}$$

By using $\xi = \delta x - \rho t,$ it has been result

$$\begin{cases} -\rho u' + \delta^2 v'' + \delta v v' - \delta u u' = 0, \\ -\rho v_t - \delta^2 u'' + \delta u v' + \delta v u' = 0. \end{cases} \tag{39}$$

Presumably $u(\xi)$ can be illustrated in the form (4). Equating $v'', v v'$ and $u u'$, leads to $m = n = 1.$ So

$$\begin{cases} u(\xi) = a_1 \left(\frac{\varphi'}{\varphi} \right) + a_0, \quad a_1 \neq 0, \\ v(\xi) = b_1 \left(\frac{\varphi'}{\varphi} \right) + b_0, \quad b_1 \neq 0, \end{cases} \tag{40}$$

Putting (40) in (39) and equalizing coefficient $\varphi^{-i}(\xi)$ to zero, leads to

$$\delta^2 b_1 \varphi''' + (\delta b_0 b_1 - \delta a_0 a_1 - \rho a_1) \varphi'' = 0, \tag{41}$$

$$(-3\delta^2 b_1 - \delta a_1^2 + \delta b_1^2) \varphi' \varphi'' + (-\delta b_0 b_1 + \delta a_0 a_1 + \rho a_1) \varphi'^2 = 0, \tag{42}$$

$$(2\delta^2 b_1 + \delta a_1^2 - \delta b_1^2) \varphi'^3 = 0, \tag{43}$$

$$-\delta^2 a_1 \varphi''' + (\delta a_0 b_1 + \delta b_0 a_1 - \rho b_1) \varphi'' = 0, \tag{44}$$

$$(3\delta^2 a_1 + 2\delta b_1 a_1) \varphi' \varphi'' + (-\delta a_0 b_1 - \delta b_0 a_1 + \rho b_1) \varphi'^2 = 0, \tag{45}$$

$$(-2\delta^2 a_1 - 2\delta b_1 a_1) \varphi'^3 = 0, \tag{46}$$

By solving Eq. (46), we drive

$$b_1 = -\delta, \tag{47}$$

By substituting above result into (43), we get

$$a_1 = \sqrt{3} \delta. \quad (48)$$

Replacing (47) and (48) into (42), leads to

$$\delta^3 \varphi'' + (\delta^2 b_0 + \sqrt{3} \delta^2 a_0 + \sqrt{3} \delta \rho) \varphi' = 0, \quad (49)$$

By solving Eq. (49), we obtain

$$\varphi' = A e^{-\frac{\delta b_0 + \sqrt{3} \delta a_0 + \sqrt{3} \rho}{\delta^2} \xi}, \quad (50)$$

Where A is a arbitrary constant. By substituting (47), (48) and (50) into eqs. (41), (44) and (45), we get

$$2\sqrt{3} \delta^2 b_0 + 2\delta^2 a_0 + 4\rho \delta = 0, \quad (51)$$

By solving the above equation, we derive

$$\rho = -\frac{1}{2} \delta (\sqrt{3} b_0 + a_0),$$

Therefore, we have

$$\varphi' = A e^{\frac{b_0 - \sqrt{3} a_0}{\delta} \xi},$$

So,

$$\varphi = \frac{A \delta}{b_0 - \sqrt{3} a_0} e^{\frac{b_0 - \sqrt{3} a_0}{\delta} \xi} + B,$$

placement φ and φ' into Eq. (40), the general solution of Eq. (38) has been obtained

$$\left\{ \begin{array}{l} u(\xi) = \frac{\sqrt{3} \delta A e^{(b_0 - \sqrt{3} a_0)(x + \frac{1}{2}(\sqrt{3} b_0 + a_0)t)}}{\frac{A \delta}{b_0 - \sqrt{3} a_0} e^{(b_0 - \sqrt{3} a_0)(x + \frac{1}{2}(\sqrt{3} b_0 + a_0)t)} + B} + a_0, \\ v(\xi) = \frac{-\delta A e^{(b_0 - \sqrt{3} a_0)(x + \frac{1}{2}(\sqrt{3} b_0 + a_0)t)}}{\frac{A \delta}{b_0 - \sqrt{3} a_0} e^{(b_0 - \sqrt{3} a_0)(x + \frac{1}{2}(\sqrt{3} b_0 + a_0)t)} + B} + b_0, \end{array} \right. \quad (52)$$

The plots of above solution for some value of parameters illustrated in Fig. 3.

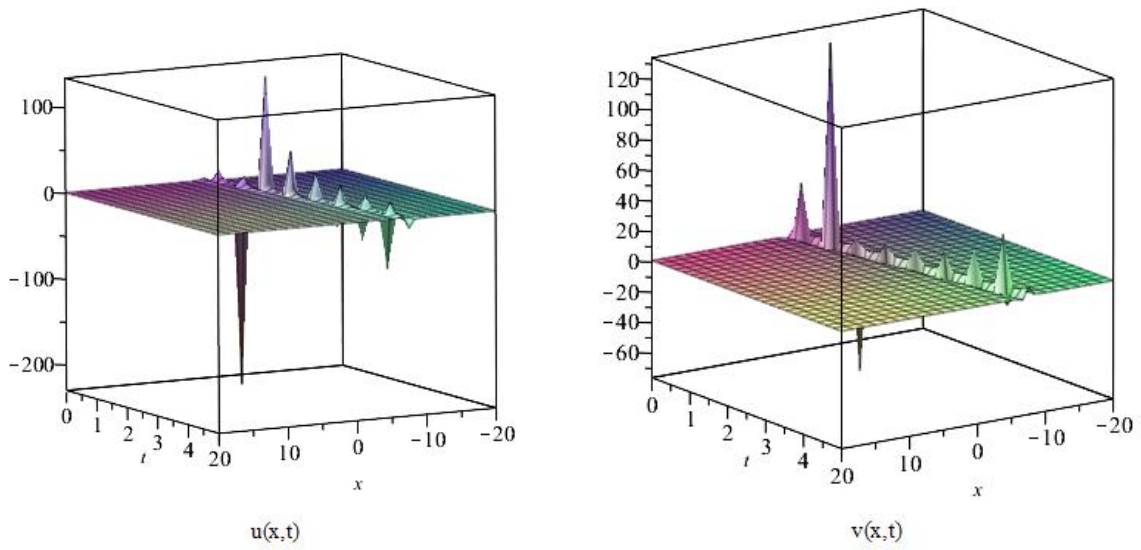


Fig.12: plots of solutions (52) for $A=1, \delta=1, B=2, a_0=1, b_0=1$.

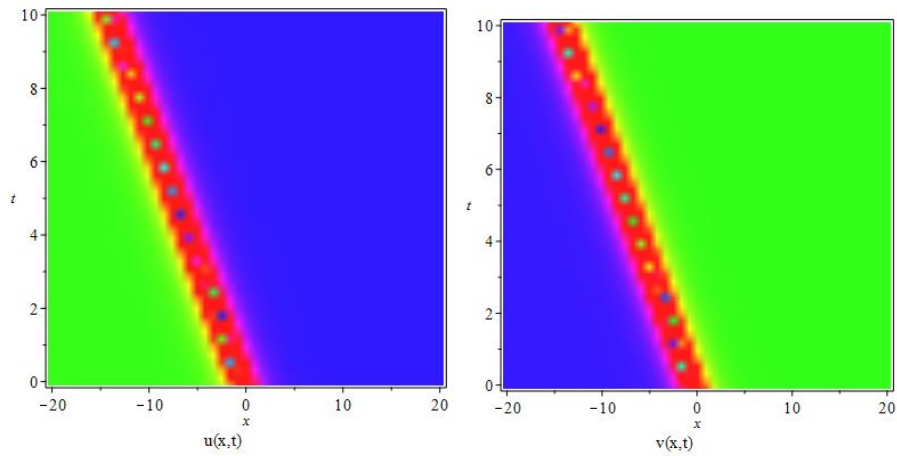


Fig.13: densityplots of solutions (52) for $A=1, \delta=1, B=2, a_0=1, b_0=1$.

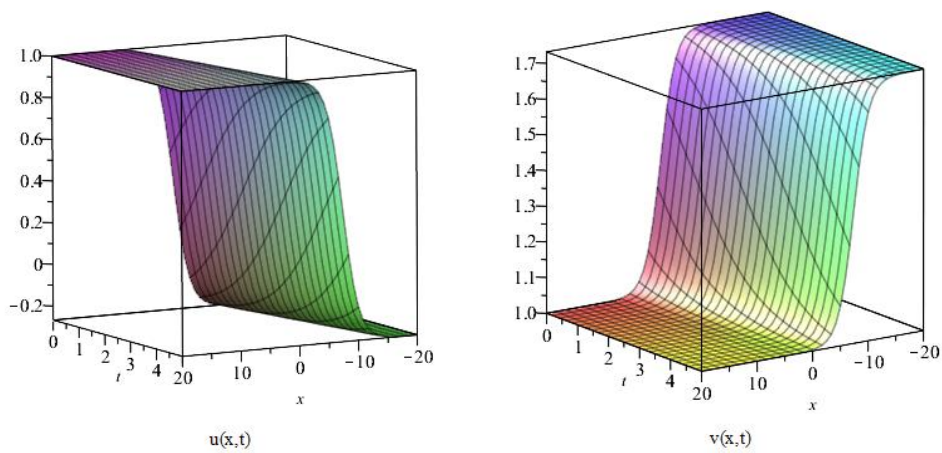


Fig.14: plots of solutions (52) for $A=1, \delta=1, B=-2, a_0=1, b_0=1$.

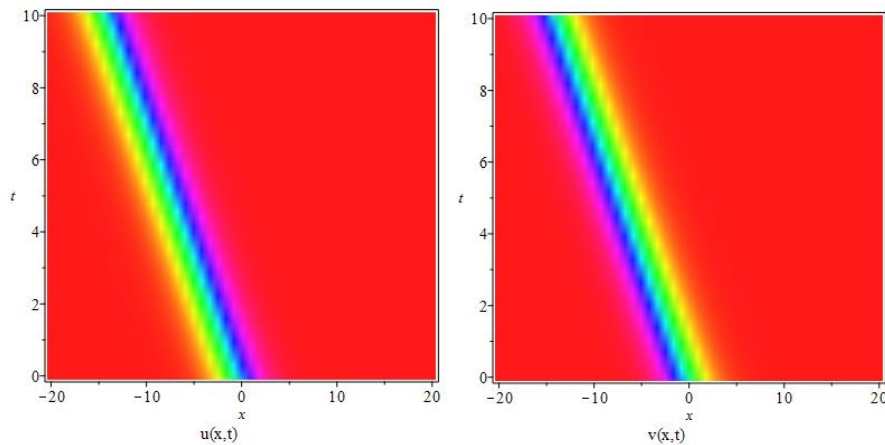


Fig.15 : densityplots of solutions (52) for $A = 1, \delta = 1, B = -2, a_0 = 1, b_0 = 1$.

5. Conclusions

In the current paper, new modification of a well-known method has been expanded to attain the generalized solutions of a system of PDE called coupled modified simple equation. The introduced method is applied to search the exact answers of the coupled generalized Schrödinger–Boussinesq system and the Boussinesq-type coupled system. The outcomes indicate that the coupled modified simple equation method is a powerful method for solving coupled system. Among the main advantages of the presented methods the following features can be considered precisely include simplicity, directness, reliability and being computerizable. As it was stated previously, the primary merits of procedure used in this research over other analytical methods is that $\varphi(\xi)$ is not satisfied in any special equation. So, the probability to get new and varied solutions by using this method increases outstandingly.

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