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SOME RESULTS ON DOMINATION IN THE GENERALIZED TOTAL GRAPH OF A COMMUTATIVE RING

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Abstract. Let R be a commutative ring with nonzero identity and H be a nonempty proper subset of R such that R/H is a saturated multiplicatively closed subset of R. Anderson and Badawi [4] introduced the generalized total graph of R as an undirected simple graph $GT_H(R)$ with vertex set as R and any two distinct vertices x and y are adjacent if and only if $x + y \in H$. The main objective of this paper is to study the domination properties of the graph $GT_H(R)$. We determine the domination number of $GT_H(R)$ and its induced subgraphs $GT_H(H)$ and $GT_H(R/H)$. We establish a relationship between the domination number of $GT_H(R)$ and the same of $GT_H(R/H)$. We also establish a relationship between diameter and domination number of $GT_H(R/H)$. In addition, we obtain the bondage number of $GT_H(R)$. Finally, a relationship between girth and bondage number of $GT_H(R/H)$ has been established.

1. Introduction

The study of algebraic structures by associating a graph has become an exciting research topic in the last two decades, leading to many fascinating results and questions. Many fundamental papers assigning graphs to rings and modules have appeared recently, for instance see,[1,2,5,8,17]. In 2008, Anderson and Badawi [3] have introduced the total graph of a commutative ring and later on this notion has been

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generalised in many different ways (see [4,7,11,12]).

The concepts of dominating sets and domination numbers play a vital role in graph theory. Dominating sets are the focus of many books of graph theory, for example see [13] and [14]. But not much research has been done on the domination properties of graphs associated to algebraic structures in terms of algebraic properties. However, some works on domination of graphs associated to rings and modules have appeared recently, for instance see,[10,16,18,19].

The study of multiplicative prime subsets and multiplicatively closed subsets of a ring is one of the important aspects of ring theory. Let R be a commutative ring with nonzero identity and H be a nonempty proper subset of R such that R/H is a saturated multiplicatively closed subset of R. Anderson and Badawi [4] have generalised the notion of total graph [3] by introducing the generalized total graph of R, denoted by $GT_H(R)$, to be an undirected simple graph with all elements of R as vertices and for distinct $x, y \in R$, the vertices x and y are adjacent if and only if $x + y \in H$. Let $GT_H(H)$ be the (induced) subgraph of $GT_H(R)$ with vertices H and let $GT_H(R/H)$ be the (induced) subgraph of $GT_H(R)$ with vertices R/H. They have studied the characteristics of $GT_H(R)$ and its two induced subgraphs $GT_H(H)$ and $GT_H(R/H)$ by considering two cases, H is an ideal of R or is not an ideal of R.

In this paper an attempt has been made to study the domination properties of the graph $GT_H(R)$ when H is a prime ideal of R . The domination number of $GT_H(R)$ and its induced subgraphs $GT_H(H)$ and $GT_H(R/H)$ has been determined. A relationship between the domination number of $GT_H(R)$ and the same of $GT_H(R/H)$ has been established. A relationship between diameter and domination number of $GT_H(R/H)$ has also been established. In addition, the bondage number of $GT_H(R)$ has been determined. Finally, a relationship between girth and bondage number of $GT_H(R/H)$ has been established.

2. Preliminaries

In this section, we recall the definitions, concepts and results which is needed in the later sections.

Throughout this paper R is commutative ring with non-zero identity. A non-empty proper subset H of R is said to be a multiplicative-prime subset of R if the following two conditions hold:(i) $ab \in H$ for every $a \in H$ and $b \in R$; (ii) if $ab \in H$ for $a, b \in R$, then either $a \in H$ or

$b \in H$. Again a multiplicative closed set is a subset S of a ring R such that the following two conditions holds: (i) $1 \in S$; (ii) $xy \in S$ for all $x, y \in S$. An ideal P of a commutative ring R is prime if $ab \in P$ implies either $a \in P$ or $b \in P$ for $a, b \in R$. If H is prime ideal of R or H is union of prime ideals of R then H is multiplicative prime subset of R. Also, we have H is multiplicative prime subset of R if and only if R/H is saturated multiplicative closed subset of R. Hence H is a multiplicative- prime subset of R iff H is union of prime ideal of R. For any undefined terminology of ring theory we refer to [6,15].

By a graph G, we mean a simple undirected graph without loops. For a graph G, we denote by $V(G)$ and $E(G)$ the set of all vertices and edges respectively. We recall that a graph is finite if both $V(G)$ and $E(G)$ are finite sets, and we use the symbol $|G|$ to denote the number of vertices in the graph G. We say that G is a null graph if $E(G) = \phi$. Two vertices x and y of a graph G are connected if there is a path in G connecting them. Also, a graph G is connected if there is a path between any two distinct vertices. A graph G is disconnected if it is not connected. A graph G is complete if any two distinct vertices are adjacent. We denote the complete graph on n vertices by K_n. If the vertex set $V(G)$ of the graph G are partitioned into two non-empty disjoint sets X and Y of cardinality $|X| = m$ and $|Y| = n$, and two vertices are adjacent if and only if they are not in the same partite set, then G is called a bipartite graph. A graph G is called a complete bipartite graph if every vertex in X is connected to every vertex in Y. We denote the complete bipartite graph on m and n vertices by $K_{m,n}$. For vertices $x, y \in G$ one defines the distance $d(x, y)$, as the length of the shortest path between x and y, if the vertices $x, y \in G$ are connected and $d(x, y) = \infty$, if they are not. Then, the diameter of the graph G is

$$diam(G) = sup\{d(x, y)|x, y \in G\}.$$

The cycle is a closed path which begins and ends in the same vertex. The cycle of n vertices is denoted by C_n. The girth of the graph G,denoted by $gr(G)$ is the length of the shortest cycle in G and $gr(G) = \infty$ if G has no cycles.
For a subset $S \subseteq V$, $< S >$ denotes the subgraph of G induced by S. For a vertex $v \in V$, $deg(v)$ is the degree of the vertex v, $N(v) = \{u \in V \mid u \text{ is adjacent to } v\}$ and $N[v] = N(v) \cup \{v\}$. A subset S of V is called a dominating set if every vertex in $V - S$ is adjacent to atleast one vertex in S. The domination number $\gamma(G)$ of G is defined to be minimum cardinality of a dominating set in G and

such a dominating set is called γ-set of G. If G is a trivial graph, then $\gamma(G) = 0$. A graph G is called excellent if for every vertex $v \in V(G)$, there exists a γ-set S containing v. A domatic partition of G is a partition of $V(G)$ into dominating sets in G. The maximum number of classes of a domatic partition of G is called the domatic number of G and is denoted by $d(G)$. The bondage number $b(G)$ is the minimum number of edges whose removal increases the domination number. For basic definitions and results in domination we refer to [9,13] and for any undefined graph-theoretic terminology we refer to [9].

Now we summarize some results on domination number and bondage number of a graph which will be useful for the later sections.

Lemma 2.1:[8,13]

(i) If G is a graph of order n, then $1 \leq \gamma(G) \leq n$. A graph G of order n has domination number 1 if and only if G contains a vertex v of degree $n-1$; while $\gamma(G) = n$ if and only if $G \cong \overline{K_n}$.
(ii) $\gamma(K_n) = 1$ for a complete graph K_n, but the converse is not true, in general and $\gamma(\overline{K_n}) = n$ for a null graph $\overline{K_n}$.
(iii) Let G be a complete r-partite graph $(r \geq 2)$ with partite sets $V_1, V_2, ..., V_r$. If $|V_i| \geq 2$ for $1 \leq i \leq r$, then $\gamma(G) = 2$; because one vertex of V_1 and one vertex of V_2 dominate G. If $|V_i| = 1$ for some i, then $\gamma(G) = 1$.
(iv) $\gamma(K_{1,n}) = 1$ for a star graph $K_{1,n}$.
(v) If G is a partition of disjoint subgraphs $G_1, G_2, ..., G_k$, then $\gamma(G) = \gamma(G_1) + \gamma(G_2) + ... + \gamma(G_k)$.
(vi) Domination number of a bistar graph is 2; because the set consisting of two centres of the graph is a minimal dominating set.

Lemma 2.2:[13,14]

(i) If G is a simple graph of order n, then $1 \leq b(G) \leq n-1$.
(ii) $b(K_n) = n-1$ for a complete graph K_n, but the converse is not true, in general and $b(\overline{K_n}) = 0$ for a null graph $\overline{K_n}$.
(iii) Let G be a complete r-partite graph with partite sets $V_1, V_2, ..., V_r$. Then $b(G) = min\{|V_1|, |V_2|, ..., |V_r|\}$. In particular, $b(K_{m,n}) = min\{m, n\}$.
(iv) If G is a partition of disjoint subgraphs $G_1, G_2, ..., G_k$, then $b(G) = min\{b(G_1), b(G_2), ..., b(G_k)\}$.

3. Domination number of $GT_H(R)$ and induced subgraphs

In this section, an attempt has been made to study the domination properties of the graph $GT_H(R)$. In particular, the domination number of $GT_H(R)$ and its induced subgraphs $GT_H(R/H)$ and $GT_H(H)$ have been determined.

We begin with the following Theorem.

Theorem 3.1: (Theorem 2.1, [4]) Let H be a prime ideal of a commutative ring R. Then $GT_H(H)$ is a complete (induced) subgraph of $GT_H(R)$ and $GT_H(H)$ is disjoint from $GT_H(R/H)$. In particular, $GT_H(H)$ is connected and $GT_H(R)$ is never connected.
From the above theorem we can easily observe that $\gamma(GT_H(H)) = 1$. The next theorem gives a complete description of $GT_H(R)$. We allow α, β to be infinite, then of course $\beta - 1 = \dfrac{(\beta - 1)}{2} = \beta$.

Theorem 3.2: (Theorem 2.2, [4]) Let H be a prime ideal of a commutative ring R such that $|H| = \mu$ and $|R/H| = \eta$.

(1) If $2 = 1_R + 1_R \in H$ then $GT_H(R/H)$ is the union of $\eta - 1$ disjoint K_μ's.
(2) If $2 = 1_R + 1_R \notin H$ then $GT_H(R/H)$ is the union of $(\eta - 1)/2$ disjoint $K_{\mu,\mu}$'s.

Proposition 3.3: Let H be a prime ideal of a commutative ring R such that $|H| = \mu$ and $|R/H| = \eta$, then $\gamma(GT_H(R)) = \eta$.

Proof. Let us consider the following two cases for H .
Case 1: Suppose that $2 = 1_R + 1_R \in H$. Then we have from theorem 3.2(1) that the graph $GT_H(R/H)$ is the union $\eta - 1$ disjoint K_μ's which yields $\gamma(GT_H(R/H)) = \eta - 1$. But $GT_H(H)$ is complete by theorem 3.1, so $\gamma(GT_H(H)) = 1$. Also by theorem 3.1, $GT_H(H)$ is disjoint from $GT_H(\frac{R}{H})$. Consequently, $\gamma(GT_H(R)) = \gamma(GT_H(H)) + \gamma(GT_H(R/H)) = 1 + \eta - 1 = \eta$.
Case 2: Suppose that $2 = 1_R + 1_R \notin H$. Then again we have from theorem 3.2(2) that the graph $GT_H(R/H)$ is the union of $(\eta - 1)/2$ disjoint $K_{\mu,\mu}$'s which implies $\gamma(GT_H(R/H)) = (\eta - 1)/2 \times 2 = \eta - 1$. But $GT_H(H)$ is complete by theorem 3.1, so $\gamma(GT_H(H)) = 1$. Also, by theorem 3.1, $GT_H(H)$ is disjoint from $GT_H(R/H)$. Therefore, $\gamma(GT_H(R)) = \gamma(GT_H(H)) + \gamma(GT_H(R/H)) = 1 + (\eta - 1) = \eta$.

Example 1 : Let us consider the ring $R = \mathbb{Z}$ and H is a prime ideal of R. Then $GT_H(R/H)$ is complete if and only if $H = 2\mathbb{Z}$. Then $\gamma(GT_H(R/H)) = 1$.

Example 2 : Let us consider the ring $R = \mathbb{Z}$. Then clearly $H = 7\mathbb{Z}$ is a prime ideal of R, $2 \notin H$ and $R/H \cong \mathbb{Z}_7$. Therefore, $|R/H| = 7$. It can be easily observed that $GT_H(R/H)$ is union of $(7-1)/2 = 3$ disjoint $K_{1,1}$'s . Thus, $\gamma(GT_H(R/H)) = 3\gamma(K_{1,1}) = 3 \times 2 = 6$.

Proposition 3.4: Let R be an integral domain such that H is a prime ideal of R, then $\gamma(GT_H(R)) = |R| - 1$.

Proof. Since R is an integral domain, so $H = \{0\}$. therefore $|R/H| = |R| = \eta$(say) and $2 = 1_R + 1_R \notin H$. Then from theorem 3.2(2), we have the graph $GT_H(R/H)$ is the union of $(\eta - 1)/2$ disjoint $K_{1,1}$'s which implies $\gamma(GT_H(R/H)) = (\eta-1)/2 \times 2 = \eta - 1$. Thus, $\gamma(GT_H(R)) = \gamma(GT_H(R/H)) = \eta - 1 = |R| - 1$.

Example 3 : Let us consider the integral domain $R = \mathbb{Z}_5$. Then clearly $H = \{0\}$ is a prime ideal of R and $R/H \cong R$. Therefore, $|R/H| = |R| = 5$. It can be easily observed that $GT_H(R/H)$ is union of $(5-1)/2 = 2$ disjoint $K_{1,1}$'s . Thus, $\gamma(GT_H(R)) = \gamma(GT_H(R/H)) = 2\gamma(K_{1,1}) = 2 \times 2 = 4 = 5 - 1 = |R| - 1$.

Proposition 3.5: Let H be a prime ideal of a commutative ring R such that $|H| = \mu$ and $|R/H| = \eta$, then

(1) $GT_H(R)$ is excellent
(2) $d(GT_H(R)) = \mu$

Proof. Since by Proposition 3.3, we have $\gamma(GT_H(R)) = \eta$, so the results follow trivially.

Theorem 3.6:(Theorem 2.3, [4]) Let H be a prime ideal of a commutative ring R. Then

(1) $GT_H(R/H)$ is complete if and only if either $R/H \cong Z_2$ or $R \cong Z_3$.
(2) $GT_H(R/H)$ is connected if and only if either $R/H \cong Z_2$ or $R/H \cong Z_3$.
(3) $GT_H(R/H)$ (and hence $GT_H(H)$) and $GT_H(R)$ are totally disconnected if and only if $H = \{0\}$ (thus R is an integral domain) and $char(R) = 2$.

Theorem 3.7:(Theorem 2.5(1), [4]) Let H be a prime ideal of a commutative ring R . Then

(1) $diam(GT_H(R/H)) = 0$ if and only if $R \cong Z_2$.

(2) $diam(GT_H(R/H))) = 1$ if and only if either $R/H \cong Z_2$ and $R \ncong Z_2$ (*i.e.* $R/H \cong Z_2$ and $|H| \geq 2$), or $R \cong Z_3$.
(3) $diam(GT_H(R/H) = 2$ if and only if $R/H \cong Z_3$ and $R \ncong Z_3$ (*i.e.* $R/H \cong Z_3$ and $|H| \geq 2$).
(4) Otherwise, $diam(GT_H(R/H)) = \infty$.

We now establish a relationship between the domination number of $GT_H(R)$ and the same of $GT_H(R/H)$.

Proposition 3.8: Let H be a prime ideal of a commutative ring R. Then the following are equivalent:
(1) $\gamma(GT_H(R)) = 2$.
(2) $\gamma(GT_H(R/H)) = 1$.
(3) $R/H \cong Z_2$ or $R/H = R \cong Z_3$.

Proof. (1) $\Leftrightarrow$ (2): Since H is prime ideal of R, so by theorem 3.1, $GT_H(H)$ and $GT_H(R/H)$ are disjoint and $GT_H(H)$ is complete. Therefore, $\gamma(GT_H(H)) = 1$ and hence $\gamma(GT_H(R)) = \gamma(GT_H(H)) + \gamma(GT_H(R/H))$ which yields $\gamma(GT_H(R)) = 1 + \gamma(GT_H(R/H))$.

(2) $\Rightarrow$ (3): Suppose $\gamma(GT_H(R/H)) = 1$. Then clearly $GT_H(R/H)$ is connected. If $2 \in H$, then $\eta - 1 = 1$ and hence $\eta = 2$, where $\eta = |R/H|$, by theorem 3.2(1) . Thus $|R/H| = 2$ which gives $R/H \cong Z_2$.
If $2 \notin H$, then $(\eta - 1)/2 = 1$ and so $\eta = |R/H| = 3$, by theorem 3.2(2) . Also , by assumption, $\mu = |H| = 1$ and hence $H = \{0\}$. Thus $|R/H| = |R| = 3$ which implies $R/H = R \cong Z_3$.

(3) $\Rightarrow$ (2): Assume $R/H \cong Z_2$ or $R/H = R \cong Z_3$. Then by theorem 3.6(1), $GT_H(R/H)$ is complete and hence $\gamma(GT_H(R/H)) = 1$.

In the following a relationship between diameter and domination number of $GT_H(R/H)$ has been established.

Corollary 3.9: Let H be a prime ideal of a commutative ring R. Then
(1) $diam(GT_H(R/H)) = 1$ if and only if $\gamma(GT_H(R/H)) = 1$.
(2) $diam(GT_H(R/H)) = 2$ if and only if $\gamma(GT_H(R/H)) = 2$.

Proof. (1) It is clear by theorem 3.7(2) and proposition 3.8 .
(2) If $diam(GT_H(R/H)) = 2$, then $R/H \cong Z_3$ and $R \ncong Z_3$, by theorem 3.7(3) . Hence $GT_H(R/H)$ is connected, by theorem 3.6(2) . Therefore $GT_H(R/H)$ is a complete bipartite graph $K_{\mu,\mu}$ with $\mu \geq 2$. So $\gamma(GT_H(R/H))) = 2$.

Conversely, if $\gamma(GT_H(R/H))) = 2$, then $GT_H(R/H))$ is the union of two K_μ's or is a complete bipartite graph $K_{\mu,\mu}$ with $\mu \geq 2$, by theorem 3.2(1) and theorem 3.2(2) respectively . So $\eta - 1 = 2$ or $(\eta - 1)/2 = 1$. In either case, $|R/H| = 3$ and $|H| \geq 2$. Thus $R/H \cong Z_3$ and $R \ncong Z_3$. Hence, by theorem 3.7(3) we have $diam(GT_H(R/H))) = 2$.

4. Bondage number of $GT_H(R)$ and induced subgraphs

In this section, the bondage number of the graph $GT_H(R)$ has been studied. We begin with the following proposition.

Proposition 4.1: Let H be a prime ideal of a commutative ring R such that $|H| = \mu$ and $|R/H| = \eta$. Then $b(GT_H(R)) = \mu - 1$.

Proof. Suppose that $2 = 1_R + 1_R \in H$. Then by theorem 3.2(1), the graph $GT_H(R/H)$ is the union of $(\eta - 1)$ disjoint K_μ's and we know that $b(K_\mu) = \mu - 1$. Hence $b(GT_H(R/H)) = min\{b(K_\mu), b(K_\mu), ...,$ $b(K_\mu)\}_{(\eta-1)copies} = \mu - 1$. Also H is a prime ideal of R, so $GT_H(H)$ is complete, by theorem 3.1 . Thus, $b(GT_H(H)) = \mu - 1$. On the other hand, $GT_H(H)$ and $GT_H(R/H)$ are disjoint, by theorem 3.1 . Therefore, $b(GT_H(R)) = min\{b(GT_H(H)), b(GT_H(R/H))\} = \mu - 1$.

Now, suppose that $2 = 1_R + 1_R \notin H$. Then, by theorem 3.2(2), $GT_H(R/H)$ is the union of $(\eta - 1)/2$ disjoint $K_{\mu,\mu}$'s and we know that $b(K_{\mu,\mu}) = \mu$.
Thus $b(GT_H(R/H) = min\{b(K_{\mu,\mu}), b(K_{\mu,\mu}), ..., b(K_{\mu,\mu})\}_{(\eta-1)/2copies} = \mu$. But $GT_H(H)$ is complete and disjoint from $GT_H(R/H)$, by theorem 3.1 . So, $b(GT_H(H)) = \mu - 1$.
Hence $b(GT_H(R)) = min\{b(GT_H(R/H), b(GT_H(H))\} = min\{\mu, \mu - 1\} = \mu - 1$.

Corollary 4.2: Let H be a prime ideal of a commutative ring R such that $|H| = \mu$ and $|R/H| = \eta$. Then $b(GT_H(R)) = d(GT_H(R)) - 1$.

Proof. The result follows from Propositions 3.5 and 4.1 directly.

Theorem 4.3:(Theorem 2.5(2), [4]) Let H be a prime ideal of a commutative ring R. Then the following hold:

(1) $gr(GT_H(R/H) = 3$ if and only if $2 \in H$ and $|H| \geq 3$.
(2) $gr(GT_H(R/H) = 4$ if and only if $2 \notin H$ and $|H| \geq 2$.
(3) Otherwise, $gr(GT_H(R/H) = \infty$.

In the following a relationship between girth and bondage number of $GT_H(R/H)$ has been established.

Proposition 4.4: Let H be a prime ideal of a commutative ring R such that $|H| = \mu$ and $|R/H| = \eta$. Then

(1) $gr(GT_H(R/H) = 3$ if and only if $b(GT_H(R/H) = \mu - 1$ and $|H| \geq 3$.
(2) $gr(GT_H(R/H) = 4$ if and only if $b(GT_H(R/H) = \mu$ and $|H| \geq 2$.

Proof.

(1) If $gr(GT_H(R/H) = 3$, then $2 \in H$ and $|H| \geq 3$, by theorem 4.3(1) . Since $2 \in H$ so $GT_H(R/H)$ is the union of $\eta - 1$ disjoint K_μ's , by theorem 3.2(1) . Therefore, $b(GT_H(R/H) = \mu - 1$.
Now assume that $b(GT_H(R/H) = \mu - 1$ and $|H| \geq 3$. If $2 \notin H$, then $GT_H(R/H)$ is the union of $(\eta - 1)/2$ disjoint $K_{\mu,\mu}$'s, by theorem 3.2(2) and thus $b(GT_H(R/H) = \mu$, a contradiction by assumption.
Therefore $2 \in H$, and then $gr(GT_H(R/H) = 3$, by theorem 4.3(1) .
(2) If $gr(GT_H(R/H) = 4$, then $2 \notin H$ and $|H| \geq 2$, by theorem 4.3(2).
So $b(GT_H(R/H) = \mu$, by the same argument to above.
Now, let $b(GT_H(R/H) = \mu$ and $|H| \geq 2$. If $2 \in H$, then $b(GT_H(R/H) = \mu - 1$, by theorem 3.2(1), a contradiction. So $2 \notin H$. Therefore, $GT_H(R/H)$ is the union of $K_{\mu,\mu}$'s, where $\mu \geq 2$. Thus $gr(K_{\mu,\mu}) = 4$ and hence $gr(GT_H(R/H) = 4$.

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