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POSET PROPERTIES WITH RESPECT TO SEMI-IDEAL BASED ZERO-DIVISOR GRAPH

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Abstract. In this paper, we discuss some properties of poset P determined by semi-ideal based zero-divisor graph $G_K(P)$, for a semi-ideal K of P. We investigate some interesting properties if $G_K(P)$ contains a cycle. Further, we prove that if $V(G_K(P))$ is a generalized tree, then $(V(G_K(P))\setminus S_x)\cup K$ and $(V(G_K(P))\setminus S_{\geq x})\cup K$ are prime semi-ideals of P.

1. Preliminaries

Throughout this paper, $(P, \leq)$ denotes a poset with zero element 0 and $G_K(P)$ denotes the semi-ideal based zero-divisor graph for a semi-ideal K of P. For $K \subseteq P$, let $(K)^l := \{s \in P : s \leq k$ for all $k \in K\}$ denotes the lower cone of K, and dually let $(K)^u := \{s \in P : k \leq s$ for all $k \in K\}$ be the upper cone of K. For $K_1, K_2 \subseteq P$, we write $(K_1, K_2)^l$ instead of $(K_1 \cup K_2)^l$. A non-empty $K \subseteq P$ is said to be semi-ideal if $k_2 \in K$ and $k_1 \leq k_2$, then $k_1 \in K$. A semi-ideal $K \subset P$ is said to be prime if for any $k_1, k_2 \in P$, $(k_1, k_2)^l \subseteq K$ implies $k_1 \in K$ or $k_2 \in K$.

In [4], I. Beck proposed the notion of a zero-divisor graph structure of a commutative ring with identity element, and concentrated mainly on coloring. In [3], D. F. Anderson et al. discussed the subgraph $\Gamma(R)$ of $G(R)$ whose vertices are set of all nonzero zero-divisors of R and any two distinct vertices r and t are connected by an edge if $rt = 0$. In [22], S. P. Redmond extended the zero-divisor graph structure for an ideal K of R as follows: An undirected graph $\Gamma_K(R)$ with vertices

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$\{r \in R\backslash K : rt \in K$ for some $t \in R\backslash K\}$, and any two distinct vertices r and t are adjacent if and only if $rt \in K$.

In [6], A. Das et al. determined number of some families of cubic graphs. In [10], F.Esmaeili Khalil saraei studied the annihilator graph of module M over a commutative ring R, and shown that $AG(M)$ is connected with diameter at most two and girth at most four, and investigated some properties of the zero-divisor graph of reduced multiplication-like R-modules. In [20], K. Porselvi et al. studied the notion of extended ideal based zero divisor graph $\overline{\Gamma}_I(R)$ for an ideal I of a commutative ring R and investigated their properties. Several authors explore graph structures for different algebraic structures ([1], [7], [8], [11], [14], [17], [21], [23] and [24]).

In [13], R. Halas et al. introduced the notion of graph structure for P. The zero-divisor graph of P, denoted by $\Gamma(P)$, is an undirected graph with the elements of P as vertices and two distinct vertices k_1 and k_2 are adjacent if and only if $(k_1, k_2)^l = \{0\}$, and discussed some interesting results using chromatic number and clique. In [16], D. Lu et al. studied the zero-divisor graph $\Gamma(P)$ of P is the graph whose vertices are nonzero zero-divisors of P, in which k_1 is adjacent to k_2 if and only if $(k_1, k_2)^l = 0$. But this graph structure is slightly different from the one defined in [13].

In [15], V. Joshi presented the zero divisor graph $G_K(P)$ of P with respect to an ideal K as follows: An undirected graph whose vertices are the set $\{k_1 \in P\backslash K \ : \ (k_1, k_2)^l \subseteq K$ for some $k_2 \in P\backslash K\}$ and distinct vertices k_1 and k_2 are connected by an edge if and only if $(k_1, k_2)^l \subseteq K$. If $K = \{0\}$, then $G_K(P) = G_{\{0\}}(P)$, and $G_K(P) = \phi$ if and only if K is a prime semi-ideal of P. Following [15], we have studied the above graph structure for a semi-ideal K of P in [9], and investigated the relationship between the graph-theoretic properties of $G_K(P)$ for a semi-ideal K of P and poset properties of P, and also topological properties of $Spec(P)$ where $Spec(P)$ is intersection of all prime semi-ideals of P.

Following [12], let K be a semi-ideal in P. Then the set $< x, \ K >= \{k \in P \ : \ (x, k)^l \subseteq K\}$ is the extension of K by $x \in P$. The neighborhood of $x \in V(G_K(P))$ is the set $N(x) = \{k \in V(G_K(P)) \ : \ (x, k)^l \subseteq K\}$. It is evident that $< x, K >= N(x) \cup K$. A path is a sequence of vertices and edges with no repeated vertices. The distance between two vertices in a graph is the number of edges in a shortest or minimal path. A cycle is a circuit in which no vertex except the first (which is also the last) appears more than once. A maximal complete subgraph of a graph is said to be clique of a graph. The girth $gr(G_K(P))$ is the length of the shortest cycle in $G_K(P)$. For any adjacent vertices k_1, k_2 in $V(G_K(P))$, we denote $C(k_1, k_2) = \{x \in V(G_K(P)) \mid N(x) = \{k_1, k_2\}\}$ and let T_k denote the set of all end vertices of $G_K(P)$ adjacent to k (see [18]). In [19], we introduced z-semi-ideal of P, and discussed many interesting results related to prime and z-semi-ideal of P.

For undefined terms and notations of graph theory, refer [5], and for posets, refer[12].

2. Main results

In this section, we explore properties of P related to a single vertex in a graph $G_K(P)$ for a semi-ideal K of P. We provide some condition under which S_x and $S_{\geq x}$ are identical. Also, we discuss interesting results if $V(G_K(P))$ is a generalized tree.

Theorem 2.1. *([15],[16]) Let $G_K(P)$ be a graph of P for a semi-ideal K in P. Then $G_K(P)$ is connected and $diam(G_K(P)) \leq 3$.*

Lemma 2.2. *Let $G_K(P)$ be a graph of P for a semi-ideal K in P and $r, s \in V(G_K(P))$. If $d(r, s) = 3$, then $(r, s)^l \cap (V(G_K(P))\backslash\{r, s\}) \neq \phi$ and $G_K(P)$ contains at least $|(r, s)^l \cap V(G_K(P))\backslash\{r, s\}|$ induced subgraph of $G_K(P)$ with $C(a, b) \neq \phi$ for some $a, b \in V(G_K(P))$.*

Proof. Suppose $r - a - b - s$ is a path of length 3. Then there is $t \in ((r, s)^l \cap V(G_K(P)))\backslash\{r, s\}$ such that $t - a - b - t$. □

Theorem 2.3. *Let $G_K(P)$ be a graph of P for a semi-ideal K in P. If $G_K(P)$ does not contain a cycle, then $G_K(P)$ is a connected subgraph of two star graphs whose centers are connected by a single edge.*

Proof. If $G_K(P)$ has a path of maximum length ≤ 2, then $G_K(P)$ is a star graph. Assume that $u-r-s-v$ is a path in $G_K(P)$ and $G_K(P)$ does not contain a cycle. Let $c \in V(G_K(P))$ such that $c \notin \{u, r, s, v\}$. If $u-c$ or $c-v$ in $G_K(P)$ (say, $u-c$), then there exist a path $c-u-r-s-v$ of length 4, a contradiction. So every vertex c in $G_K(P)$ should adjacent with either r or s, but not both. Hence $G_K(P)$ is a two star graph. □

Corollary 2.4. *Let $G_K(P)$ be a graph of P for a semi-ideal K in P. If $G_K(P)$ has no cycle and $b \in V(G_K(P))$, then $(u, v)^l \subseteq T_b \cup K$ for any $u, v \in T_b \cup K$.*

Proof. Let $u, v \in T_b \cup K$ and $t \in (u, v)^l\backslash K$. If $G_K(P)$ is a star graph, then $t \in T_b$. Suppose $G_K(P)$ is a two-star graph. Then $G_K(P)$ has two centres one of which should be b and other say a. So we have a path $r - a - b - s$ for some $r \in V(G_K(P))$ and $s \in V(G_K(P)) \cup \{u, v\}$. If $t \notin T_b$, then for some $k \in V(G_K(P))$, $r - a - b - t - k$ is a path of length 4, a contradiction. □

Proposition 2.5. *Let $G_K(P)$ be a graph of P for a semi-ideal K in P and $b \in V(G_K(P))$. Then the following assertions hold:*

(i) If $G_K(P)$ has a cycle and for any $u \in T_b \cup K$, if $(u)^l \cap (y)^u = \phi$ for all $y \in V(G_K(P)) \backslash T_b$, then $(u,v)^l \subseteq T_b \cup K$, for any $v \in T_b \cup K$.

(ii) If $T_b \neq \phi$, then $K \cup \{b\}$ is semi-ideal.

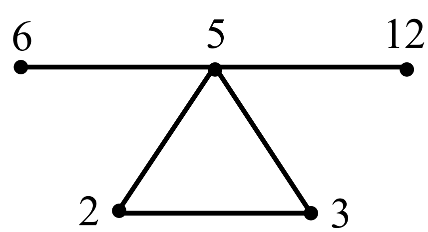
(iii) If S is a clique in $G_K(P)$ such that $(b)^l \cap (V(G_K(P))\backslash S) = \phi$ for all $b \in S$, then $S \cup K$ is semi-ideal.

Proof. (i) Suppose that $(u,v)^l \nsubseteq T_b \cup K$ for some $v \in T_b \cup K$. Then there is $c \in (u,v)^l$ such that $c \notin T_b \cup K$ which implies $c \in (u)^l \cap (c)^u \neq \phi$, which is a contradiction.

(ii) Let $a \in T_b$ and let $r, s \in P$ with $r \leq s$ and $s \in K \cup \{b\}$. If $s = b$, then $(r,a)^l \subseteq K$ which implies $r = b$. Hence $K \cup \{b\}$ is semi-ideal.

(iii) Let $r, s \in P$ with $r \leq s$ and $s \in S \cup K$. If $s \in S$, then $r \notin S$, since S is clique in $G_K(P)$. Now $r \in (s)^l \cap (V(G_K(P))\backslash S) \neq \phi$, a contradiction. □

Example 2.6. Let $P = \{1,2,3,5,6,12\}$ be a poset under the relation division and $K = \{1\}$. Then K is a semi-ideal of P and the graph $G_K(P)$ is given below:

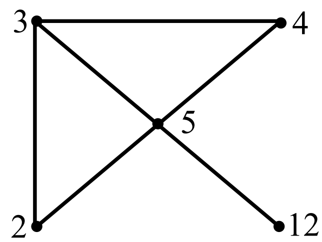


Here $(6)^l \cap (2)^u \neq \phi$ for $6 \in T_5$. But $(6,12)^l \nsubseteq T_5 \cup K$. Also note that $T_6 = \{\phi\}$, but $K \cup \{6\}$ is not a semi-ideal of P. This shows that one can not drop the conditions $(u)^l \cap (y)^u = \{\phi\}$ and $T_b \neq \{\phi\}$ from Proposition 2.5 (i) and (ii) respectively.

The following example shows that the condition $(b)^l \cap (V(G_K(P))\backslash S) = \phi$ for all $b \in S$ is necessary in Proposition 2.5 (iii).

Example 2.7. Let $P = \{1,2,3,4,5,12\}$ be a poset under the relation division and $K = \{1\}$ be a semi-ideal of P. The graph $G_K(P)$ is given below:

Here $S = \{3,4,5\}$ is a clique of $G_K(P)$ and $(4)^l \cap (V(G_K(P))\backslash S) \neq \phi$, but $S \cup K$ is not a semi-ideal of P.



Following [25], we denote $S_x = \{s \in V(G_K(P)) \mid N(s) = N(x)\}$ and $S_{\geq x} = \{s \in V(G_K(P)) \mid N(s) \subseteq N(x)\}$. It is clear that $S_x \subseteq S_{\geq x}$ for any $x \in V(G_K(P))$. But $S_{\geq x}$ is not necessary to be a subset of S_x. In Example 2.7, for $2 \in V(G_K(P))$, we have $S_{\geq 2} \nsubseteq S_2$.

Theorem 2.8. *Let $G_K(P)$ be a graph of P for a semi-ideal K of P and $x \in V(G_K(P))$. Then $(V(G_K(P))\backslash S_{\geq x}) \cup K$ is a semi-ideal of P.*

Proof. Let $r, s \in P$ with $r \leq s$ and $s \in (V(G_K(P))\backslash S_{\geq x}) \cup K$. If $r \notin K$ and $s \in V(G_K(P))\backslash S_{\geq x}$, then $N(s) \subseteq N(r)$ and $N(s) \nsubseteq N(x)$ which imply $r \notin S_{\geq x}$. □

As an immediate consequence, we have the following.

Corollary 2.9. *Let $G_K(P)$ be a graph of P for a semi-ideal K in P and $x \in V(G_K(P))$. Then the following hold:*

(i) If $G_K(P)$ is complete graph, then $(V(G_K(P))\backslash S_x) \cup K$ is semi-ideal.

(ii) If x is an end vertex, then $(V(G_K(P))\backslash S_x) \cup K$ is semi-ideal.

Following [2], P is said to be a generalized tree if for any $r \in P$ and for every $x_1, x_2 \in (r)^l$, either $x_1 \leq x_2$ or $x_2 \leq x_1$. For a non-empty $Q \subseteq P$, an element $r \in Q$ is said to be a minimal element of Q, denoted by $Min(Q)$, whenever $s \in Q$ and $s \leq r$ implies that $s = r$.

Theorem 2.10. [2] *Let $V(G_K(P))$ be a generalized tree and for $k \geq 2$, $|Min(P\backslash K)| = k$. If $(x)^l\backslash K$ has a minimal element for every $x \in P\backslash K$, then $G_K(P)$ is a complete k-partite graph.*

Theorem 2.11. *Let $G_K(P)$ be a graph of P for a semi-ideal K in P. Let $V(G_K(P))$ be a generalized tree and for $k \geq 2$, $|Min(P\backslash K)| = k$. If $(x)^l\backslash K$ has a minimal element for every $x \in P\backslash K$, then the sets S_x and $S_{\geq x}$ are same.*

Proof. Let $s \in S_{\geq x}$. Then $N(s) \subseteq N(x)$. Suppose that $N(x) \nsubseteq N(s)$. Then there exists $t \in N(x)\backslash N(s)$. So t and s are in same partition of $V(G_K(P))$ since $G_K(P)$ is a complete k-partite graph by Theorem 2.10. Since $t \in N(x)$, we have $s \in N(x)$ which gives $x \in N(s) \subseteq N(x)$

and so $x \in K$, a contradiction. Thus $N(x) \subseteq N(s)$ which gives $s \in S_x$ and hence S_x and $S_{\geq x}$ are same. □

Theorem 2.12. *Let $G_K(P)$ be a graph of P for a semi-ideal K in P and $x \in V(G_K(P))$. If $V(G_K(P))$ is a generalized tree, then for any $v \in P$, we have $(u,v)^l \subseteq S_{\geq x} \cup K$ for all $u \in S_{\geq x} \cup K$.*

Proof. Let $u \in S_{\geq x}$ and suppose that $(u,v)^l \nsubseteq S_{\geq x} \cup K$ for some $v \in P$. Then there is $t \in (u,v)^l \backslash S_{\geq x} \cup K$ such that $N(t) \backslash (N(x) \cup N(u)) \neq \phi$. Now let $s \in N(t) \backslash (N(x) \cup N(u))$. Then there is $a \in (s,u)^l \backslash K$ such that $a \leq t$ or $t \leq a$ as $\{a,t\} \subseteq (u)^l$ and $V(G_K(P))$ is generalized tree. Since $(a,t)^l \subseteq K$, we get $a \in K$ or $t \in K$, which is a contradiction. So $(u,v)^l \subseteq S_{\geq x} \cup K$. □

In view of Theorem 2.12, we have the following:

Corollary 2.13. *Let $G_K(P)$ be a graph of P for a semi-ideal K in P and $x \in V(G_K(P))$. If $V(G_K(P))$ is a generalized tree, then $S_{\geq x} \cup K$ is a semi-ideal of P.*

Remark 2.14. If $V(G_K(P))$ is not a generalized tree, then Theorem 2.12 will fail . Indeed, in Example 2.7, for $K = \{1\}$, $S_{\geq 12} = \{12\}$, but $(12)^l \nsubseteq S_{\geq 12} \cup K$.

Theorem 2.15. *Let $G_K(P)$ be a graph for a semi-ideal K in P and $x \in V(G_K(P))$. If $V(G_K(P))$ is a generalized tree, then $(V(G_K(P)) \backslash S_x) \cup K$ is semi-ideal. Moreover, for any $k_1, k_2 \in V(G_K(P)) \cup K$, if $(k_1,k_2)^l \subseteq (V(G_K(P)) \backslash S_x) \cup K$, we have $k_1 \in (V(G_K(P)) \backslash S_x) \cup K$ or $k_2 \in (V(G_K(P)) \backslash S_x) \cup K$.*

Proof. Let $u, v \in P$ with $u \leq v$ and $v \in (V(G_K(P)) \backslash S_x) \cup K$. Suppose $v \in V(G_K(P)) \backslash S_x$ and $u \in S_x$. Then $N(u) = N(x)$ which implies $N(v) \subset N(x)$. Now let $s \in N(x) \backslash N(v)$. Then there exists $k_1 \in (s,v)^l \backslash K$ such that $k_1 \leq u$ or $u \leq k_1$ as $\{k_1, u\} \subseteq (v)^l$ and $V(G_K(P))$ is generalized tree. Since $(k_1,u)^l \subseteq K$, we have $k_1 \in K$ or $u \in K$, a contradiction. So $u \notin S_x$ and hence $(V(G_K(P)) \backslash S_x) \cup K$ is semi-ideal.

Let $(k_1,k_2)^l \subseteq (V(G_K(P)) \backslash S_x) \cup K$ and $k_1, k_2 \notin \{(V(G_K(P)) \backslash S_x) \cup K\}$. Then $N(k_1) = N(x)$ and $N(k_2) = N(x)$.

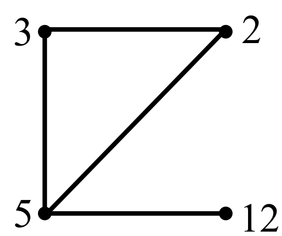
Case(i) : If $(k_1,k_2)^l \subseteq K$, then $k_2 \in N(k_1) = N(k_2)$ which implies $k_2 \in K$, a contradiction.

Case(ii) : If $(k_1,k_2)^l \nsubseteq K$, then there exists $t \in (k_1,k_2)^l$ such that $t \in V(G_K(P)) \backslash S_x$ which implies $N(t) \nsubseteq N(x)$. Now let $s \in N(t) \backslash (N(x) \cup N(k_1) \cup N(k_2))$. Then there exists $d \in (s,k_1)^l \backslash K$ such that $d \leq t$ or $t \leq d$ as $\{d,t\} \subseteq (k_1)^l$ and $V(G_K(P))$ is a generalized tree. Since $(d,t)^l \subseteq K$, we have $d \in K$ or $t \in K$, a contradiction. So $k_1 \in (V(G_K(P)) \backslash S_x) \cup K$ or $k_2 \in (V(G_K(P)) \backslash S_x) \cup K$. □

Corollary 2.16. *Let $G_K(P)$ be a graph of P for a semi-ideal K of P and $x \in V(G_K(P))$. If $V(G_K(P))$ is a generalized tree and $V(G_K(P)) = P\backslash K$, then $(V(G_K(P))\backslash S_x) \cup K$ is a prime semi-ideal of P.*

The following example shows that we can not drop the condition $V(G_K(P)) = P\backslash K$ in Corollary 2.16.

Example 2.17. Let $P = \{1, 2, 3, 5, 12, 60, 90\}$ be a poset under the relation division and $K = \{1\}$. Then K is a semi-ideal of P and the graph $G_K(P)$ is given below:



Here $S_{12} = \{12\}$ and $(V\backslash S_{12}) \cup K = \{1, 2, 3, 5\}$. But $(V\backslash S_{12}) \cup K$ is not prime as $(60, 90)^l = \{1, 2, 3, 5\} \subseteq (V\backslash S_{12}) \cup K$ with $60, 90 \notin (V\backslash S_{12}) \cup K$.

Theorem 2.18. *Let $G_K(P)$ be a graph of P for a semi-ideal K in P and $x \in V(G_K(P))$. If $V(G_K(P))$ is a generalized tree, then the following assertions hold:*

(i) For any $v \in P$, $(u, v)^l \subseteq S_x \cup K$ for all $u \in S_x \cup K$. In particular, $S_x \cup K$ is semi-ideal.

(ii) $(V(G_K(P))\backslash S_{\geq x}) \cup K$ is a semi-ideal of P. For any $a, b \in V(G_K(P)) \cup K$, if $(a, b)^l \subseteq (V(G_K(P))\backslash S_{\geq x}) \cup K$, we have $a \in (V(G_K(P))\backslash S_{\geq x}) \cup K$ or $b \in (V(G_K(P))\backslash S_{\geq x}) \cup K$. In particular, $(V(G_K(P))\backslash S_{\geq x}) \cup K$ is prime semi-ideal.

Proof. It is evident from Theorems 2.11, 2.12 and 2.15. □

Recall that the set of associated primes of P, denoted by $Ass(P)$, is the set of prime semi-ideals Q in P such that there is some $x \in P$ with $Q =< x, K >$ for a semi-ideal K in P.

Theorem 2.19. *Let $G_K(P)$ be a graph of P for a semi-ideal K in P. Then following assertions hold:*

(i) If $|Ass(P)| \geq 2$ and $P_1 =< r, K >$, $P_2 =< s, K >$ are distinct elements of $Ass(P)$, then $(r, s)^l \subseteq K$.

(ii) If $|Ass(P)| \geq 3$, *then* $gr(G_K(P)) = 3$.
(iii) If $|Ass(P)| \geq 5$, *then* $G_K(P)$ *is non-planar.*

Proof. (i) Suppose $P_1 \neq P_2$. Then there exists $x \in P_1 \backslash P_2$ such that $(x, r)^l \subseteq K \subseteq P_2$. Since P_2 is prime, $r \in P_2$ which implies $(r, s)^l \subseteq K$.

(ii) Let $P_1 =< r, K >,\ P_2 =< s, K >$ and $P_3 =< t, K >$ be distinct elements of $Ass(P)$. Then by (i), $r - s - t - r$ is a cycle in $G_K(P)$ and so $gr(G_K(P)) = 3$.

(iii) Since $|Ass(P)| \geq 5$, the complete graph K_5 is a subgraph of $G_K(P)$. By Kuratowski's theorem, $G_K(P)$ is non-planar. □

Theorem 2.20. [2] *Let* $G_K(P)$ *be a graph of* P *for a semi-ideal* K *in* P. *Then following assertions are equivalent:*

(i) There exist nonzero prime semi-ideals P_1 *and* P_2 *in* P *such that* $P_1 \cap P_2 = K$.

(ii) $G_K(P)$ *is a complete bipartite graph.*

(iii) $G_K(P)$ *is a bipartite graph.*

Theorem 2.21. *Let* $G_K(P)$ *be a graph of* P *for a semi-ideal* K *in* P. *If* $|Ass(P)| = \{P_1, P_2\}$ *and* $|P_1| \geq 3,\ |P_2| \geq 3$ *with* $P_1 \cap P_2 = K$, *then* $gr(G_K(P)) = 4$.

Proof. Let $P_1 =< r, K >$ and $P_2 =< s, K >$ with $x \in P_1 \backslash (K \cup \{s\})$ and $y \in P_2 \backslash (K \cup \{r\})$. Since $(x, y)^l \subseteq K$, we have $x - r - s - y - x$ and so $gr(G_K(P)) \leq 4$. By Theorem 2.20, $G_K(P)$ is a bipartite graph. Hence $gr(G_K(P)) = 4$. □

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