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NEARRINGS OF FUNCTIONS WITHOUT IDENTITY DETERMINED BY A SINGLE SUBGROUP

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Abstract. Let $(G, +)$ be a finite group, written additively with identity 0, but not necessarily abelian, and let H be a nonzero, proper subgroup of G. Then the set $M = \{f : G \to G \mid f(G) \subseteq H \text{ and } f(0) = 0\}$ is a right, zero-symmetric nearring under pointwise addition and function composition. We find necessary and sufficient conditions for M to be a ring and determine all ideals of M, the center of M, and the distributive elements of M.

1. Introduction

A right nearring is a triple $(N, +, \cdot)$ where $(N, +)$ is a group, written additively with identity 0, but not necessarily abelian, $(N, \cdot)$ is a semigroup, and the right distributive law holds. Thus, nearrings are generalizations of rings. A nearring N is zero-symmetric if $0n = n0 = 0$ for all $n \in N$. For more information on nearrings, see [7], [11], and [12].

Natural examples of nearrings occur when considering sets of functions under addition and composition. Numerous papers have investigated various nearrings of functions, including [2], [3], [4], [5], and [6]. A special class of nearrings of functions, the centralizer nearrings, have received particular attention since every nearring with identity is isomorphic to a centralizer nearring ([7], Theorem 14.3). Studies of centralizer nearrings can be found in [1], [9], and [10].

We continue the investigation of nearrings of functions. Let $(G, +)$ be a finite group with identity 0, though not necessarily abelian, and

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let H be a nontrivial, proper subgroup of G. Define $M = \{f : G \to G \mid f(G) \subseteq H \text{ and } f(0) = 0\}$. Then M is a zero-symmetric nearring without identity under function addition and composition. We investigate the internal structure of this nearring. We determine when M is abelian and distributive, identify all ideals of M, and characterize when M is a ring.

We let 0 denote the identity element in $(G, +)$ and the zero function from G to G, but its use should be obvious from the context. We also let $End\, H$ denote the set of endomorphisms of H. Most of the results of this paper appear in [8].

2. General Results

In this section we investigate basic properties of the nearring $M = \{f : G \to G \mid f(G) \subseteq H \text{ and } f(0) = 0\}$. We first determine when M is abelian, i.e., when $f_1 + f_2 = f_2 + f_1$ for all $f_1, f_2 \in M$.

Theorem 2.1. *The nearring M is abelian if and only if the subgroup H is abelian.*

Proof. Assume M is abelian. Thus for all $f_1, f_2 \in M$ and every $g \in G$, $f_1(g) + f_2(g) = (f_1 + f_2)(g) = (f_2 + f_1)(g) = f_2(g) + f_1(g)$.

Fix $0 \neq g \in G$ and let $h_1, h_2 \in H$. For $i = 1, 2$, define $f_i(x) = \begin{cases} h_i & \text{if } x = g \\ 0 & \text{if } x \neq g \end{cases}$. Then $f_1, f_2 \in M$ and $h_1 + h_2 = f_1(g) + f_2(g) = f_2(g) + f_1(g) = h_2 + h_1$ and H is abelian.

Now assume H is abelian. Let $f_1, f_2 \in M$ and $g \in G$. Then $f_1(g), f_2(g) \in H$ and $(f_1 + f_2)(g) = f_1(g) + f_2(g) = f_2(g) + f_1(g) = (f_2 + f_1)(g)$. Hence $f_1 + f_2 = f_2 + f_1$ and M is abelian. Therefore M is abelian if and only if H is abelian. □

Next, we identify the distributive elements of M and find necessary and sufficient conditions for M to be a ring. First we need some definitions.

Definition 2.2. Let N be a nearring. We define $N_d = \{n \in N \mid n(x + y) = nx + ny \text{ for all } x, y \in N\}$, the set of all (left) distributive elements of N. If $N = N_d$, we say that N is a distributive nearring.

Theorem 2.3. *Let M_d be the set of distributive elements in M. Then $f \in M_d$ if and only if $f|_H \in End\, H$.*

Proof. Assume $f \in M_d$. As in the proof of Theorem 2.1, fix $0 \neq g \in G$ and let $h_1, h_2 \in H$. For $i = 1, 2$, define $f_i(x) = \begin{cases} h_i & \text{if } x = g \\ 0 & \text{if } x \neq g \end{cases}$. Then $f_1, f_2 \in M$. Since $f \in M_d$, we have $f(h_1 + h_2) = f(f_1(g) + f_2(g)) =$

$[f(f_1+f_2)](g) = [f\circ f_1 + f\circ f_2](g) = (f\circ f_1)(g) + (f\circ f_2)(g) = f(f_1(g)) + f(f_2(g)) = f(h_1) + f(h_2)$. Thus $f|_H \in End\,H$.

Now assume $f \in M$ such that $f|_H \in End\,H$. Let $n_1, n_2 \in M$ and $g \in G$. Thus $n_1(g), n_2(g) \in H$. It follows that $[f(n_1+n_2)](g) = f(n_1(g)+n_2(g)) = f(n_1(g)) + f(n_2(g)) = (f\circ n_1)(g) + (f\circ n_2)(g) = [f\circ n_1 + f\circ n_2](g)$. Hence $f \in M_d$. We conclude that $f \in M_d$ if and only if $f|_H \in End\,H$. □

Theorem 2.4. *The following are equivalent:*

(i) *M is a ring;*
(ii) *M is distributive;*
(iii) $|H| = 2$.

Proof. Condition (i) clearly implies condition (ii). We prove (ii) implies (iii) via the contrapositive. So assume $|H| \geq 3$, say $0, a, b \in H$ with all three elements distinct. Fix $0 \neq g \in G$. Define the functions $f_1(x) = \begin{cases} b & \text{if } x = b \\ a+b & \text{if } x = a+b \\ 0 & \text{else} \end{cases}$, $f_2(x) = \begin{cases} a & \text{if } x = g \\ 0 & \text{if } x \neq g \end{cases}$, and $f_3(x) = \begin{cases} b & \text{if } x = g \\ 0 & \text{if } x \neq g \end{cases}$. So $f_1, f_2, f_3 \in M$. Then $[f_1\circ(f_2+f_3)](g) = f_1(f_2(g) + f_3(g)) = f_1(a+b) = a+b \neq b = 0+b = f_1(a) + f_1(b) = f_1(f_2(g)) + f_1(f_3(g)) = [f_1\circ f_2](g) + [f_1\circ f_3](g)$, and M is not distributive. Thus, if $|H| \neq 2$, then M is not distributive, and (ii) implies (iii).

Now assume $|H| = 2$, say $H = \{0, h\}$. Note that for any $f \in M$ and $g \in G$, $f(g) \in H$ and $f(g) + f(g) = 0$. In particular, $h + h = 0$. We show that M is distributive. To this end, let $f_1, f_2, f_3 \in M$ and $g \in G$.

If $f_2(g) = 0 = f_3(g)$, then $[f_1\circ(f_2+f_3)](g) = f_1(f_2(g)+f_3(g)) = f_1(0+0) = f_1(0) = 0 = 0+0 = f_1(0) + f_1(0) = f_1(f_2(g)) + f_1(f_3(g)) = [f_1\circ f_2](g) + [f_1\circ f_3](g)$.

If $f_2(g) = h = f_3(g)$, then $[f_1\circ(f_2+f_3)](g) = f_1(f_2(g)+f_3(g)) = f_1(h+h) = f_1(0) = 0 = f_1(h) + f_1(h) = f_1(f_2(g)) + f_1(f_3(g)) = [f_1\circ f_2](g) + [f_1\circ f_3](g)$.

If $f_2(g) = 0$ and $f_3(g) = h$, then $[f_1\circ(f_2+f_3)](g) = f_1(f_2(g) + f_3(g)) = f_1(0+h) = f_1(h) = 0 + f_1(h) = f_1(0) + f_1(h) = f_1(f_2(g)) + f_1(f_3(g)) = [f_1\circ f_2](g) + [f_1\circ f_3](g)$.

If $f_2(g) = h$ and $f_3(g) = 0$, the proof is similiar to that of the previous case. In all cases, $[f_1\circ(f_2+f_3)](g) = [f_1\circ f_2](g) + [f_1\circ f_3](g)$, and M is distributive. Hence, (iii) implies (ii).

Again assume $|H| = 2$. Then H is abelian. It follows that M is abelian from Theorem 2.1. From above, M is also distributive. Therefore M is a distributive, abelian nearring, making M a ring. Thus (iii) implies (i) and the proof is complete. □

For the final result of this section, we characterize the center of M.

Definition 2.5. The center of a nearring N is $C(N) = \{c \in N \mid cn = nc \text{ for all } n \in N\}$.

In general, the center of a nearring N is not a subnearring of N (See [2] and [6]). For the nearring M, the center is always a subnearring of M.

Theorem 2.6. *The center of M is $C(M) = \{0\}$. Therefore $C(M)$ is a subnearring of M.*

Proof. Let $c \in C(M)$. Fix $g \notin H$ and let $h \in H$. Define $f_1(x) = \begin{cases} h & \text{if } x = g \\ 0 & \text{if } x \neq g \end{cases}$. Then $f_1 \in M$. Since $c(g) \in H$ and $g \notin H$, we conclude that $c(g) \neq g$. Thus $0 = f_1(c(g)) = c(f_1(g)) = c(h)$. Since $h \in H$ is arbitrary, $C(M) \subseteq \{f \in M \mid f(H) = 0\}$.

Now define $f_2(x) = \begin{cases} x & \text{if } x \in H \\ 0 & \text{if } x \notin H \end{cases}$. Then $f_2 \in M$. Let $g \notin H$. Thus $c(g) \in H$ and $0 = c(0) = c(f_2(g)) = f_2(c(g)) = c(g)$. Since $g \notin H$ is arbitrary, $C(M) \subseteq \{f \in M \mid f(G\backslash H) = 0\}$. It follows that $C(M) \subseteq \{f \in M \mid f(H) = 0\} \cap \{f \in M \mid f(G\backslash H) = 0\} = \{0\}$. Since M is zero-symmetric, it is clear that $0 \in C(M)$. Thus $C(M) = \{0\}$ and $C(M)$ is a subnearring of M. □

3. Ideals

In this section, we characterize all ideals of M.

Definition 3.1. Let N be a nearring and let $(I, +)$ be a normal subgroup of $(N, +)$. If $IN \subseteq I$, we say I is a right ideal of N. If for all $n_1, n_2 \in N$ and for all $i \in I$, $n_1(n_2 + i) - n_1 n_2 \in I$, we say I is a left ideal of N. If I is both a right ideal and a left ideal of N, we say I is an ideal of N.

Assume N is a zero-symmetric nearring and let I be an ideal of N. Then for $n, 0 \in N$ and $i \in I$, the left ideal property yields $n(0+i) - n0 = ni - 0 = ni \in I$. Thus $NI \subseteq I$. We will use this property in our work without necessarily referencing it.

Definition 3.2. Let S be a subset of G. We define $Ann\, S = \{f \in M \mid f(S) = 0\}$.

Theorem 3.3. *Let S be a subset of G with $H \subseteq S$. Then $Ann\, S$ is an ideal of M contained in $Ann\, H$.*

Proof. Let $f_1, f_2 \in Ann\, S$ and $s \in S$. Then $(f_1 - f_2)(s) = f_1(s) - f_2(s) = 0 - 0 = 0$. Hence $f_1 - f_2 \in Ann\, S$, and $Ann\, S$ is a subgroup of M.

Let $n \in M$, $f \in Ann\, S$, and $s \in S$. Then $(-n + f + n)(s) = -n(s) + f(s) + n(s) = -n(s) + 0 + n(s) = 0$. Thus $-n + f + n \in Ann\, S$, and $Ann\, S$ is a normal subgroup of M. Also, note that $n(s) \in H \subseteq S$. It follows that $(f \circ n)(s) = f(n(s)) = 0$. So $f \circ n \in Ann\, S$, and $Ann\, S$ is a right ideal of M.

Now let $n_1, n_2 \in M$, $f \in Ann\, S$, and $s \in S$. Then $(n_1 \circ (n_2 + f) - n_1 \circ n_2)(s) = n_1(n_2(s) + f(s)) - n_1(n_2(s)) = n_1(n_2(s) + 0) - n_1(n_2(s)) = 0$. Hence $n_1 \circ (n_2 + f) - n_1 \circ n_2 \in Ann\, S$, and $Ann\, S$ is a left ideal of M. It follows that $Ann\, S$ is an ideal of M. Since $H \subseteq S$, it is clear that $Ann\, S \subseteq Ann\, H$. □

Note that the above theorem shows that $Ann\, H$ is an ideal of M. Also, for any two functions $f_1, f_2 \in Ann\, S$, $f_1 \circ f_2 = 0$. Therefore the entire multiplication table for $Ann\, S$ consists solely of the zero function.

We next show that $Ann\, H$ is the only maximal ideal of M.

Theorem 3.4. *Let I be an ideal of M such that $I \not\subseteq Ann\, H$. Then $I = M$. Thus $Ann\, H$ is the unique maximal ideal of M.*

Proof. Let I be an ideal of M such that $I \not\subseteq Ann\, H$. To prove that $I = M$, we only need to show that $M \subseteq I$. Let $f \in M$ be given by $f(x) = \begin{cases} h_i & \text{if } x = g_i \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$, where $g_i \in G$, $h_i \in H$, $i = 1, 2, \ldots, w$. Since $I \not\subseteq Ann\, H$, there exists $f_1 \in I$ and $k, b \in H$ such that $f_1(k) = b \neq 0$.

Define $n_i(x) = \begin{cases} k & \text{if } x = g_i \\ 0 & \text{if } x \neq g_i \end{cases}$, and $m_i(x) = \begin{cases} h_i & \text{if } x = b \\ 0 & \text{if } x \neq b \end{cases}$ for $i = 1, 2, \ldots, w$. Then $n_i, m_i \in M$. Define $\alpha_i = m_i \circ f_1 \circ n_i$. Since $I \circ M \subseteq I$ and $M \circ I \subseteq I$, we conclude that $\alpha_i \in I$. By the definition of α_i, we get $\alpha_i(x) = \begin{cases} h_i & \text{if } x = g_i \\ 0 & \text{if } x \neq g_i \end{cases}$. Note that $f = \alpha_1 + \alpha_2 + \cdots + \alpha_w \in I$. Hence $M \subseteq I$ and $I = M$. The rest of the theorem follows immediately. □

Now we consider the separate cases of whether $|H| \geq 3$ or $|H| = 2$.

Theorem 3.5. *Assume $|H| \geq 3$. Let I be any ideal contained in $Ann\, H$. Then $I = Ann\, S$ for some subset S of G with $H \subseteq S$.*

Proof. Assume that $I \subseteq Ann\, H$ is an ideal and that $0, h$, and $\bar{h}$ are distinct elements in H. Let $S = \{g \in G \mid f(g) = 0 \text{ for all } f \in I\}$. Then $H \subseteq S$ and $I \subseteq Ann\, S$. Assume $G \backslash S = \{g_1, g_2, \ldots, g_w\}$. To show $Ann\, S \subseteq I$, let $f \in Ann\, S$, say $f(x) = \begin{cases} h_i & \text{if } x = g_i \notin S \\ 0 & \text{if } x \in S \end{cases}$, where $h_i \in H$ for every $i = 1, 2, \ldots, w$.

For $g_1 \in G \backslash S$, there exists $f_1 \in I$ such that $f_1(g_1) = k_1$ for some $0 \neq k_1 \in H$. Assume $f_1(g_i) = k_i \in H$ for $i = 2, 3, \ldots, w$. So $f_1(x) = \begin{cases} k_i & \text{if } x = g_i \\ 0 & \text{else} \end{cases}$.

Define $n_1(x) = \begin{cases} h & \text{if } x = k_i \neq 0 \\ 0 & \text{else} \end{cases}$. Then $n_1 \in M$. Hence $f_h(x) := (n_1 \circ f_1)(x) = \begin{cases} h & \text{if } x = g_i \text{ and } k_i \neq 0 \\ 0 & \text{else} \end{cases}$. Since I is an ideal of M, it follows that $f_h \in I$.

Now define the functions $n_2(x) = \begin{cases} h_1 & \text{if } x = h \\ 0 & \text{else} \end{cases}$, and $n_3(x) = \begin{cases} \bar{h} & \text{if } x = g_i \text{ with } i \geq 2 \\ 0 & \text{else} \end{cases}$. Then $n_2, n_3 \in M$. Define $\alpha_1(x) = (n_2 \circ (n_3 + f_h) - n_2 \circ n_3)(x)$. Since I is an ideal of M and $f_h \in I$, we conclude that $\alpha_1 \in I$.

Note that $\alpha_1(g_1) = (n_2 \circ (n_3 + f_h) - n_2 \circ n_3)(g_1) = n_2(n_3(g_1) + f_h(g_1)) - n_2(n_3(g_1)) = n_2(0 + h) - n_2(0) = n_2(h) = h_1$.

For $g_i \in G \backslash S$, $i \geq 2$ with $k_i \neq 0$, $\alpha_1(g_i) = (n_2 \circ (n_3 + f_h) - n_2 \circ n_3)(g_i) = n_2(n_3(g_i) + f_h(g_i)) - n_2(n_3(g_i)) = n_2(\bar{h} + h) - n_2(\bar{h}) = 0 - 0 = 0$.

For $g_i \in G \backslash S$, $i \geq 2$ with $k_i = 0$, $\alpha_1(g_i) = (n_2 \circ (n_3 + f_h) - n_2 \circ n_3)(g_i) = n_2(n_3(g_i) + f_h(g_i)) - n_2(n_3(g_i)) = n_2(\bar{h} + 0) - n_2(\bar{h}) = 0$. Thus, $\alpha_1(x) = \begin{cases} h_1 & \text{if } x = g_1 \\ 0 & \text{if } x \neq g_1 \end{cases}$.

For each $g_i \in G \backslash S$, $i \geq 2$, there exists $f_i \in I$ such that $f_i(g_i) = z_i$ for some $0 \neq z_i \in H$. Using the same procedure as above, we can construct $\alpha_i \in I$ such that $\alpha_i(x) = \begin{cases} h_i & \text{if } x = g_i \\ 0 & \text{if } x \neq g_i \end{cases}$.

We see that $f = \alpha_1 + \alpha_2 + \cdots + \alpha_w \in I$. Thus $Ann\, S \subseteq I$ and $I = Ann\, S$. □

Corollary 3.6. *Assume $|H| \geq 3$. Then the ideals of M are $\{0\}$, M, and $Ann\, S$ with $H \subseteq S$.*

Now we assume $|H| = 2$. Note that by Theorem 2.4, M is a ring in this case.

Lemma 3.7. *Assume $|H| = 2$ and let $f \in Ann\, H$. Then $I = \{0, f\}$ is an ideal of the ring M.*

Proof. Since $|H| = 2$, it follows that $f + f = 0$. Thus $(I, +)$ is a subgroup of $(M, +)$.

Let $n \in M$ and $g \in G$. For $0 \in I$, we get $0 \circ n = 0 \in I$. Also, since $n(g) \in H$, it follows that $(f \circ n)(g) = f(n(g)) = 0$. Thus $f \circ n = 0 \in I$. Therefore I is a right ideal of M.

Next we verify that I is a left ideal of M. Let $n \in M$ and $g \in G$. For $0 \in I$, we get $n \circ 0 = 0 \in I$. For $f \in I$, we only need to show that $n \circ f = 0$ or $n \circ f = f$. Let $H = \{0, h\}$. If $n(h) = 0$, then $n(H) = 0$. Since $f(g) \in H$, it follows that $(n \circ f)(g) = n(f(g)) = 0$. Thus $n \circ f = 0 \in I$.

Now assume $n(h) = h$. If $f(g) = 0$, then $(n \circ f)(g) = n(f(g)) = n(0) = 0 = f(g)$. If $f(g) = h$, then $(n \circ f)(g) = n(f(g)) = n(h) = h = f(g)$. Therefore $n \circ f = f \in I$, and I is a left ideal of M. Hence I is an ideal of M. □

Theorem 3.8. *Assume $|H| = 2$ and let $J \subseteq Ann\, H$. Then J is an ideal of the ring M if and only if $(J, +)$ is a subgroup of $(Ann\, H, +)$.*

Proof. If J is an ideal of M, then clearly $(J, +)$ is a subgroup of $(M, +)$. Since $J \subseteq Ann\, H$, we also have that $(J, +)$ is a subgroup of $(Ann\, H, +)$. Now assume $(J, +)$ is a subgroup of $(Ann\, H, +)$. Then $(J, +)$ is a subgroup of $(M, +)$.

Let $0 \neq f \in J$. Let $I = \{0, f\}$. By Lemma 3.7, I is an ideal of M. Thus for all $n \in M$, $f \circ n \in I \subseteq J$ and $n \circ f \in I \subseteq J$. It follows that J is an ideal of M. □

Corollary 3.9. *Assume $|H| = 2$. Then the ideals of M are $\{0\}$, M, and J where $(J, +)$ is a subgroup of $(Ann\, H, +)$.*

In the case where $|H| \geq 3$, every proper ideal was of the form $Ann\, S$ with $H \subseteq S$. This is not the case when $|H| = 2$, as the next proposition illustrates.

Proposition 3.10. *Assume $|H| = 2$, and let $f \in Ann\, H$ such that $f(g_1), f(g_2) \neq 0$ for some distinct $g_1, g_2 \notin H$. Then the ideal $I = \{0, f\}$ is not of the form $Ann\, S$ with $H \subseteq S$.*

Proof. Assume $I = Ann\, S$ for some $H \subseteq S$. Since $f(g_1), f(g_2) \neq 0$, we conclude that $g_1, g_2 \notin S$. Let $H = \{0, h\}$. Define a function $f_1(x) = \begin{cases} 0 & \text{if } x \in S \cup \{g_1\} \\ h & \text{otherwise} \end{cases}$. Then $f_1 \in Ann\, S$. Since $f_1(g_1) = 0$, $f_1 \neq f$. Also, $f_1(g_2) = h \neq 0$ implies that $f_1 \neq 0$. Therefore $f_1 \in Ann\, S$, but $f_1 \notin I$, a contradiction. Thus I is not of the form $Ann\, S$ with $H \subseteq S$. □

Lastly, we describe elements of the factor nearring $M/Ann\, S$. To this end, we define $T = \{f : S \to H \mid f(0) = 0\}$, where $H \subseteq S$. Then T is a zero-symmetric nearring.

Theorem 3.11. *Let S be a subset of G with $H \subseteq S$. As nearrings, $M/Ann\, S \cong T$.*

Proof. Consider the mapping $\varphi : M \to T$ given by $\varphi(f) = f|_S$. It is straightforward to verify that φ is a nearring homomorphism with kernel $Ann\, S$. Let $f_1 \in T$. Define $f_2 : G \to G$ by $f_2(x) = \begin{cases} f_1(x) & \text{if } x \in S \\ 0 & \text{if } x \notin S \end{cases}$. Then $f_2 \in M$ and $\varphi(f_2) = f_2|_S = f_1$. Hence φ is an epimorphism. By the First Isomorphism Theorem ([11], Theorem 1.27), $M/Ann\, S \cong T$. □

We have examined the internal structure of the nearring $M = \{f : G \to G \mid f(G) \subseteq H \text{ and } f(0) = 0\}$. A possibility for future study is the non-zero-symmetric nearring $P = \{f : G \to G \mid f(G) \subseteq H\}$. The same properties explored in this paper could be considered for the nearring P. A generalization of the nearring M could also be investigated. Let $H_1 \geq H_2 \geq \cdots \geq H_n$ be a collection of subgroups of G and consider the nearring $Q = \{f : G \to G \mid f(0) = 0 \text{ and } f(G) \subseteq H_1, f(H_1) \subseteq H_2, \ldots, f(H_{n-1}) \subseteq H_n\}$. The study of the nearring Q may produce interesting results.

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