

Optimization of Forestry, Infrastructure and Fire Management

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ABSTRACT

Forests, sensitive to fires, cover large parts of our planet. Rational protection of forests against fires, forest fire management, is a very important topic area. Our planet is facing the serious problem of global warming. The probabilities of long dry periods and strong winds are increasing functions of a warmer climate. Heat, dry conditions and strong winds increase the probabilities that fires start. Furthermore, if a fire starts, the stronger winds make the fires spread more rapidly and the destruction increases. Under the influence of global warming, we may expect more severe problems in forestry caused by wild fires. For all of these reasons, it is essential to investigate and optimize the general principles of the combined forestry and wild fire management problem. In this process, we should integrate the infrastructure and the fire fighting resources in the system as decision variables in the optimization problem. First, analytical methods are used to determine general results concerning how the optimal decisions are affected by increasing wind speed. The total system is analyzed with one dimensional optimization. Then, different combinations of decisions are optimized. The importance of optimal coordination is demonstrated. Finally, a particular numerical version of the optimization problem is constructed and studied. The main results, under the influence of global warming, are the following: In order to improve the expected total results, we should reduce the stock level in the forests, increase the level of fuel treatment, increase the capacity of fire fighting resources and increase the density of the road network. The total expected present value of all activities in a forest region are reduced even if optimal adjustments are made. These results are derived via analytical optimization and comparative statics analysis. They have also been confirmed via a numerical nonlinear programming model where all decisions simultaneously are optimized.

Keywords: Optimization, Fire Management, Forestry, Infrastructure, Coordination.

INTRODUCTION

Forests, sensitive to fires, cover large parts of our planet. Rational protection of forests against fires, forest fire management, is a very important topic area. FAO (2001) is an international handbook on forest fire protection. This contains most aspects of fire theories and practical ways to handle real problems. Detailed examples from regulations and instructions in several countries are included. The report however contains no proofs that the methods are optimal and no fundamental empirical investigations are described. The focus is to give practical guidelines to real decision makers. The fundamental theories of forest fires have gained considerable attention during many years. Green (1983) investigates the shapes of simulated fires with different kinds of fuels. He finds that fires develop near-elliptic shapes in continuous fuels. If the fuels are discrete and patchy, then less regular patterns are typical. Alexander (1985) develops the study of elliptical fire growth models further. He determines a nonlinear function for the length to breadth ratio as a function of the wind speed. If the wind speed increases, then the length increases more than the breadth and the shape of the elliptical forest fire is adjusted. Later, it became popular to study forest fires via cellular automata models. Russo *et al.* (2014) developed such a model that could describe the fire development over time as a function of fuel and landscape properties and meteorological data. In the cellular automata study by Zheng *et al.* (2017), the impact of wind velocity on the

forest fire spreading pattern is investigated and described. Cruz *et al.* (2019) determine a very simple expression for the spread rate of a forest fire as a function of the wind speed. More detailed dynamic forest fire functions were derived by Crawl *et al.* (2017). Many different kinds of problems can be defined within the forest fire class. It is even possible to investigate and optimize tactical fire fighting principles based on fundamental cellular automata fire models. This has been shown by Alexandridis *et al.* (2011), who combine the fire model with a model of water bomber airplanes and attack rules. It is important to describe, define and handle the relevant decision problems correctly. An optimization model has to be defined where there is an objective function and a relevant description of the system under analysis. One central contribution is Finney (2005). He writes that fire effects must be evaluated on a common scale. The values of infrastructure, ecological values etc. that may be destroyed by fires must be evaluated in the same currency. Otherwise, the total planning of fire management can not be optimized. Rideout *et al.* (2008) continue in this direction. They argue for a unified economic theory of fire program analysis and optimization. They write that different fire program components, such as suppression, fuels and prevention, should not be handled in independent planning processes. An integrated fire system can give a better total solution for the same total cost.

The central idea, that fully integrated planning processes give better total results than partially integrated or decoupled planning processes, is also supported by Nemati *et al.* (2017) in a different kind of application. Other decision areas, that if possible should be coordinated, are maintenance management and statistical process control, as described by Rasay *et al.* (2018). These theoretical studies, focusing on different application areas, have obvious implications also in the area of forest fire management. When better total results are obtained, which often are the results of more coordinated decisions in different connected parts of the systems under analysis, then it is important that the shares of the extra profits obtained are efficiently distributed to the different part of the system. It is rational to motivate different actors to contribute to the best total solution. Often, the coordinated solutions give not only better economic results but also improvements for the environment.

This is reported by Hosseini-Motlagh *et al.* (2020). In forest fire management, under difficult conditions, we sometimes face dramatic fires that can not be managed by local resources. Then, it is important to realize this fact in time and to call for external fire fighting resources without extra delays. This kind of decision flexibility is present in the model developed in this paper. Similar problems are also handled by Yadegari *et al.* (2015) who design a flexible logistics network with different delivery paths. Yousefnejad *et al.* (2019) handle a related problem which includes finding the optimal strategy for acceptance or rejection of incoming orders. Of course, we may consider such orders to include orders to handle forest fires in different locations. Some of the decisions that may be adjusted, in order to handle the system including forests and forest fires, in a rational way, are connected to the "classical forestry variables", such as the stock level, the harvest level and the time interval between harvests. Even if these decisions do not necessarily affect the fire probabilities or the speed of fire spread, they can still be important to adjust since the total result is affected by the expected present value of the profits of forestry. If fires for some reason become more severe, it is rational to adjust the investment intensity in forest production, since the expected present value of the future harvest is reduced. Optimal forest management principles under the influence of exogenous stochastic processes are found in Lohmander (2000, 2007, 2019a and 2019b). These principles are relevant to the present study. In this analysis, however, the stochastic forest fires, are partly affected by endogenous variables that may be optimized. With different decisions affecting fuel treatment, the density of the road network and the capacity of the local fire fighting resources, the expected areas of burned forests change. What are the optimal ways to influence the future forest fires? Finney (2004) focuses on fuel treatment optimization. He finds that the spatial arrangement of the individual treatment units is very important to the expected results. Random patterns are inefficient. The relative fire spread rate is much lower if the fuel treatment is made in parallel strips. The relative spread rate is also a decreasing function of the fraction of the landscape that is treated. Palma *et al.* (2007) find that the harvesting of forest stands can reduce landscape flammability by fragmenting fuel continuity.

This makes it more difficult for fires to spread. Furthermore, the harvested stands can serve as anchor points for fire fighters during suppression operations. Wei *et al.* (2008) continue the optimization of fuel treatment and defines the objective function as the expected fire loss. The model locates fuel treatments by using a fire risk reduction map calculated via fire simulations. They have a total budget constraint and optimize the treatments based on alternative assumptions concerning the wind directions.

Introduction to the concrete application problem

The general problem is the following:

We want to optimize the expected present value of the complete system under analysis and control. We start by considering a presently not utilized forest area where the trees have a more or less natural size distribution. Our first decisions are to build new roads and to harvest a part of the trees, according to the CCF principles. These essentially say that you repeatedly make "thinning from above", which means that the largest trees are harvested. The rest of the trees continue to grow until some later point in time when the process is repeated. The forest continues to produce new seeds and trees via natural regeneration. In the first period, we face the costs of road building and the profits from the harvest of the large trees. We invest in local fire fighting resources, and fuel treatment is initiated in the area. During the following years, some fires are expected to appear. These imply costs of different kinds. Some parts of the fire fighting costs are caused by the local resources and other parts, during events of dramatic fires, are caused by the occasional use of external resources. Over time, the area of not earlier burnt forest decreases. Generally, the economic net values of the trees are strongly reduced after fires. The main focus of this study is the complete system and how the optimal decisions in this system should be adjusted to handle a warmer climate. As results of the warmer climate, stronger winds and longer periods with dry conditions are expected. Since more dry conditions and stronger winds influence the probabilities and spreads of wild fires in the same direction, this study focuses on the optimal changes of rational decisions under the influence of increasing wind speed. The general results should for these reasons be given a wider interpretation. They answer the questions: How are the optimal decisions and the expected total results affected by global warming?

First, analytical methods will be used to determine general results concerning how the optimal decisions are affected by increasing wind speed. We study the total system and consider one decision at a time. Then, we optimize different combinations of decisions. The importance of optimal coordination is demonstrated. In many countries, the different decisions concerning forest management, infrastructure and fire management are handled as separate issues. Many different organizations and decision processes are involved and rational coordination of the complete system can not be expected. Some regions in Sweden, where severe forest fires occurred in the year 2018, have completely changed and centralized the control of the fire management resources. SvT (2020).

It is important to be aware that the results derived with the analytical model are quite general. They are true for all possible parameter values, as long as the functional forms are relevant. Finally, numerical nonlinear optimization is used to simultaneously optimize all decisions. This includes the already defined structure of the analytical model and is expanded to cover even more aspects of the problem. The numerical results confirm the already proved general analytical comparative statics effects of a warmer climate. A numerical model makes it possible to rapidly and automatically derive optimal answers to more detailed versions of the problem. Locally optimal decisions, however, require locally relevant data and functions. Hopefully, the new methodology will soon be tested and used with data from many different forest regions of our planet.

Methods

The expected burned area and the dynamics of the unburned area

Each year, the expected burned area is B . This is randomly distributed over all forest areas, irrespective of if these areas have already been burned, or not.

The initial not burned area is A_0 .

Directly after the first year, year 0, the expected burned area of not earlier burned area, divided by the not earlier burned area, is k_0 .

$$k_0 = \frac{B(1-k)^0}{A_0(1-k)^0} = \frac{B}{A_0} \quad (1)$$

Next year, the corresponding ratio is k_1 .

$$k_1 = \frac{B(1-k)^1}{A_0(1-k)^1} = \frac{B}{A_0} \quad (2)$$

In some arbitrary future period, n , the ratio is k_n .

$$k_n = \frac{B(1-k)^n}{A_0(1-k)^n} = \frac{B}{A_0} \quad (3)$$

Hence, we see that $k_0 = k_1 = \dots = k_n = k = \frac{B}{A_0}$.

The area of unburned (not earlier burned) forest in the beginning of an arbitrary period $n, n \geq 0$, is.

$$A_U = A_0(1-k)^n \quad (4)$$

Now, we consider continuous time.

$$A_U = A_0(1-k)^t = A_0e^{\gamma t} \quad (5)$$

$$A_0e^{\gamma t} = A_0(1-k)^t, A_0 \neq 0 \quad (6)$$

$$e^{\gamma t} = (1-k)^t \quad (7)$$

$$e^{\gamma} = (1-k) \quad (8)$$

$$\ln(e^{\gamma}) = \ln(1-k) \quad (9)$$

$$\gamma = \ln(1-k) \quad (10)$$

$$\gamma \approx -k, 0 \leq k \leq 0.05 \quad (11)$$

Let

$$g = -\gamma \quad (12)$$

$$A_U(t) = A_0e^{-gt}, g \approx k \quad (13)$$

Decision variables

The following decision variables are considered for optimization. In different situations, some of them are considered as fixed or exogenous and in other cases, they are free variables that can be optimized.

S (m³/ha), the equilibrium stock level per hectare, is controlled via thinning from above. Of course, in an even more detailed analysis, the periodic changes of the stock level can be included. F (index) is the level of fuel treatment. D (hm) denotes the distance between parallel roads in the network and L (index) is the capacity of the local fire fighting unit.

Functions and parameters

w Expected wind speed during fires (m/s).

A_0 The initial not burned area (ha).

$A_U(t)$ The unburned (not earlier burned) area (ha).

$B = B(D, F, L, w)$ The expected burned area each year (ha/year).

$$B(.) = e^{(B_0 + B_D D + B_F F + B_L L + B_w w)}, B_D > 0, B_F < 0, B_L < 0, B_w > 0$$

r Real rate of interest in the capital market. $r > 0$.

t Time (years).

T Time horizon (years).

Z_0 Objective function in the analytical discrete time problem: The expected total present value (Euro).

Z Objective function in the analytical continuous time problem: The expected total present value (Euro).

$I = I(D, L)$ Investment cost (Euro).

$\pi_0 = \pi_0(D, S)$ Profit from forestry in year 0 (Euro).

$\pi_U = \pi_U(D, S)$ Profit from forestry in unburned areas (Euro/ha/year).

π_B Profit from forestry in burned areas (Euro/ha/year). Default value = 0.

$C_L = C_L(B, F)$ Local fire management cost (Euro/year).

$\lambda = \lambda(D, F, L, w)$ Expected number of missions when external fire fighting resources are used (N/year).

$$\lambda(.) = e^{(\lambda_0 + \lambda_D D + \lambda_F F + \lambda_L L + \lambda_w w)}, \lambda_D > 0, \lambda_F < 0, \lambda_L < 0, \lambda_w > 0$$

C_E Expected external cost of each fire fighting mission when external resources are used (Euro). (Note that, in the objective function, λ and C_E only appear as the product λC_E . As an alternative, we may consider the product to be a function $\lambda G_E = \Phi(D, F, L, w)$. This product is the expected cost of utilized external resources (Euro). The derived results would still be the same if

$$\Phi(.) = e^{(\Phi_0 + \Phi_D D + \Phi_F F + \Phi_L L + \Phi_w w)}, \Phi_D > 0, \Phi_F < 0, \Phi_L < 0, \Phi_w > 0.)$$

$g = g(D, F, L, w)$ Relative burned area per year.

$$g(.) = e^{(g_0 + g_D D + g_F F + g_L L + g_w w)}, g_D > 0, g_F < 0, g_L < 0, g_w > 0$$

The optimization problem

Let us start by defining the optimization problem in discrete time, with periods representing one year each. The objective function in discrete time is Z_0 , the total expected present value. Z_0 is negatively affected by the investment cost, I , the local fire management cost, C_L and the expected cost of utilized external resources, λC_E . The positive contributions to Z_0 come from π_0 , the profit from forestry in year 0, $\pi_U A_U(t)$, the profit from forestry in unburned areas, and $\pi_B (A_0 - A_U(t))$, the profits from forestry in burned areas. We consider a time horizon of T years and r is the rate of interest in continuous time in the capital market.

$$Z_0 = -I + \pi_0 + \sum_{t=0}^T e^{-rt} (\pi_U A_U(t) + \pi_B (A_0 - A_U(t)) - C_L - \lambda C_E) \quad (14)$$

The following reformulations simplify the later analysis.

$$Z_0 = -I + \pi_0 + \sum_{t=0}^T e^{-rt} (\pi_U A_0 e^{-gt} + \pi_B A_0 (1 - e^{-gt}) - C_L - \lambda C_E) \quad (15)$$

$$Z_0 = -I + \pi_0 + \sum_{t=0}^T e^{-rt} (A_0 (\pi_U e^{-gt} + \pi_B (1 - e^{-gt}))) - C_L - \lambda C_E \quad (16)$$

$$Z_0 = -I + \pi_0 + \sum_{t=0}^T e^{-rt} (A_0 (\pi_B + (\pi_U - \pi_B) e^{-gt})) - C_L - \lambda C_E \quad (17)$$

Transformation of the problem to continuous time simplifies the following analysis even further.

Here is the objective function in continuous time:

$$Z = -I + \pi_0 + \int_0^T e^{-rt} (A_0 (\pi_B + (\pi_U - \pi_B) e^{-gt}) - C_L - \lambda C_E) dt \quad (18)$$

$$Z = -I + \pi_0 + \int_0^T e^{-rt} (A_0 \pi_B - C_L - \lambda C_E) dt + \int_0^T e^{-(r+g)t} A_0 (\pi_U - \pi_B) dt \quad (19)$$

$$Z = -I + \pi_0 + (A_0 \pi_B - C_L - \lambda C_E) \int_0^T e^{-rt} dt + A_0 (\pi_U - \pi_B) \int_0^T e^{-(r+g)t} dt \quad (20)$$

This is the form of the objective function that will be used in the following analysis:

$$Z = -I + \pi_0 + (A_0 \pi_B - C_L - \lambda C_E) \left(\frac{1 - e^{-rT}}{r} \right) + A_0 (\pi_U - \pi_B) \left(\frac{1 - e^{-(r+g)T}}{r + g} \right) \quad (21)$$

Optimal Response to a Changing Climate via General Functions

Optimal consideration of the changing climate from the forestry perspective:

Optimal response via S .

The first order optimum condition is:

$$\frac{dZ}{dS} = \frac{d\pi_0}{dS} + A_0 \left(\frac{1 - e^{-(r+g)T}}{r+g} \right) \frac{d\pi_U}{dS} = 0 \quad (22)$$

This economically interpretable result follows:

$$-\frac{d\pi_0}{dS} = A_0 \left(\frac{1 - e^{-(r+g)T}}{r+g} \right) \frac{d\pi_U}{dS} \quad (23)$$

Denote the left hand side, LHS, and the right hand side, RHS.

The LHS can be interpreted as the "marginal cost of the stock level, S , invested at time 0", since it shows the marginal reduction of the profit at time 0 as a function of S . The RHS is the "expected marginal revenue", or more explicitly, the expected marginal increase of the expected present value of profits during time interval $(0, T)$. In the RHS expression, the rate of interest in the capital market and the gradually decreasing area of still not burned forest, are taken into account. Hence, the fundamental economic principle, that the marginal cost should equal the marginal revenue in an optimal solution, is confirmed.

Let $h_0 > 0$ denote the harvest in year 0. S_0 is the initial stock level, instantly before h_0 takes place. $P(D) > 0$ is the net price (= the price reduced by the costs of harvesting and terrain transport) per cubic meter as a function of the distance between roads.

$$\pi_0 = P(D)h_0A_0 \quad (24)$$

$$S = S_0 - h_0 \quad (25)$$

$$h_0 = S_0 - S \quad (26)$$

$$\pi_0 = P(D)(S_0 - S)A_0 \quad (27)$$

$$\frac{d\pi_0}{dS} = -P(D)A_0 < 0 \quad (28)$$

h_1 is the equilibrium harvest level after the first harvest h_0 .

$$\pi_U = P(D)h_1 \quad (29)$$

h_1 is determined from a logistic growth equation. m_1 is the intrinsic growth rate and K is the carrying capacity. The principles of the logistic growth equation are well presented by Braun (1983).

$$h_1 = m_1 S \left(1 - \frac{S}{K}\right) \quad (30)$$

$$h_1 = m_1 S - m_2 S^2, \quad m_1 > 0, m_2 = \frac{m_1}{K} > 0 \quad (31)$$

$$\pi_U = P(D)(m_1 S - m_2 S^2) \quad (32)$$

$$\frac{d\pi_U}{dS} = P(D)(m_1 - 2m_2 S) \quad (33)$$

$$\frac{dZ}{dS} = -P(D)A_0 + A_0 \left(\frac{1 - e^{-(r+g)T}}{r+g} \right) P(D)(m_1 - 2m_2 S) \quad (34)$$

The first order optimum condition is:

$$\frac{dZ}{dS} = P(D)A_0 \left(-1 + \left(\frac{1 - e^{-(r+g)T}}{r+g} \right) (m_1 - 2m_2 S) \right) = 0 \quad (35)$$

$$\frac{d^2Z}{dS^2} = -2P(D)A_0 m_2 \left(\frac{1 - e^{-(r+g)T}}{r+g} \right) \quad (36)$$

$$(P(D)A_0 m_2 > 0 \wedge (r+g)T > 0) \Rightarrow \frac{d^2Z}{dS^2} < 0 \quad (37)$$

The second order condition of a unique interior maximum is satisfied. Hence, the following equation, derived from the first order optimum condition, gives a unique maximum with respect to the stock level:

$$-1 + \left(\frac{1 - e^{-(r+g)T}}{r+g} \right) (m_1 - 2m_2 S) = 0 \quad (38)$$

$$\left(\frac{1 - e^{-(r+g)T}}{r+g} \right) (m_1 - 2m_2 S) = 1 \quad (39)$$

$$m_1 - 2m_2 S = \left(\frac{r+g}{1 - e^{-(r+g)T}} \right) \quad (40)$$

$$-2m_2 S = \left(\frac{r+g}{1 - e^{-(r+g)T}} \right) - m_1 \quad (41)$$

$$2m_2S = m_1 - \left(\frac{r+g}{1-e^{-(r+g)T}} \right) \quad (42)$$

We denote optimal values by stars. It is possible to derive an explicit function for the optimal stock level:

$$S^* = \frac{\left(m_1 - \left(\frac{r+g}{1-e^{-(r+g)T}} \right) \right)}{2m_2} \quad (43)$$

Observation:

Consider the following alternative problem:

Assume that we are interested to maximize the average harvest volume per year, h_x , after the initial harvest h_0 . The same growth function is applied. Denote the optimal stock level in this problem S_x .

$$\max_{S_x} h_x = m_1 S_x - m_2 S_x^2, \quad m_1 > 0, m_2 > 0 \quad (44)$$

The first order optimum condition is:

$$\frac{dh_x}{dS_x} = m_1 - 2m_2 S_x = 0 \quad (45)$$

The second order condition of a unique interior maximum is always satisfied:

$$\frac{d^2 h_x}{dS_x^2} = -2m_2 < 0 \quad (46)$$

$$\left(\frac{dh_x}{dS_x} = 0 \right) \Rightarrow \left(S_x^* = \frac{m_1}{2m_2} \right) \quad (47)$$

We note that

$$S_x^* > S^* = \frac{\left(m_1 - \left(\frac{r+g}{1-e^{-(r+g)T}} \right) \right)}{2m_2}, \quad 0 < (r+g), 0 < T \quad (48)$$

From the expression above, we instantly see that S^* increases if T increases.

The average harvest level h_x is an increasing function of S_x if $S_x < \frac{m_1}{2m_2}$.

$$(S_x^* > S^*) \Rightarrow (h_x > h_1), \quad 0 < (r+g), 0 < T \quad (49)$$

Hence, the optimal solution in the original problem is to keep the stock at a level that is strictly lower than the stock level that would maximize the average harvest level.

Furthermore, in the original problem, if T increases, we consider the value of future harvesting over a longer period. Then, we should harvest less in year 0, in order to increase the value of future harvesting.

Let us study how the following expression is affected by increasing $(r + g)$:

$$\eta = \left(\frac{1 - e^{-(r+g)T}}{r + g} \right) \quad (50)$$

$$\eta = \left(\frac{1 - e^{-aT}}{a} \right), \quad a = r + g > 0 \quad (51)$$

$$\frac{d\eta}{da} = \frac{1}{a^2} (Te^{-aT}a - (1 - e^{-aT})) \quad (52)$$

$$\frac{d\eta}{da} = \frac{1}{a^2} (Te^{-aT}a - 1 + e^{-aT}) \quad (53)$$

$$\frac{d\eta}{da} = \frac{1}{a^2} (e^{-aT}(1 + aT) - 1) \quad (54)$$

$$\frac{d\eta}{da} = \frac{e^{-aT}}{a^2} ((1 + aT) - e^{aT}) \quad (55)$$

Observation:

$$\zeta = aT \quad (56)$$

A Taylor approximation gives:

$$e^\zeta \approx \frac{e^0}{0!} + \frac{e^0}{1!}\zeta + \frac{e^0}{2!}\zeta^2 + \frac{e^0}{3!}\zeta^3 + \dots \quad (57)$$

$$e^\zeta \approx 1 + \zeta + \frac{1}{2!}\zeta^2 + \frac{1}{3!}\zeta^3 + \dots > (1 + \zeta), \zeta > 0 \quad (58)$$

$$(aT > 0) \Rightarrow ((1 + aT) - e^{aT} < 0) \quad (59)$$

$$\frac{d\eta}{da} = a^{-2} e^{-rT} \left((1+aT) - e^{aT} \right) < 0 \quad (60)$$

$$> 0 > 0 \quad < 0 \Big|_{aT > 0}$$

$$\left(\frac{d\eta}{da} < 0 \right) \Rightarrow \frac{d \left(\frac{1 - e^{-(r+g)T}}{r+g} \right)}{dg} < 0 \quad (61)$$

In the later part of this paper, we will also need to know the sign of $\frac{d^2\eta}{da^2}$.

$$\frac{d^2\eta}{da^2} = -2a^{-3} \left((1+aT)e^{-aT} - 1 \right) + a^{-2} \left(Te^{-aT} - T(1+aT)e^{-aT} \right) \quad (62)$$

$$\frac{d^2\eta}{da^2} = -a^{-2} \left(-2a^{-1} \left((1+aT)e^{-aT} - 1 \right) + Te^{-aT} \left(1 - (1+aT) \right) \right) \quad (63)$$

$$\frac{d^2\eta}{da^2} = -a^{-2} \left(-2a^{-1} \left((1+aT)e^{-aT} - 1 \right) - aT^2 e^{-aT} \right) \quad (64)$$

$$\frac{d^2\eta}{da^2} = -a^{-3} \left(2 \left((1+aT)e^{-aT} - 1 \right) + (aT)^2 e^{-aT} \right) \quad (65)$$

Let

$$\varphi = aT > 0 \quad (66)$$

$$\frac{d^2\eta}{da^2} = -a^{-3} \left(2 \left((1+\varphi)e^{-\varphi} - 1 \right) + \varphi^2 e^{-\varphi} \right) \quad (67)$$

$$\frac{d^2\eta}{da^2} = -a^{-3} \left((2 + 2\varphi + \varphi^2)e^{-\varphi} - 2 \right) \quad (68)$$

$$\frac{d^2\eta}{da^2} = -2a^{-3} e^{-\varphi} \left(\left(1 + \varphi + \frac{\varphi^2}{2} \right) - e^{\varphi} \right) > 0 \quad (69)$$

$$< 0 \Big|_{\varphi > 0}$$

$$\left(\frac{d^2\eta}{da^2} > 0 \right) \Rightarrow \frac{d^2 \left(\frac{1 - e^{-(r+g)T}}{r+g} \right)}{dg^2} > 0 \quad (70)$$

Now, we may use the knowledge concerning the function $\eta(\cdot)$ to determine the properties of $S^*(\cdot)$.

$$S^* = \frac{(m_1 - (\eta^{-1}))}{2m_2} \quad (71)$$

$$\frac{dS^*}{d\eta} = \frac{\eta^{-2}}{2m_2} > 0 \quad (72)$$

$$\frac{dS^*}{da} = \frac{dS^*}{d\eta} \frac{d\eta}{da} < 0 \quad (73)$$

$$\left((a = r + g) \wedge \frac{dS^*}{da} < 0 \right) \Rightarrow \left(\frac{dS^*}{dr} < 0 \wedge \frac{dS^*}{dg} < 0 \right) \quad (74)$$

Hence, if the rate of interest in the capital market and/or the relative burned area per year, for some reason(s) increase, then the optimal stock level decreases.

These results were derived via the explicit function of $S^*(\cdot)$. In the case of alternative growth functions, it is possible that an explicit function $S^*(\cdot)$ can not be determined. In such cases, we may also use the implicit method. We differentiate the first order optimum condition. Again, let $a = r + g$.

$$\frac{dZ}{dS} = P(D)A_0 \left(-1 + \left(\frac{1 - e^{-aT}}{a} \right) (m_1 - 2m_2S) \right) = 0 \quad (75)$$

$$d \left(\frac{dZ}{dS} \right) = \frac{d^2Z}{dS^2} dS^* + \frac{d^2Z}{dSda} da = 0 \quad (76)$$

$$\frac{d^2Z}{dSda} = P(D)A_0 (m_1 - 2m_2S) \left(\frac{d \left(\frac{1 - e^{-aT}}{a} \right)}{da} \right) \quad (77)$$

The existence of a solution to the first order optimum condition gives:

$$\left(\frac{dZ}{dS} = 0 \right) \Rightarrow \left(-1 + \left(\frac{1 - e^{-aT}}{a} \right) (m_1 - 2m_2S) = 0 \right) \Rightarrow ((m_1 - 2m_2S) > 0) \quad (78)$$

$$\operatorname{sgn}\left(\frac{d^2Z}{dSda}\right) = \operatorname{sgn}\left(\frac{d\left(\frac{1-e^{-aT}}{a}\right)}{da}\right) = \operatorname{sgn}\left(\frac{d\eta}{da}\right) < 0 \quad (79)$$

$$\left(d\left(\frac{dZ}{dS}\right) = 0\right) \Rightarrow \left(\frac{d^2Z}{dS^2} dS^* = -\frac{d^2Z}{dSda} da\right) \quad (80)$$

$$\frac{dS^*}{da} = -\frac{\left(\frac{d^2Z}{dSda}\right)}{\left(\frac{d^2Z}{dS^2}\right)} < 0 \quad (81)$$

Hence, we know that the optimal stock level, S^* , is a decreasing function of g and of r .

We recall that: $g = g(D, F, L, w)$ and that $\frac{dg}{dD} > 0$, $\frac{dg}{dF} < 0$, $\frac{dg}{dL} < 0$, $\frac{dg}{dw} > 0$.

Hence,

$$\frac{dS^*}{dD} < 0, \frac{dS^*}{dF} > 0, \frac{dS^*}{dL} > 0, \frac{dS^*}{dw} < 0 \quad (82)$$

This means that the optimal stock level is a decreasing function of the distance between roads and of the expected wind speed. The optimal stock level is an increasing function of the level of fuel treatment and of the capacity of the local fire fighting resources.

Optimal response via F .

The first order optimum condition is:

$$\frac{dZ}{dF} = \left(-\frac{dC_L}{dF} - \frac{d\lambda}{dF} C_E\right) \left(\frac{1-e^{-rT}}{r}\right) + A_0 (\pi_U - \pi_B) \frac{d\left(\frac{1-e^{-(r+g)T}}{r+g}\right)}{dg} \frac{dg}{dF} = 0 \quad (83)$$

We may extract the following equation:

$$\frac{dC_L}{dF} \left(\frac{1-e^{-rT}}{r}\right) = -\frac{d\lambda}{dF} C_E \left(\frac{1-e^{-rT}}{r}\right) + A_0 (\pi_U - \pi_B) \frac{d\left(\frac{1-e^{-(r+g)T}}{r+g}\right)}{dg} \frac{dg}{dF} \quad (84)$$

The LHS is the "marginal cost of F ", or more explicitly, the present value of the marginally increased local fire management cost in the time interval $(0, T)$, that follows from a marginally increased F . The RHS contains two components that together represent the "marginal revenue of F ". The first component is the expected present value of the marginal reduction of external costs of fire fighting in time interval $(0, T)$, resulting from the marginal reduction of the probabilities that fires become so many and so large that it is necessary to call for the external resources. The second component on the RHS is the expected present value of the marginally increased areas of harvesting of forests that have not burned. Of course, if F increases, the areas of such forests increases. Hence, also in this decision dimension, the economic principle that the marginal cost should equal the marginal revenue in optimal solutions, is confirmed. We assume that the second order condition of a unique interior maximum is satisfied. This can be demonstrated under particular functional assumptions. Such details may be handled within numerically specified model versions of relevance to local conditions and locally collected empirical data sets.

Hence,

$$\frac{d^2Z}{dF^2} < 0 \tag{85}$$

How is F^* affected if w increases?

$$\begin{aligned} \frac{d^2Z}{dFdW} = & - \left(\frac{d^2C_L}{dFdW} + \frac{d^2\lambda}{dFdW} C_E \right) \left(\frac{1-e^{-rT}}{r} \right) \\ & (0) \quad (<0) (>0) \quad (>0) \\ & + A_0 (\pi_U - \pi_B) \left(\frac{d^2 \left(\frac{1-e^{-(r+g)T}}{r+g} \right)}{dg^2} \frac{dg}{dw} \frac{dg}{dF} + \frac{d \left(\frac{1-e^{-(r+g)T}}{r+g} \right)}{dg} \frac{d^2g}{dFdW} \right) \tag{86} \\ & (>0) \quad (>0) \quad (>0) (<0) \quad (<0) \quad (<0) \\ & (small) \end{aligned}$$

$$\begin{aligned} \Xi_1 = & \frac{d^2 \left(\frac{1-e^{-(r+g)T}}{r+g} \right)}{dg^2} \frac{dg}{dw} \frac{dg}{dF} + \frac{d \left(\frac{1-e^{-(r+g)T}}{r+g} \right)}{dg} \frac{d^2g}{dFdW} \tag{87} \\ = & \frac{d^2\eta}{da^2} g_w g(\cdot) g_F g(\cdot) + \frac{d\eta}{da} g_F g_w g(\cdot) \end{aligned}$$

$$\Xi_1 = g_F g_w g(\cdot) \left(\frac{d^2\eta}{da^2} g(\cdot) + \frac{d\eta}{da} \right) \tag{88}$$

For typical parameter values, we have:

$$\frac{d^2\eta}{da^2} g(.) + \frac{d\eta}{da} < 0 \tag{89}$$

$$\left(g_F < 0 \wedge g_w > 0 \wedge g(.) > 0 \wedge \left(\frac{d^2\eta}{da^2} g(.) + \frac{d\eta}{da} \right) < 0 \right) \Rightarrow \Xi_1 > 0 \tag{90}$$

$$\frac{d^2Z}{dFdW} = - \frac{d^2\lambda}{dFdW} C_E \left(\frac{1-e^{-rT}}{r} \right) + A_0 (\pi_U - \pi_B) \Xi_1 > 0 \tag{91}$$

(-) < 0 > 0 > 0 > 0 > 0 > 0

$$\frac{d^2Z}{dFdW} > 0 \tag{92}$$

$$d \left(\frac{dZ}{dF} \right) = \frac{d^2Z}{dF^2} dF^* + \frac{d^2Z}{dFdW} dW = 0 \tag{93}$$

$$\frac{dF^*}{dW} = \frac{- \left(\frac{d^2Z}{dFdW} \right)}{\left(\frac{d^2Z}{dF^2} \right)} > 0 \tag{94}$$

Optimal response via S and F in combination.

Now, we have to satisfy two first order optimum conditions.

$$\begin{cases} \frac{dZ}{dS} = 0 \\ \frac{dZ}{dF} = 0 \end{cases} \tag{95}$$

The optimal changes of the decision variables under the influence of a changing climate can be determined via differentiation of the first order optimum conditions with respect to S^* , F^* and w .

$$\begin{cases} \frac{d^2Z}{dS^2} dS^* + \frac{d^2Z}{dSdF} dF^* + \frac{d^2Z}{dSdW} dW = 0 \\ \frac{d^2Z}{dFdS} dS^* + \frac{d^2Z}{dF^2} dF^* + \frac{d^2Z}{dFdW} dW = 0 \end{cases} \tag{96}$$

In this process, we need to know the signs of $\frac{d^2Z}{dSdF}$ and $\frac{d^2Z}{dFdS}$. Of course, $\frac{d^2Z}{dSdF} = \frac{d^2Z}{dFdS}$.

$$\frac{dZ}{dS} = \frac{d\pi_0}{dS} + A_0 \left(\frac{1 - e^{-(r+g)T}}{r+g} \right) \frac{d\pi_U}{dS} \quad (97)$$

Observation:

$$\frac{d\pi_U}{dS} > 0 \text{ since } \frac{d\pi_U}{dS} = P(D)(m_1 - 2m_2S) \text{ and } m_1 - 2m_2S = \left(\frac{r+g}{1 - e^{-(r+g)T}} \right) > 0.$$

$$\frac{d^2Z}{dSdF} = \frac{d^2\pi_0}{dSdF} + A_0 \left(\frac{d \left(\frac{1 - e^{-(r+g)T}}{r+g} \right)}{dg} \frac{dg}{dF} \right) \frac{d\pi_U}{dS} > 0 \quad (98)$$

(= 0) (> 0) (< 0) (< 0) (> 0)

$$\frac{d^2Z}{dSdF} = \frac{d^2Z}{dFdS} > 0 \quad (99)$$

$$\begin{bmatrix} \frac{d^2Z}{dS^2} & \frac{d^2Z}{dSdF} \\ \frac{d^2Z}{dFdS} & \frac{d^2Z}{dF^2} \end{bmatrix} \begin{bmatrix} dS^* \\ dF^* \end{bmatrix} = \begin{bmatrix} -\frac{d^2Z}{dSdw} dw \\ -\frac{d^2Z}{dFdW} dw \end{bmatrix} \quad (100)$$

$$\begin{bmatrix} \frac{d^2Z}{dS^2} & \frac{d^2Z}{dSdF} \\ \frac{d^2Z}{dFdS} & \frac{d^2Z}{dF^2} \end{bmatrix} \begin{bmatrix} \frac{dS^*}{dw} \\ \frac{dF^*}{dw} \end{bmatrix} = \begin{bmatrix} -\frac{d^2Z}{dSdw} \\ -\frac{d^2Z}{dFdW} \end{bmatrix} \quad (101)$$

$$\frac{dS^*}{dw} = \frac{\begin{vmatrix} -\frac{d^2Z}{dSdw} & \frac{d^2Z}{dSdF} \\ -\frac{d^2Z}{dFdW} & \frac{d^2Z}{dF^2} \end{vmatrix}}{\begin{vmatrix} \frac{d^2Z}{dS^2} & \frac{d^2Z}{dSdF} \\ \frac{d^2Z}{dFdS} & \frac{d^2Z}{dF^2} \end{vmatrix}} = \frac{-\frac{d^2Z}{dSdw} \frac{d^2Z}{dF^2} + \frac{d^2Z}{dFdW} \frac{d^2Z}{dSdF}}{\frac{d^2Z}{dS^2} \frac{d^2Z}{dF^2} - \frac{d^2Z}{dFdS} \frac{d^2Z}{dSdF}} \quad (102)$$

Observation:

Recall that $a = r + g$.

$$\left(\operatorname{sgn} \left(\frac{d^2 Z}{dSdw} \right) = \operatorname{sgn} \left(\frac{d^2 Z}{dSda} \right) \right) \Rightarrow \left(\frac{d^2 Z}{dSdw} < 0 \right) \quad (103)$$

$$\left(\text{If } \frac{d^2 Z}{dFdS} = 0 \right) \Rightarrow \left(\frac{dS^*}{dw} = - \frac{\frac{d^2 Z}{dSdw}}{\frac{d^2 Z}{dS^2}} < 0 \right) \quad (104)$$

$$\frac{dF^*}{dw} = \frac{\left| \begin{array}{cc} \frac{d^2 Z}{dS^2} & - \frac{d^2 Z}{dSdw} \\ \frac{d^2 Z}{dFdS} & - \frac{d^2 Z}{dFdW} \end{array} \right|}{\left| \begin{array}{cc} \frac{d^2 Z}{dS^2} & \frac{d^2 Z}{dSdF} \\ \frac{d^2 Z}{dFdS} & \frac{d^2 Z}{dF^2} \end{array} \right|} = \frac{- \frac{d^2 Z}{dS^2} \frac{d^2 Z}{dFdW} + \frac{d^2 Z}{dFdS} \frac{d^2 Z}{dSdw}}{\frac{d^2 Z}{dS^2} \frac{d^2 Z}{dF^2} - \frac{d^2 Z}{dFdS} \frac{d^2 Z}{dSdF}} \quad (105)$$

Observation:

$$\left(\text{If } \frac{d^2 Z}{dFdS} = 0 \right) \Rightarrow \left(\frac{dF^*}{dw} = \frac{- \frac{d^2 Z}{dFdW}}{\frac{d^2 Z}{dF^2}} > 0 \right) \quad (106)$$

A deeper investigation of $\left(\frac{dF^*}{dw}, \frac{dS^*}{dw} \right)$:

We differentiate the two first order optimum conditions with respect to S , F and w .

$$\frac{d^2 Z}{dS^2} dS^* + \frac{d^2 Z}{dSdF} dF^* + \frac{d^2 Z}{dSdw} dw = 0 \quad (a1) \quad (107)$$

$$\frac{d^2 Z}{dFdS} dS^* + \frac{d^2 Z}{dF^2} dF^* + \frac{d^2 Z}{dFdW} dw = 0 \quad (a2) \quad (108)$$

Let $dw = 0$.

\mathcal{G}_1 is the slope, $\frac{dS}{dF}$, of equation (a1).

$$\frac{d^2 Z}{dS^2} dS + \frac{d^2 Z}{dSdF} dF = 0 \quad (109)$$

$$\mathcal{G}_1 = \frac{dS}{dF} = \frac{-\frac{d^2Z}{dSdF}}{\frac{d^2Z}{dS^2}} > 0 \quad (110)$$

\mathcal{G}_2 is the slope, $\frac{dS}{dF}$, of equation (a2).

$$\frac{d^2Z}{dFdS} dS + \frac{d^2Z}{dF^2} dF = 0 \quad (111)$$

$$\mathcal{G}_2 = \frac{dS}{dF} = \frac{-\frac{d^2Z}{dF^2}}{\frac{d^2Z}{dFdS}} > 0 \quad (112)$$

If we have a unique interior maximum, the following condition has to be fulfilled:

$$\begin{vmatrix} \frac{d^2Z}{dS^2} & \frac{d^2Z}{dSdF} \\ \frac{d^2Z}{dFdS} & \frac{d^2Z}{dF^2} \end{vmatrix} = \frac{d^2Z}{dS^2} \frac{d^2Z}{dF^2} - \frac{d^2Z}{dFdS} \frac{d^2Z}{dSdF} > 0 \quad (113)$$

$$\left(\frac{d^2Z}{dS^2} \right) \left(\frac{d^2Z}{dFdS} \right) \left(\frac{\left(\frac{d^2Z}{dF^2} \right)}{\left(\frac{d^2Z}{dFdS} \right)} - \frac{\left(\frac{d^2Z}{dSdF} \right)}{\left(\frac{d^2Z}{dS^2} \right)} \right) > 0 \quad (114)$$

(< 0) (> 0)

Hence,

$$\left(\frac{\left(\frac{d^2Z}{dF^2} \right)}{\left(\frac{d^2Z}{dFdS} \right)} - \frac{\left(\frac{d^2Z}{dSdF} \right)}{\left(\frac{d^2Z}{dS^2} \right)} \right) < 0 \quad (115)$$

$$\mathcal{G}_1 = \frac{-\left(\frac{d^2Z}{dSdF} \right)}{\left(\frac{d^2Z}{dS^2} \right)} < \frac{-\left(\frac{d^2Z}{dF^2} \right)}{\left(\frac{d^2Z}{dFdS} \right)} = \mathcal{G}_2 \quad (116)$$

Observation:

The intersection of (a1) and (a2) gives the optimal combination of adjustments of S^* and F^* . If $\frac{d^2Z}{dSdF} = 0$, then, $\frac{dS^*}{dw} < 0$ and $\frac{dF^*}{dw} > 0$ and can be derived as the one dimensional optimization results in the earlier sections. If we have a unique interior maximum of the objective function with respect to S and F , and $\frac{d^2Z}{dSdF} > 0$, then the absolute values of the adjustments of S^* and F^* under the influence of increasing w are reduced. Hence, $\frac{dS^*}{dw}$ is strictly negative, but less negative than if $\frac{d^2Z}{dSdF}$ is zero. $\frac{dF^*}{dw}$ is strictly positive, but less positive than if $\frac{d^2Z}{dSdF}$ is zero. Compare Fig. 1.

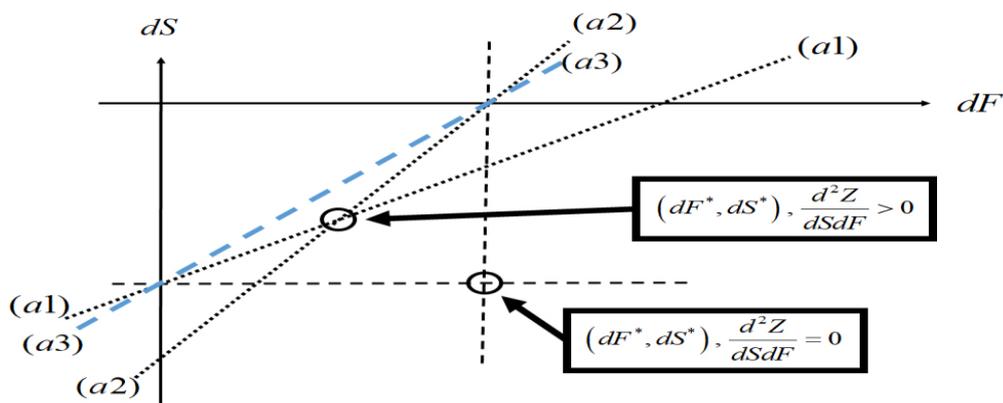


Fig. 1. Optimal adjustments of F^* and S^* if w increases, and the point (F^*, S^*) gives a unique interior maximum of Z .

$dF^* > 0$ and $dS^* < 0$. The absolute values of dF^* and dS^* are strictly decreasing functions of $\frac{d^2Z}{dSdF}$. Along line (a3), $\mathcal{G}_1 = \mathcal{G}_2$. Hence, optimal adjustment combinations are always located below line (a3).

Optimal consideration of the climate from the fire brigade perspective:

Optimal response via L . The first order optimum condition is:

$$\frac{dZ}{dL} = -\frac{dI}{dL} - \frac{d\lambda}{dL} C_E \left(\frac{1-e^{-rT}}{r} \right) + A_0 (\pi_U - \pi_B) \frac{d \left(\frac{1-e^{-(r+g)T}}{r+g} \right)}{dg} \frac{dg}{dL} = 0 \tag{117}$$

The next equation follows:

$$\frac{dI}{dL} = -\frac{d\lambda}{dL} C_E \left(\frac{1-e^{-rT}}{r} \right) + A_0 (\pi_U - \pi_B) \frac{d \left(\frac{1-e^{-(r+g)T}}{r+g} \right)}{dg} \frac{dg}{dL} \tag{118}$$

The LHS is the marginal investment cost in local fire fighting capacity. The RHS, can be interpreted as the "marginal revenue", containing two parts: The first part is the expected marginal present value of reduced costs of external fire fighting missions in time interval $(0, T)$. The second part is the marginal expected increase of the present value of profits from forestry because of the marginal reduction of burned areas in the same time interval. As expected, the "marginal cost" equals the "marginal revenues" also in this decision dimension. We assume that the optimum is a unique maximum with respect to L .

$$\begin{aligned} \frac{d^2Z}{dLdw} = & -\frac{d^2\lambda}{dLdw} C_E \left(\frac{1-e^{-rT}}{r} \right) \\ & + A_0 (\pi_U - \pi_B) \left(\frac{d^2 \left(\frac{1-e^{-(r+g)T}}{r+g} \right)}{dg^2} \frac{dg}{dL} \frac{dg}{dw} + \frac{d \left(\frac{1-e^{-(r+g)T}}{r+g} \right)}{dg} \frac{d^2g}{dLdw} \right) \end{aligned} \quad (119)$$

$$\begin{aligned} \frac{d^2Z}{dLdw} = & -\frac{d^2\lambda}{dLdw} C_E \left(\frac{1-e^{-rT}}{r} \right) \\ & (-) < 0 > 0 > 0 \\ & + A_0 (\pi_U - \pi_B) \left(\frac{d^2 \left(\frac{1-e^{-(r+g)T}}{r+g} \right)}{dg^2} \frac{dg}{dL} \frac{dg}{dw} + \frac{d \left(\frac{1-e^{-(r+g)T}}{r+g} \right)}{dg} \frac{d^2g}{dLdw} \right) \\ & > 0 > 0 > 0 < 0 > 0 < 0 < 0 \end{aligned} \quad (120)$$

$$\begin{aligned} \Xi_2 = & \frac{d^2 \left(\frac{1-e^{-(r+g)T}}{r+g} \right)}{dg^2} \frac{dg}{dL} \frac{dg}{dw} + \frac{d \left(\frac{1-e^{-(r+g)T}}{r+g} \right)}{dg} \frac{d^2g}{dLdw} \\ = & \frac{d^2\eta}{da^2} g_L g_w g(.) + \frac{d\eta}{da} g_L g_w g(.) \end{aligned} \quad (121)$$

$$\Xi_2 = g_L g_w g(.) \left(\frac{d^2\eta}{da^2} g(.) + \frac{d\eta}{da} \right) \quad (122)$$

For typical parameter values, we have:

$$\frac{d^2\eta}{da^2} g(.) + \frac{d\eta}{da} < 0 \quad (123)$$

$$\left(g_L < 0 \wedge g_w > 0 \wedge g(.) > 0 \wedge \left(\frac{d^2\eta}{da^2} g(.) + \frac{d\eta}{da} \right) < 0 \right) \Rightarrow \Xi_2 > 0 \quad (124)$$

$$\frac{d^2Z}{dLdw} = -\frac{d^2\lambda}{dLdw} C_E \left(\frac{1-e^{-rT}}{r} \right) + A_0 (\pi_U - \pi_B) \Xi_2 > 0$$

$$(-) < 0 > 0 > 0 > 0 > 0 > 0$$
(125)

$$d \left(\frac{dZ}{dL} \right) = \frac{d^2Z}{dL^2} dL^* + \frac{d^2Z}{dLdw} dw = 0$$

$$< 0 > 0$$
(126)

$$\frac{d^2Z}{dL^2} dL^* = -\frac{d^2Z}{dLdw} dw$$
(127)

$$\frac{dL^*}{dw} = \frac{-\left(\frac{d^2Z}{dLdw} \right)}{\left(\frac{d^2Z}{dL^2} \right)} > 0$$
(128)

Hence, a clear conclusion is that the optimal capacity of local fire fighting capacity is an increasing function of the different consequences of a warmer climate.

Observation

The share of costs for forest fire fighting is critical to the optimal investment intensity. It is common that the costs of fire damages in forests are much lower than the costs of other types of fires in areas with high population densities.

The capacity of the fire fighting resources should be optimized for both forest fires and the other types of fires. Hence, close to large cities, the fire fighting capacity is mostly much higher than in remote forest areas. Forestry close to cities benefits from this extra security level and the forest damages can be expected to be much lower than in sparsely populated areas, even if no special forest fire fighting investments have been made. On the other hand, close to cities, forest fires are sometimes caused by people, via campfires, accidents, smoking etc.

Optimal consideration of the climate from the infrastructure perspective:

Optimal response via D .

The first order optimum condition is:

$$\frac{dZ}{dD} = -\frac{dI}{dD} + \frac{d\pi_0}{dD} - \frac{d\lambda}{dD} C_E \left(\frac{1-e^{-rT}}{r} \right)$$

$$+ A_0 \left(\frac{d\pi_U}{dD} \left(\frac{1-e^{-(r+g)T}}{r+g} \right) + (\pi_U - \pi_B) \frac{d \left(\frac{1-e^{-(r+g)T}}{r+g} \right)}{dg} \frac{dg}{dD} \right)$$
(129)

As a consequence we get following expression:

$$-\frac{dI}{dD} = -\frac{d\pi_0}{dD} + \frac{d\lambda}{dD} C_E \left(\frac{1-e^{-rT}}{r} \right) - A_0 \frac{d\pi_U}{dD} \left(\frac{1-e^{-(r+g)T}}{r+g} \right) - A_0 (\pi_U - \pi_B) \frac{d \left(\frac{1-e^{-(r+g)T}}{r+g} \right)}{dg} \frac{dg}{dD} \quad (130)$$

The LHS is the "marginal cost", more explicitly, the *cost of marginal reduction of distance* between parallel roads. In this way, the marginal cost is the marginal cost of marginally increasing the density of the road network, which means that more roads will be constructed.

The RHS is the "marginal revenue" of a more dense road network, containing four components. These are:

- A marginally increasing initial profit from forestry (at time 0), caused by shorter terrain transports.
- A marginal reduction of the present value of expected external fire fighting missions, since the fires will be easier to keep small with local fire fighting resources via the marginally more dense road network.
- Marginal expected present value of increased profits in forestry because of shorter terrain transport distances during time interval $(0, T)$.
- Marginal expected present value of increased profits from forestry because of the marginal reduction of the size of the burned areas in time interval $(0, T)$.

The famous economic principle that the marginal costs should equal the marginal revenues, is satisfied also in the road density dimension. We assume that the optimum is a unique maximum with respect to D .

$$\frac{d^2Z}{dDdw} = -\frac{d\lambda^2}{dDdw} C_E \left(\frac{1-e^{-rT}}{r} \right) + A_0 \frac{d\pi_U}{dD} \frac{d\eta}{da} \frac{da}{dg} \frac{dg}{dw} + A_0 (\pi_U - \pi_B) \left(\frac{d^2\eta}{da^2} \frac{da}{dg} \frac{dg}{dD} \frac{da}{dg} \frac{dg}{dw} + \frac{d\eta}{da} \frac{da}{dg} \frac{d^2g}{dDdw} \right) \quad (131)$$

$$\frac{da}{dg} = 1 \quad (132)$$

$$\begin{aligned} \frac{d^2Z}{dDdw} &= -\frac{d\lambda^2}{dDdw} C_E \left(\frac{1-e^{-rT}}{r} \right) + A_0 \frac{d\pi_U}{dD} \frac{d\eta}{da} \frac{dg}{dw} \\ &(-) > 0 > 0 > 0 > 0 < 0 < 0 > 0 \\ &+ A_0 (\pi_U - \pi_B) \left(\frac{d^2\eta}{da^2} \frac{dg}{dD} \frac{dg}{dw} + \frac{d\eta}{da} \frac{d^2g}{dDdw} \right) \\ &> 0 > 0 > 0 > 0 > 0 < 0 > 0 \end{aligned} \quad (133)$$

$$\Xi_3 = \frac{d^2\eta}{da^2} \frac{dg}{dD} \frac{dg}{dw} + \frac{d\eta}{da} \frac{d^2g}{dDdw} = \frac{d^2\eta}{da^2} g_D g(\cdot) g_w g(\cdot) + \frac{d\eta}{da} g_D g_w g(\cdot) \quad (134)$$

$$\Xi_3 = g_D g_w g(\cdot) \left(\frac{d^2\eta}{da^2} g(\cdot) + \frac{d\eta}{da} \right) \quad (135)$$

For typical parameter values, we have:

$$\frac{d^2\eta}{da^2} g(.) + \frac{d\eta}{da} < 0 \tag{136}$$

$$\left(g_D > 0 \wedge g_w > 0 \wedge g(.) > 0 \wedge \left(\frac{d^2\eta}{da^2} g(.) + \frac{d\eta}{da} \right) < 0 \right) \Rightarrow \Xi_3 < 0 \tag{137}$$

$$\frac{d^2Z}{dDdw} = -\frac{d\lambda^2}{dDdw} C_E \left(\frac{1-e^{-rT}}{r} \right) + A_0 \frac{d\pi_U}{dD} \frac{d\eta}{da} \frac{dg}{dw} + A_0 (\pi_U - \pi_B) \Xi_3 \tag{138}$$

$(-) > 0 \quad > 0 \quad > 0 \quad > 0 < 0 < 0 > 0 \quad > 0 \quad > 0 \quad < 0$

Mostly, when distances between roads are close to optimal, the forestry profit is not very sensitive to marginal changes of the distances between roads.

$$\left| \frac{d\pi_U}{dD} \right| \text{small} \Rightarrow \frac{d^2Z}{dDdw} < 0 \tag{139}$$

$$d \left(\frac{dZ}{dD} \right) = \frac{d^2Z}{dD^2} dD^* + \frac{d^2Z}{dDdw} dw = 0 \tag{140}$$

$< 0 \quad < 0$

$$\frac{d^2Z}{dD^2} dD^* = -\frac{d^2Z}{dDdw} dw \tag{141}$$

$$\frac{dD^*}{dw} = \frac{-\left(\frac{d^2Z}{dDdw} \right)}{\left(\frac{d^2Z}{dD^2} \right)} < 0 \tag{142}$$

Hence, in typical cases, the optimal distances between parallel roads are decreasing functions of the different consequences of a warmer climate.

Optimal consideration of the climate from the forest, infrastructure and fire brigade perspective:

Optimal response via S , F , D and L in combination.

In case the optimal levels of all decision variables are strictly positive, $(S^* > 0 \wedge F^* > 0 \wedge L^* > 0 \wedge D^* > 0)$

, which is not always obvious, then:, the first order optimum conditions are:

$$\begin{cases} \frac{dZ}{dS} = 0 \\ \frac{dZ}{dF} = 0 \\ \frac{dZ}{dL} = 0 \\ \frac{dZ}{dD} = 0 \end{cases} \quad (143)$$

The second order conditions of a unique interior maximum are:

$$\left| Z_{SS} \right| < 0, \begin{vmatrix} Z_{SS} & Z_{SF} \\ Z_{FS} & Z_{FF} \end{vmatrix} > 0, \begin{vmatrix} Z_{SS} & Z_{SF} & Z_{SL} \\ Z_{FS} & Z_{FF} & Z_{FL} \\ Z_{LS} & Z_{LF} & Z_{LL} \end{vmatrix} < 0, \begin{vmatrix} Z_{SS} & Z_{SF} & Z_{SL} & Z_{SD} \\ Z_{FS} & Z_{FF} & Z_{FL} & Z_{FD} \\ Z_{LS} & Z_{LF} & Z_{LL} & Z_{LD} \\ Z_{DS} & Z_{DF} & Z_{DL} & Z_{DD} \end{vmatrix} > 0 \quad (144)$$

RESULTS

A numerical version of the analytical model was developed and tested. In the numerical analysis, all decisions were simultaneously optimized. The optimal decisions were calculated with nonlinear programming for alternative values of the expected wind speed parameter. This, in turn, is an indicator of the level of global warming. Of course, the many particular numerical parameter values in the numerical optimization model influence the model results. However, the numerically specific changes of the optimal decisions and expected values under the influence of stronger winds and a warmer climate confirm the general analytical results derived in the earlier sections of this paper. Below, we will compare the analytical and numerical results. The numerical model is found in the Appendix. We know, from the analytical one and two-dimensional optimization and comparative statics analysis, that and how the optimal stock level in the forest is affected by different parameters,

including the expected wind speed. $\frac{dS^*}{dD} < 0$, $\frac{dS^*}{dF} > 0$, $\frac{dS^*}{dL} > 0$, $\frac{dS^*}{dw} < 0$. As expected, the numerical

multidimensional optimization model confirms that $\frac{dS^*}{dw} < 0$. This is found in Fig. 2.

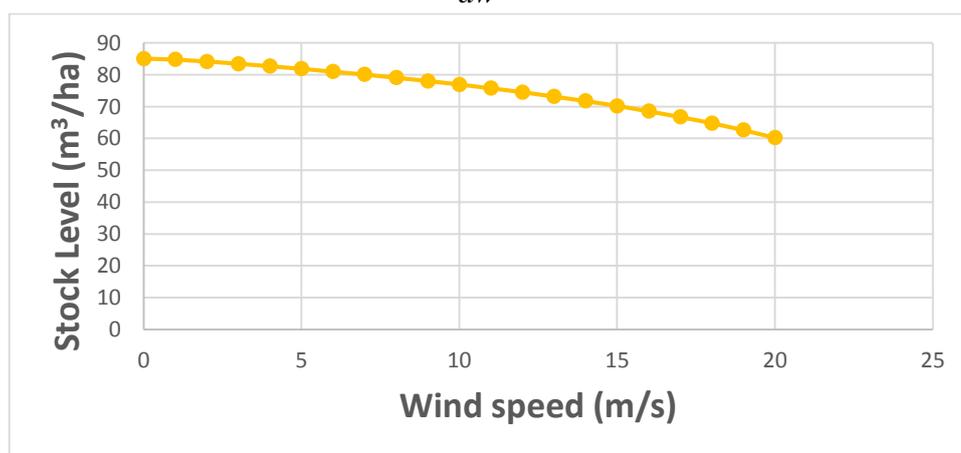


Fig. 2. The optimal stock level in the forest as a function of the expected wind speed. Result of numerical optimization.

Via analytical one and two dimensional optimization with comparative statics analysis, we know that $\frac{dF^*}{dw} > 0$

This is also confirmed via the numerical optimization. Figure 3. shows that the optimal fuel treatment level increases if the expected wind speed increases, which in turn can be considered to be an effect of global warming.

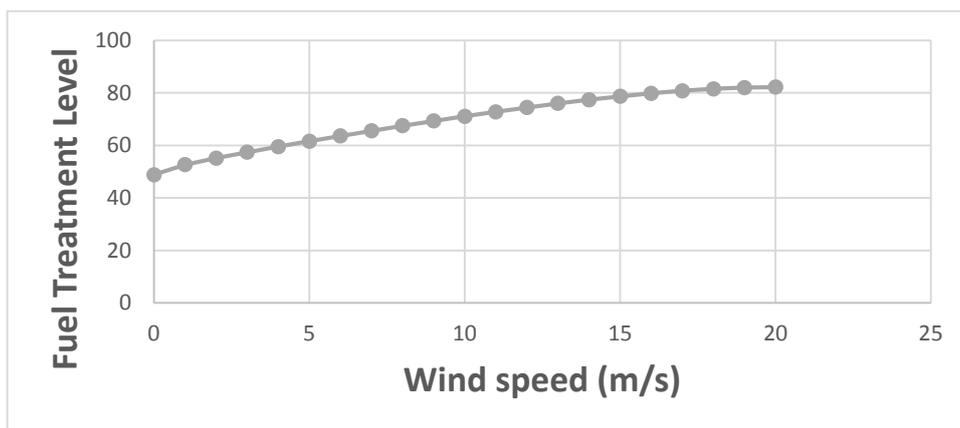


Fig. 3. The optimal fuel treatment level as a function of the expected wind speed. Result of numerical optimization. With more frequent and larger fires, it is rational to invest in more fire fighting capacity. This is also consistent with the analytical finding, $\frac{dL^*}{dw} > 0$ and the optimal numerical results found in Fig. 4.

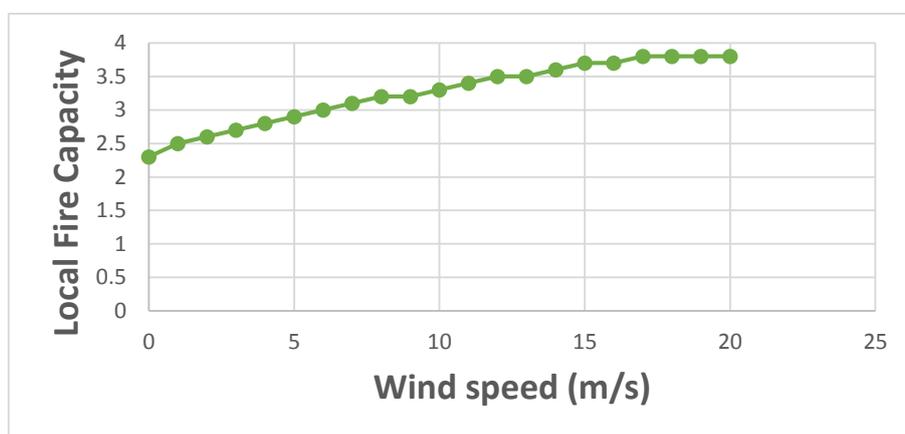


Fig. 4. The optimal level of the local fire fighting capacity as a function of the expected wind speed. Result of numerical optimization.

If the density of the road network increases, it is easier to fight the fires and to reduce the size of them. Hence, when the probabilities of more and larger fires increase, because of global warming and stronger winds, we should invest in a more dense road network. This is what the analytical results tell us, $\frac{dD^*}{dw} < 0$ and this is what Figure 5. reports from the numerical optimization.

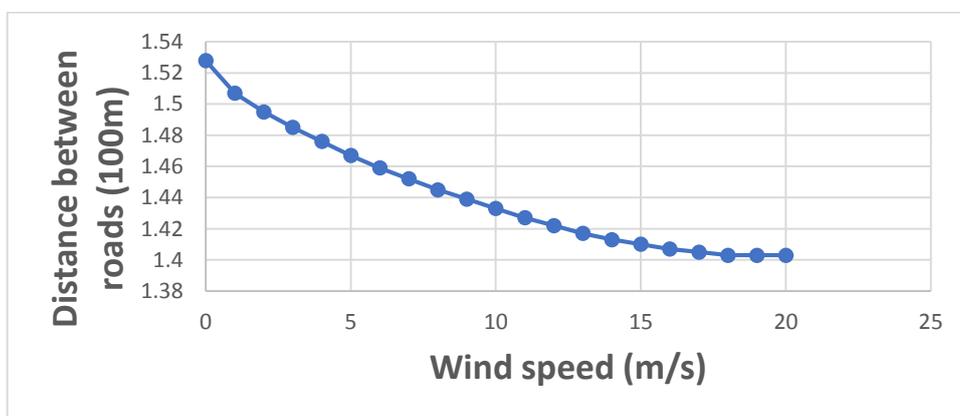


Fig. 5. The optimal distance between roads as a function of the expected wind speed. Result of numerical optimization.

As the expected wind speed increases, the numerical optimization results also tell us that the expected number of occasions when it is necessary to call for external help, maybe from other countries, to get access to water bombing airplanes and other necessary heavy equipment, increases. The particular numbers shown in Figure 6 are optimized and they are affected by many particular conditions in the optimization model. Of course, if the capacity of the local fire fighting resources would have been lower than optimal, then the expected number or times when external resources are needed would be higher. As a consequence, the total expected net present value would then have been lower than optimal.

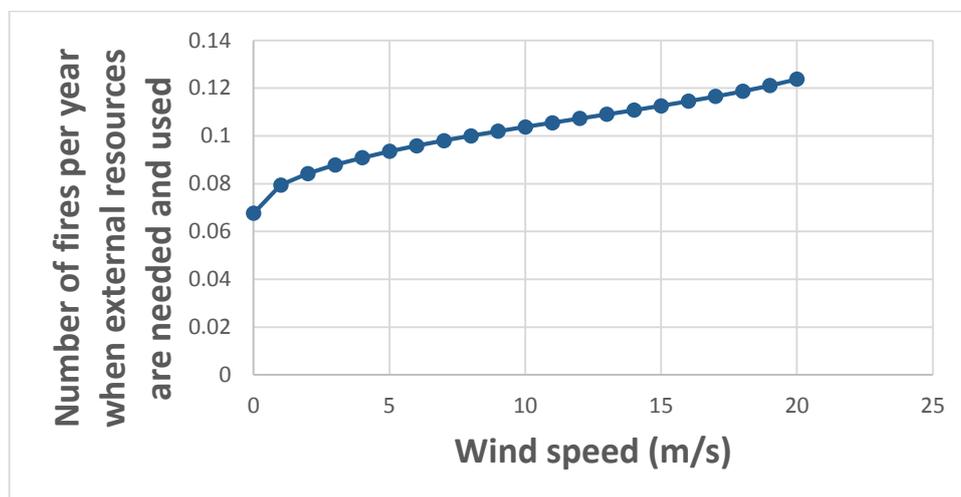


Fig. 6. The expected number of fires per year when external fire fighting resources are needed and used as a function of the expected wind speed. Result of numerical optimization.

Even if all decisions are optimally adjusted to more difficult wind conditions, Fig. 7. shows that the optimal total present value is a decreasing function of the expected wind speed. If the decisions would not have been optimized, then the expected present value would have been even more negatively affected by winds.

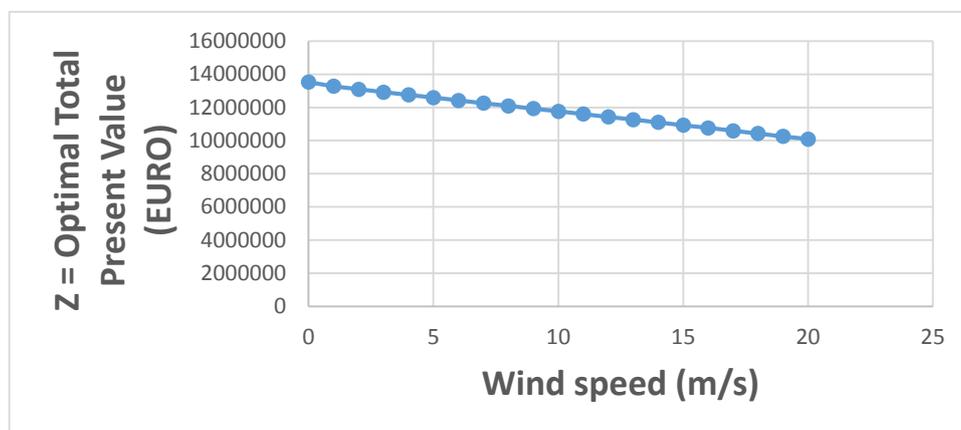


Fig. 7. The optimal expected total present value as a function of the expected wind speed. Result of numerical optimization.

DISCUSSION

In this paper, it has been found that several decisions of relevance to forestry, fire management and infrastructure should change in particular directions under the influence of global warming and stronger winds. These findings also have relevance to the struggle against global warming. Our planet is facing this serious problem. Recent research by Lohmander (2020a) has shown that the increasing level of CO₂ in the atmosphere can be explained as a dynamic function of the net emissions of CO₂. One way to reduce the CO₂ level in the atmosphere obviously is to reduce the industrial emissions and/or to capture and store the CO₂ via CCS, Carbon Capture and Storage. However, we may also increase the utilization of the large and presently almost not utilized forest areas of the world for more rational activities, as stated and proved by Lohmander (2020b). There, presently, we find more or less natural forests. There, the trees grow until they fall in storms and/or are consumed by insects and/or burn in

large wild forest fires. We find large forest areas of this type in particular in Russian Federation and Canada. Large parts of these areas can be transformed to productive forests utilized via CCF, Continuous Cover Forestry, principles. With CCF, the areas are continuously covered with trees and it is possible to maintain a rather natural type of environment, which can be acceptable as habitat for most species in these areas. In order to make this forestry rational, it is necessary to make the complete system optimal. This means several things: The intensity in forest production should be optimized. Stock levels and harvest intervals need to be decided. Infrastructure has to be available, essentially in the form of roads and railroads with the optimal capacities and densities. Since we want to avoid to destroy all of the forests by fires, we have to invest in rational solutions. In the forests, fuel treatment can be used to reduce the probabilities that fires start and rapidly spread. If we increase the road network density, it is easier to reach the fires with efficient equipment and to stop the fires along the roads, to contain them. Furthermore, some fire fighting equipment and fire fighters have to be available, and the capacities and initial locations of these resources should be optimized. Sometimes, very large fires can be expected. Then, the option to call in external resources, such as water bombing air planes and helicopters, have to be available. All of the different resources and decisions have to be combined in the best possible way in order to obtain the optimal expected total result. The expanded forestry and fire management will result in an increasing level of CO₂ absorption, which is beneficial to the general climate development. However, there are important other effects that have to be considered. First, a considerable part of the produced raw materials from the forests can be used to replace fossil fuels in the energy industry. In particular in the CHP, Combined Heat and Power, plants, it is possible and already common, to replace fossil coal and oil by forest raw materials. Furthermore, this way, electricity can be produced that is useful also in the transport sector via electrical cars and railroads. Electricity can also be useful in the production of hydrogen, another car fuel, without harmful emissions. Finally, meteorology tells us that the probabilities of long dry periods and strong winds are increasing functions of a warmer climate. Heat, dry conditions and strong winds increase the probabilities that fires start. Furthermore, if a fire starts, the stronger winds make the fires spreads more rapidly and the destruction becomes more severe.

For all of these reasons, it is and has been essential to investigate and optimize the general principles of the combined forestry and wild fire management problem. In this process, we should integrate the infrastructure and the fire fighting resources in the system as decision variables in the optimization problem. In future developments of these studies, it is also possible to increase the level of detail in the time dimension and to utilize sequential information about the latest state development in the decision process. Lohmander (2018) presents an approach that includes stochastic dynamic programming with quadratic programming as a subroutine. This numerical method may be useful, and even necessary, when such problems should be analyzed. Ideally, the investments and production in the energy industry, CCS, other industries with emissions and the transport sector, should also be endogenous. In this study, however, we have considered them as exogenous. Nevertheless, the scope of the performed analysis has to be considered as wide. In future development of the topic area, the author encourages that the scope becomes even wider. The results presented in this study originate from the specification of the optimization model, including the objective function. This function is the expected present value, which includes several components, such as the investment costs, the local fire management costs, the costs of utilized external resources and the profits from forestry. In case all decisions are centralized and taken by one well informed decision maker, and if the capital markets are perfect, maximization of the expected total present value presents no extra problems. In case different decisions are taken by many different decisions makers, the maximization of the expected total present value is still rational, if perfect markets within the total system make it rational for the many different decision makers to cooperate in the way that is most rational for the total system. Of course, most real systems with many decision makers have delays and imperfections of many kinds. This is true in the system under analysis in this paper and in almost every other system. For these reasons, it is very important that the concerned decision makers really try to focus on the total solution of importance and to avoid internal friction in the optimization of the forestry- infrastructure and fire management system of the future. Under the influence of global warming and stronger winds, several effects have been shown. The reduced optimal stock level and the increased optimal fuel treatment level certainly imply costs. In countries where the forest owners pay the fuel treatment costs, the expected present values of the profits of the forest owners are obviously reduced. On the other hand, without these actions, the expected present values would fall even more, under the influence of global warming and stronger winds. The optimal local fire capacity increases. This normally means that society has to pay more for this change, and that more taxes have to be collected from the inhabitants. On the other hand, in

some regions, fire fighting resources may originate from the actors in the region, which essentially may be the forest owners. The density of the road network should increase, which means that more roads should be constructed and maintained. If the road system is public, the government and tax payers may get higher costs. On the other hand, in several regions of the world, many forest roads are constructed and maintained by the companies and forest owners in the area. Then, these have to pay more and the expected present value of the profits of these actors are reduced. Finally, the optimal use of external fire fighting resources increases under the influence of a warmer climate and stronger winds. Such resources certainly imply costs. However, the alternatives would cost more from the total perspective.

CONCLUSIONS

Under the influence of global warming, we may expect more severe problems in forestry caused by wild fires. In order to improve the total results, we should reduce the stock level in the forests, increase the level of fuel treatment, increase the capacity of local fire fighting resources and increase the density of the road network. The total expected present value of all activities in a forest region is reduced even if optimal adjustments are made. These results have been derived via analytical optimization and comparative statics analysis. They have also been confirmed via a numerical nonlinear programming model where all decisions simultaneously were optimized.

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بهینه سازی جنگلداری، زیرساختها و مدیریت آتش سوزی

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گروه راه حل های مطلوب، یومنا، سوئد، در همکاری با دانشگاه لینه

(تاریخ دریافت: ۹۹/۰۴/۱۵ تاریخ پذیرش: ۹۹/۰۹/۰۴)

چکیده

جنگل ها حساس به آتش سوزی هستند و قسمت های زیادی از سیاره زمین را پوشانده اند. حفاظت منطقی از جنگل ها در برابر آتش سوزی و مدیریت آتش سوزی جنگل ها یک موضوع بسیار مهم است. سیاره زمین با مشکل جدی گرم شدن روبرو است. احتمال دوره های خشک طولانی و وزش باد شدید باعث افزایش عملکردهای آب و هوای گرم می شود. گرما، شرایط خشک و وزش باد شدید احتمال شروع آتش سوزی را افزایش می دهد. بعلاوه در صورت شروع سوزی، بادهای شدید باعث گسترش سریع آتش سوزی و افزایش تخریب می شوند. ممکن است انتظار داشته باشیم که مشکلات جدی تری در جنگلداری ناشی از آتش سوزی ها بر اثر گرم شدن کره زمین ایجاد شود. به خاطر این دلایل، بررسی و بهینه سازی اصول کلی جنگلداری ترکیبی و مسئله مدیریت آتش سوزی ضروری است. در این فرآیند ما باید زیرساختها و منابع اطفای حریق را در سیستم به عنوان متغیرهای تصمیم گیری در مسئله بهینه سازی ادغام کنیم. ابتدا روشهای تحلیلی برای تعیین نتایج کلی در مورد چگونگی تأثیر تصمیمات بهینه با افزایش سرعت باد استفاده می شود. کل سیستم با بهینه سازی تک بعدی تجزیه و تحلیل می شود. سپس ترکیبات مختلف تصمیم گیری بهینه سازی می شود و اهمیت هماهنگی بهینه اثبات می شود. سرانجام یک نسخه عددی خاص از مسئله بهینه سازی ساخته شده و مورد مطالعه قرار می گیرد. نتایج اصلی تحت تأثیر گرم شدن کره زمین به شرح زیر است: برای بهبود نتایج کلی مورد انتظار باید سطح موجودی جنگل ها را کاهش دهیم، سطح مراقبت از سوخت را افزایش دهیم، ظرفیت منابع آتش نشانی و تراکم شبکه جاده را افزایش دهیم. ارزش کل مورد انتظار کل فعالیتهای یک منطقه جنگلی حتی اگر تنظیمات بهینه انجام شود نیز کاهش می یابد. این نتایج از طریق بهینه سازی تحلیلی و تجزیه و تحلیل آماری مقایسه ای حاصل می شود. این نتایج همچنین از طریق یک مدل برنامه نویسی غیر خطی عددی تأیید شده اند که تمام تصمیمات به طور همزمان بهینه شده اند.

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