**Journal of Mathematical Modeling**
Vol. 9, No. 1, 2021, pp. 81–92

JMM

# An RBF approach for oil futures pricing under the jump-diffusion model

**Mohammad Karimnejad Esfahani**[†], **Abdolsadeh Neisy**[†*] , **Stefano De Marchi**[‡]

[†] *Department of Mathematics, Allameh Tabataba'i University, Iran*
[‡] *Department of Mathematics "Tullio Levi-Civita", University of Padova, Italy*

*Email(s): m.karimnejad7395@gmail.com, a_neisy@atu.ac.ir, demarchi@math.unipd.it*

**Abstract.** In this paper, our concern is to present and solve the problem of pricing oil futures. For this purpose, firstly we suggest a model based on the well-known Schwartz's model, in which the oil futures price is based on spot price of oil and convenience yield, however, the main difference here is that we have assumed that the former was imposed to some jumps, thus we added a jump term to the model of spot price. In our case, the oil future price model would be a Partial Integral Differential Equation (PIDE). Since, no closed form solution can be suggested for these kind of equations, we desire to solve our model with an appropriate numerical method. Although Finite Differences (FD) or Finite Elements (FE) is a common method for doing so, in this paper, we propose an alternative method based on Radial Basis Functions (RBF).

*Keywords*: Oil derivative market, Radial Basis Functions (RBF), oil futures, initial and boundary value problems, jump-diffusion model
*AMS Subject Classification 2010*: 34A34, 65L05.

## 1 Introduction

Nowadays, science and technology are able to improve the quality of life, making it more comfortable. In addition, they can increase capital for businesses and economic activities. Therefore, engineers, physicians, mathematicians and many other scientists have drawn their attention to the financial models. Admittedly, new mathematical models play a major part in economic and financial markets analysis. The main goal of these mathematical models is to find and represent fair price for derivatives and bonds. In this case, many useful efforts have been recently made by Neisy and Salmani in [18], Merton [16], Mohamadinejad et al. [17] as well as Safaei et al. [21]. Although the mentioned articles were mainly about options, this paper was done in an effort to discover new models for oil derivatives usage.

*Corresponding author.
Received: 7 March 2020 / Revised: 12 July 2020 / Accepted: 22 July 2020
DOI: 10.22124/jmm.2020.15948.1396

© 2021 University of Guilan http://jmm.guilan.ac.ir

It is known that in Middle Eastern Countries, oil industry is of great importance and many great firms and operations rely, directly and indirectly, on the oil price. Obviously, one way for these companies is to hedge their risk against changes that occur in the spot price of oil, using oil future contracts. As a result, it would be crucial to construct models which are able to suggest fair price for these derivatives.

Even the most valid institutions prioritize financial modeling. Thus, after studying petroleum markets and a variety of articles such as Cortazar and Schwartz [4, 5], it was indicated that petroleum futures may have a crucial place in risk hedging, and may control financial crisis.

In addition, great money and expenditures have been spent on oil sections to buy software, computer packages, and other facilities in the above-mentioned countries. It is very important that oil discovery and extraction process result in a Petroleum Sell Contract (PSC) that has been studied.

Keeping in mind the importance of this field, we firstly extend the model based on spot price of oil and convenience yield, which was introduced by Schwartz [5]. However, jump-diffusion term is added to the spot price of oil dynamic. Then, with regards to these underlying assets, using free arbitrage portfolio strategy, we obtain a model in the form of Partial-Integro Differential Equation (PIDE). Due to the existence of stochastic terms as well as jump term, the so-constructed model enables us to obtain reliable price. On the other hand, clearly, no closed form solution can be found for our model, due to the nature of PIDE models. Thus, selecting an appropriate numerical method for solving the pricing equation is another step which needs to be taken in order to find a fair price for the oil futures. Before suggesting a numerical method and going through it, in this study, we assumed that oil future price is based on two underlying assets which lead us to a two-dimensional PIDE. However, it is not far-fetched that the oil future price can be considered to be dependent on more than two underlying assets, and as a result, the desired model would be more than two dimensional. Needless to say, it would be hard to deal with three or more dimensional PIDE. Therefore, in this work, we propose the use of Radial Basis Function (RBF) approximation for solving our model, since RBF are more flexible and suitable for high dimensional problems (this is known as "dimension blindness") comparing with other common numerical methods, such as finite differences.

The remainder of the article is organized as follows. In Section 2, we present our model for pricing oil futures which is based on two underlying assets, one of which is imposed to jump. In Section 3, we firstly highlight some notions of RBF method, with hopes of achieving a reasonable price for oil futures. In Section 4 we apply it to our model, using real and simulated data (by using MATLAB), then we discuss and analyze the numerical results. Eventually, Section 5 contains conclusion as well as, some suggestions for further research.

## 2 Mathematical Formulations

In this section, we are going to discuss the future price model that includes jump and diffusion terms. The main reason behind considering a jump term is simply that a derivative price does not necessarily change in a continuous way in the real market, but there are some minor or major sudden changes in their value. Considering this explanation, it makes sense to consider a jump term in our model. Starting from Schwartz' model (cf. [5]), we develop an alternative model for the underlying assets.

For this purpose, assume $S$ as the spot price of oil and $z$ as the instantaneous convenience yield. Then, for a given time $T > 0$, the changes of the spot price and the convenience yield can be expressed as follows:

$$\begin{aligned} dS &= (r - z)Sdt + \sigma_s S dW_S + (J - 1)Sdp = dS_{SC} + dS_{JM}, \quad && 0 < t < T, \\ dz &= [\kappa(a - z) - l]dt + \sigma_z z dW_z, && 0 < t < T, \end{aligned} \tag{1}$$

where $r$ is the risk free interest rate, $\sigma_s$ and $\sigma_z$ are the volatility of the spot price and the convenience yield dynamics, respectively, $\kappa$ is the mean-reverting coefficient, $a$, the long-term convenience yield, and $l$ is the expected drift. Furthermore, $dW_z$ is the standard Wiener process under $\mathbb{Q}$ correlated with $dW_S$ such that:

$$Cov(dW_S, dW_z) = \rho dt. \tag{2}$$

The reader should note that here we can consider the dynamic of $S$ with two components. First

$$dS_{SC} = (r - z)Sdt + \sigma_s S dW_S,$$

which is for the Schwartz model, and the second part

$$dS_{JM} = (J - 1)Sdp,$$

which is the jump term, such that $p$ is an independent Poisson process with density of $\gamma > 0$. As a result, $dp$ can be expressed as follows:

$$dp = \begin{cases} 0, & \text{with probablity } 1 - \gamma dt, \\ 1, & \text{with probablity } \gamma dt. \end{cases}$$

Additionally, the total change in spot price of oil is easily represented as:

$$dS = dS_{SC} + dS_{JM}. \tag{3}$$

Before going through the future model, we would like to stress that, in this paper, there is a significant assumption which is convenience yield, $z$, can be traded in the market. Now to begin, suppose that $D$ is a portfolio containing the futures whose value is $G = G(S, z, t)$, the quantity $\Delta_1$ of the spot price of oil and $\Delta_2$ of convenience yield. Therefore, we would have:

$$D = G - \Delta_1 S - \Delta_2 z.$$

Now, any kind of change in the value of portfolio can be expressed as:

$$dD = dD_{SC} + dD_{JM}, \tag{4}$$

where

- $dD_{SC}$ is the variations in value of portfolio for underlying spot price of oil with Schwartz's model,

- $dD_{JM}$ is the variations due to the pure jump.

Then, from $Ito's$ lemma, we have:

$$\begin{aligned} dD_{SC} =& dG - \Delta_1 dS_{SC} - \Delta_2 dz \\ =& \frac{\partial G}{\partial t}dt + \frac{\partial G}{\partial S}dS_{SC} + \frac{\partial G}{\partial z}dz + \frac{1}{2}\frac{\partial^2 G}{\partial S^2}(dS_{SC})^2 + \frac{1}{2}\frac{\partial^2 G}{\partial z^2}(dz)^2 \\ &+ \frac{\partial^2 G}{\partial z \partial S}Cov(dS_{SC}, dz) - \Delta_1 dS_{SC} - \Delta_2 dz. \end{aligned} \tag{5}$$

Before continuing the discussion, let examine the $Cov(dS_{SC}, dz)$ and the second power of $dS_{SC}$ and $dz$:

$$\begin{aligned} Cov(dS_{SC}, dz) &= \sigma_s S \sigma_z z Cov(dW_S, dW_z) = \sigma_S S \sigma_z z \rho dt, \\ (dS_{SC})^2 &= (r-z)^2 S^2 (dt)^2 + 2(r-z)S^2\sigma_s(dW_S)(dt) + (\sigma_s S)^2 (dW_S)^2 = (\sigma_s S)^2 dt, \\ (dz)^2 &= [\kappa(a-z)-l]^2(dt)^2 + 2[\kappa(a-z)-l]\sigma_z z(dW_z)(dt) + (\sigma_z z)^2(dW_z)^2 = (\sigma_z z)^2 dt. \end{aligned}$$

Now by plugging (1) and above formulas into (5), we have:

$$\begin{aligned} dD_{SC} =& \Big(\frac{\partial G}{\partial t} + \frac{1}{2}(\sigma_S S)^2\frac{\partial^2 G}{\partial S^2} + \frac{1}{2}(\sigma_z z)^2\frac{\partial^2 G}{\partial z^2} + \rho\sigma_z\sigma_S S z\frac{\partial^2 G}{\partial z \partial S}\Big)dt \\ &+ \Big(\frac{\partial G}{\partial S} - \Delta_1\Big)(dS_{SC}) + \Big(\frac{\partial G}{\partial z} - \Delta_2\Big)(dz). \end{aligned} \tag{6}$$

In order to make this portfolio instantaneously risk free, we have to eliminate the risk terms containing $S$ and $z$ by setting each part equal to 0 separately. Hence, we have:

$$\frac{\partial G}{\partial S} = \Delta_1 \qquad \text{and} \qquad \frac{\partial G}{\partial z} = \Delta_2.$$

Therefore, we could rewrite the equation (6) as:

$$dD_{SC} = \Big(\frac{\partial G}{\partial t} + \frac{1}{2}(\sigma_S S)^2\frac{\partial^2 G}{\partial S^2} + \frac{1}{2}(\sigma_z z)^2\frac{\partial^2 G}{\partial z^2} + \rho\sigma_z\sigma_S S z\frac{\partial^2 G}{\partial z \partial S}\Big)dt.$$

Now, for the jump term, we would simply write:

$$dD_{JM} = [G(JS, z, t) - G(S, z, t)]dp - \frac{\partial G}{\partial S}(J-1)S dp.$$

After combining $dS_{SC}$ and $dS_{JM}$, the following would be obtained:

$$\begin{aligned} dD =& \Big(\frac{\partial G}{\partial t} + \frac{1}{2}(\sigma_S S)^2\frac{\partial^2 G}{\partial S^2} + \frac{1}{2}(\sigma_z z)^2\frac{\partial^2 G}{\partial z^2} + \rho\sigma_S\sigma_z S z\frac{\partial^2 G}{\partial z \partial S}\Big)dt \\ &+ \Big([G(JS, z, t) - G(S, z, t)] - \frac{\partial G}{\partial S}(J-1)S\Big)dp. \end{aligned} \tag{7}$$

Note that, we assume that the jumps are uncorrelated and the variance of the portfolio of portfolios is small (there is little risk). Then, the expected return of the portfolio $D$ must be:

$$E[dD] = E[dD_{SC}] + E[dD_{JM}] = rDdt.$$

Now, by taking expectation from both side of the equation (7), we find that:

$$\begin{aligned} r(G - \frac{\partial G}{\partial S}S - \frac{\partial G}{\partial z}z)dt = &\Big(\frac{\partial G}{\partial t} + \frac{1}{2}(\sigma_S S)^2\frac{\partial^2 G}{\partial S^2} + \frac{1}{2}(\sigma_z z)^2\frac{\partial^2 G}{\partial z^2} + \rho\sigma_S\sigma_z Sz\frac{\partial^2 G}{\partial z \partial S}\Big)dt \\ &+ \Big(E[G(JS, z, t)] - G(S, z, t) - \frac{\partial G}{\partial S}E[(J-1)]S\Big)\gamma dt. \end{aligned}$$

After collecting all terms on one side, the following equation is easily obtained:

$$\begin{aligned} &\frac{\partial G}{\partial t} + (\frac{\sigma_S^2 S^2}{2})\frac{\partial^2 G}{\partial S^2} + (\frac{\sigma_z^2 z^2}{2})\frac{\partial^2 G}{\partial z^2} + (rS - S\gamma h)\frac{\partial G}{\partial S} + (rz)\frac{\partial G}{\partial z} \\ &+ (\rho\sigma_S\sigma_z Sz)\frac{\partial^2 G}{\partial S \partial z} - (r+\gamma)G + \gamma I = 0, \end{aligned} \tag{8}$$

such that

$$\begin{aligned} I &= E[G(JS, z, t)] = \int_0^\infty G(JS, z, t)g(J)dJ, \\ h &= E[(J-1]. \end{aligned}$$

Needless to say, computing expectation of $G[(JS, z, t)]$ depends on knowing probability distribution function of the magnitude of an occurred jump $J$. Since most of the jumps are mainly due to political, economical issues or natural catastrophe, it is common to estimate the potential damage caused by $J$ using a continuous PDF such as *log-normal* or *Pareto distribution*. Evidently, after selecting an appropriate function, determining variables of PDF is of great importance. In equation (8), a model is represented which is free from the drift of both underlying assets and determines the evolution of the oil futures price, which is based on spot price of oil and convenience yield. However, as it was previously mentioned, when it comes to solving the PIDE, some difficulties emerge, from the numerical point of view due to the integral-part.

Additionally, before discussing the initial and boundary conditions, we suggest the change of variable $\tau = T - t$ while T is the maturity time of the contract. So, $\tau$ indicates the time to maturity. Now regarding this change of varibale, for the initial condition we simply have:

$$G(S, z, 0) = S.$$

Also, for the boundary condition, we have the following:

$$\begin{aligned} G(0, z, \tau) &= 0, \\ G(S, 0, \tau) &= Se^{-r\tau}, \\ \lim_{S\to\infty} G(S, z, \tau) &\to \infty, \\ \lim_{z\to\infty} G(S, z, \tau) &= 0. \end{aligned}$$

This is because, when convenience yield goes very high, holders of the commodity in the market try to keep their commodity and so they do not buy or sell it for any price in the future. On the other hand, when the value is very low, it means that the holders of the commodity in the market do not try to buy or sell it for a certain price in the future.

## 3 Radial basis functions for the numerical solution

In this section, our goal is to show how to solve (8) using a proper numerical method. Although Monte Carlo or Finite Differences method are more common in financial problems including American option pricing (cf. e.g [12]), in this work, Radial Basis Functions has been suggested. Similar to Monte Carlo or Finite Differences, RBF has its own advantages and disadvantages. A basic knowledge concerning the RBF approximation method is briefly discussed in this section. However, the readers are referred to the references mentioned in [9, 14].

Basically, using RBF approximation, we assume that there is a function called $\varphi : [0, \infty) \to \mathbb{R}$ and a coefficient function $\alpha$, such that, we can write any function including $G(X, \tau)$, as the form of:

$$G(X, \tau) = \sum_{j=1}^{N} \alpha(\tau)\varphi(\varepsilon||X - y_j||), \tag{9}$$

where $X$ and $y_j$ are multidimensional vectors. The centers $y_j$ are some scattered points in our multidimensional domain, say $\Omega \subset \mathbb{R}^d$, providing a discretization of $\Omega$, with no restriction on their location. Finally, $\varphi$ is called kernel and $\|\ \|$ is the Euclidean norm. $\alpha(\tau)$ are the coefficient functions of $\tau$, and should be determined with regard to the initial and boundary condition that the equation (PDE or PIDE) must satisfy. In addition, $\varepsilon$ is the *shape parameter*, which has a significant effect upon the accuracy and stability of the approximation process [7, 8, 15].

There are multiple choices for the kernel function $\varphi$ in (9). These include: Multiquadrics (MQ), Inverse Multiquadrics (IM), Inverse Quadric (IQ), each of which has its own features (for more details see [9, 14]). However, in this paper, based on later discussion, we select the Gaussian kernel which is of the form:

$$\varphi(||X - y_j||) = \exp(-\frac{||X - y_j||}{\varepsilon^2}). \tag{10}$$

Obviously, due to the norm inside the function $\varphi$ this method is completely *"dimension-blind"*, at least from a theoretical standpoint. In fact, radial basis functions depend only on distance of the points from the centers $y_j$.

Since in the financial modeling the dimension of the PDE is determined by the number of underlying assets, it is required to find a numerical method which can be used when the dimension grows, Hence, the RBF approach appears to be completely useful because, as just mentioned, only the distance between the points is involved. Furthermore, another point that makes this method attractive, is that unlike FD method no mesh is needed. In other words, the RBF method can be used for meshfree or scattered data.

On the other hand, one of the main drawbacks of this method is that, after inserting (9) in the PDE problem, the outcome is a linear system, which can be severely ill-conditioned. Although much research has been conducted in the recent years to overcome this problem, there are no clear conditions ensuring that we will not face ill-conditioning [14, 15, 20]. The most relevant solutions are in the direction of the so-called *trade-off principle* (see [9, 14]). Another solution deals with localization techniques, the most popular is the *partition of unity method* [8]. Furthermore, well-posedness of the boundary condition is of great importance, and sometimes it is the main source of problem in the numerical results. In this work, we use the method presented in [10], which enforces initial and boundary conditions to avoid these difficulties. Readers are recommended to see [6,9] for further information about these topics.

Now, regarding this concise discussion, it comes to applying the RBF method to the equation (8). As the initial step, we take an arbitrary discretization of the centers $y_j$ with $1 < j < N$ such that $y_j = (S_j, z_j)$. In addition, since we are using collocation approach, we require partial derivatives of our pricing function (in RBF's form):

$$\frac{\partial G}{\partial \tau} = \sum_{j=1}^{N} \dot{\alpha}(\tau)\varphi(||X - y_j||),$$

$$\frac{\partial G}{\partial X_i} = \sum_{j=1}^{N} \alpha(\tau)\frac{-2(X_i - y_{j,i})}{\varepsilon^2}\varphi(||X - y_j||),$$

$$\frac{\partial^2 G}{\partial X_i \partial X_k} = \sum_{j=1}^{N} \alpha(\tau)\frac{4(X_i - y_{j,i})(X_k - y_{j,k})}{\varepsilon^4}\varphi(||X - y_j||),$$

with $y_{j,i}$ indicates the $i$-th component of the $j$-th point. Moreover, for the integral part

$$I = \int_0^{\eta} G(JS, z, t)g(J)dJ,$$

we would simply use the method suggested in [1]. While the function $g$ is the PDF determining the magnitude of the occurred jump. Following the brief discussion in the previous section, here we assume that it follows a log-normal distribution function that is a continuous PDF, hence [23]:

$$g(J) = \frac{e^{-\frac{1}{2}\frac{(lnJ-\mu)^2}{\Lambda}}}{\Lambda J\sqrt{2\pi}}. \tag{11}$$

Now, consider $L$ as the distance matrix of the centers and the scattered points regarding the function $\varphi$; $L_S$ the distance matrix of the scattered points and the centers regarding the function $\varphi_S$ and so forth. By forming the other matrix and after taking variable changing ($\tau = T - t$) into consideration, we can rewrite equation (8) in the matrix form.

$$-L\dot{\alpha} + \Big(\frac{\sigma_S^2}{2}L_{SS} + \frac{\sigma_z^2}{2}L_{zz} + (r - h\gamma)L_S + rL_z + (\rho\sigma_S\sigma_z)L_{Sz} - (r + \gamma)L + \gamma L^I\Big)\alpha = 0.$$

Please note that by $L^I$, we mean the distance matrix resulted from the $I$. Now, calling with $R$ the coefficient of $\alpha$, we can rewrite the above equation in the form

$$-L\dot{\alpha} + R\alpha = 0, \tag{12}$$

which is an ODE of $\tau$. By using the well-known $\theta$-method, equation (12) can be re-written as follows.

$$-L\Big(\frac{\alpha^{n+1} - \alpha^n}{\Delta\tau}\Big) + \Theta R\alpha^{n+1} + (1 - \Theta)R\alpha^n = 0.$$

Choosing $\Theta = 0$, we get

$$L\alpha^{n+1} = \Big(L + R\Delta\tau\Big)\alpha^n. \tag{13}$$

Now, using iterative methods, we are able to solve the equation regarding the initial and the boundary conditions.

## 4 Numerical results

In this section, we present two different numerical experiments. In the first example, some parameters are obtained from [22] and some are simulated. While in the second, real data set retrieved from the well-known website www.quandl.com. In fact, in the latter, some of the important statistics are calculated with regards to the data set. Subsequently, parameter estimation techniques are applied in order to estimate the parameters of the model such as volatility. Eventually, using numerical solvers from MATLAB, equation (12) is solved for each case separately. The reader should note that, in both examples, we restrict the underlying assets to a finite value which is the interval 0 to 4. Furthermore, though there is no theoretical restriction for selecting the location of the centers (except their dissimilarity), here we consider a regular discretization in both examples. Another key aspect is that, in both cases, we will not take jump term into consideration for the sake of simplicity, particularly, no jump can be found in our selected data set which means, it would be meaningless to attempt to estimate jump term parameters. However, a very good suggestion for a further research is to look for data set, in which some unexpected jumps occurred.

Therefore, for our first numerical experiment, using 16 points, we would have the results presented in Table 1 and the obtained solution is plotted in Figure 1.

Table 1: Parameter values of the model.

| Parameter | Estimated values |
|---|---|
| $r$ | 0.1 |
| $\rho$ | 0.766 |
| $\sigma_s$ | 0.3 |
| $\sigma_z$ | 0.5 |

Regarding the shape parameter, though there are various algorithms aimed to optimize $\varepsilon$, here we didn't go through those methods. Instead, we obtained $\varepsilon = 0.533$ using trial and error, similar to the other parameters. It is worth to remind that the shape parameter affects both accuracy and stability of the approximation method. In fact, as $\varepsilon$ increases, so does the accuracy, but it leads the algorithm to an ill-conditioning system which results in instability, and vice-versa. Therefore, determining the value of the shape parameter follows, as already pointed out, the trade-off principle (for further information see [9]).

In the second experiment, we intend to explain a more detailed example. For this purpose, consider the data set consists of daily observations for five WTI crude oil futures, including G1, G2, G3, G4 and G5, where G1 is the contract closest to the maturity, G2 is the second contract close to the maturity, etc. We select the data from 23 November 2015 to 31 May 2018. Figure 2 displays the time series plots of the prices of the five futures contracts. We take the future prices from 23 November 2015 to 31 May 2018 as in-sample data and the futures prices from 01 June 2018 to 31 December 2018 as out-of-sample data. The basic statistics for these data are described in the Table 2. As already pointed out, the parameters are approximated using parameters estimation algorithms (for more information, the reader might see [19]). From the

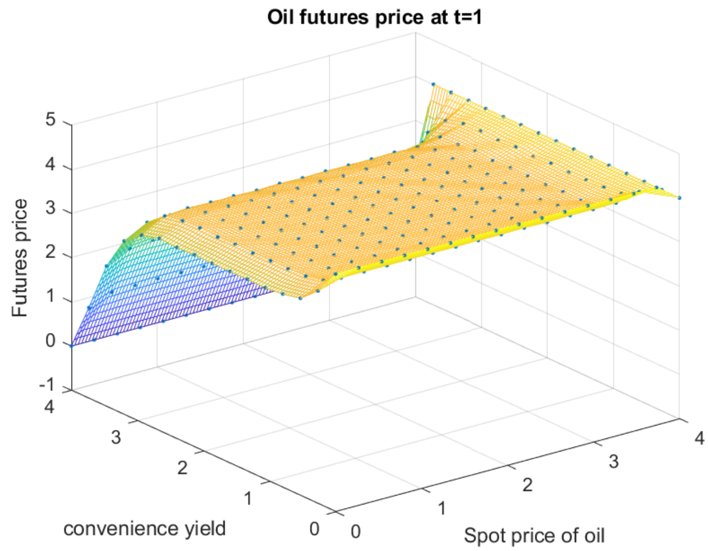


Figure 1: RBF solution using $N = 16$ points.

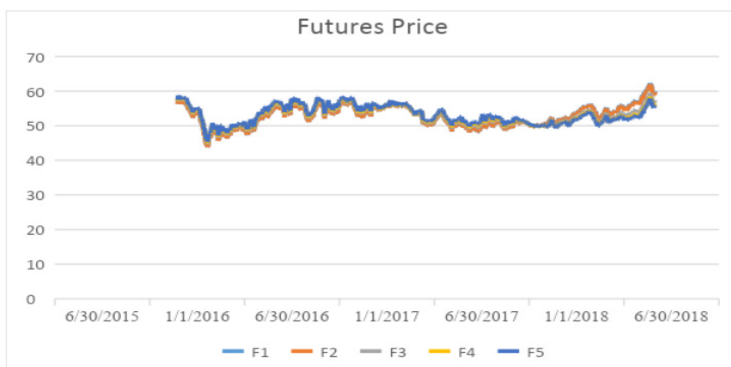


Figure 2: Future prices.

time-series, following statistics could be achieved:

Table 2: Basic statistics.

| Future contracts | Price(Mean) | Price(Standard deviation) | Maturity(Mean) | Maturity(Standard deviation) |
|---|---|---|---|---|
| $F1$ | 52.82 | 3.0911 | 3.3433 | 0.7279 |
| $F2$ | 52.77 | 3.0397 | 3.4266 | 0.7279 |
| $F3$ | 52.9014 | 2.7524 | 4.1766 | 0.7279 |
| $F4$ | 52.99 | 2.7062 | 4.5933 | 0.7279 |
| $F5$ | 53.18 | 2.7233 | 4.8433 | 0.7279 |

Now, using parameter estimation algorithms, we obtain the following:

Table 3: Estimated parameters of the Model.

| Parameter | Estimated value |
|---|---|
| $r$ | 0.3 |
| $\rho$ | 0.8042 |
| $\sigma_s$ | 0.4329 |
| $\sigma_z$ | 0.2675 |

Again, after multiple trial and error, we realized that $\varepsilon = 0.600$ could be an appropriate value for the shape parameter. Now, for 21 points, the solution is plotted in Figure 3.

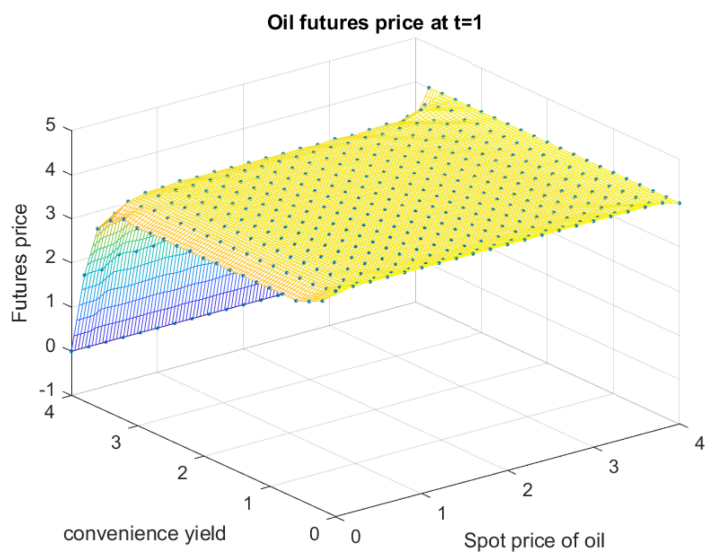


Figure 3: RBF solution using $N = 21$ points.

Comparing Figures 1 and 3, it can be easily seen that, increasing the number of centers may lead to smoother function approximation whereas, large number of centers may result in higher computational cost; so, it would not be an easy task to decide the number of centers when dealing with RBF approximation method.

Before bringing this section to an end, there is a significant point that's worth discussing which is, radial basis functions do not satisfy the boundary conditions automatically, hence at each time step, it's required to enforce the initial and boundary condition to the solution vector. Afterwards, the coefficient vector should be updated by resolving the linear system. The reason behind this is some potential problems that may come to existence at the boundary collocation points. The reader may find more comprehensive information in [10].

## 5 Conclusions and future works

This work represents a model based on the works of Schwartz in [4, 5]. Above all, it contains jump term which were not considered in the mentioned works. In the context of this study, oil futures price is based on two stochastic variables, namely the spot price of oil and the convenience yield, the former possibly being the main source of jump. It is because, as discussed previously, the spot price of oil is heavily influenced by political or economic phenomena. Therefore, it is understandable to consider jump terms in the modeling of the underlying assets. Also, for the numerical solution we proposed RBF approximation which is fairly new approach in future contracts pricing. As it was mentioned, the good thing about RBF method is that it works for high-dimensional problems. Henceforth, bringing up interest rates as a third underlying asset, the method might be considered in further and future studies. Another thing that should be highlighted is finding real data, on which sudden jumps have occurred, and solving the model with respect to that, which could be a great contribution to this field. Last but not least, since Gaussian RBF is globally supported, the resulting matrix in the linear system is full, which makes the computational cost relatively high. On the other hand, using compactly supported kernels will lead to sparse matrix, so it would increase the efficiency of this method. In summary, this paper suggests RBF method as a possible way for solving the problem of pricing futures which is based on more than one underlying asset.

## Acknowledgements

This work has been done within the bilateral agreement between the *Allameh Tabataba'i University* and *University of Padova*. We appreciated the anonymous referees that allow to improve the final version of the paper.

## References

[1] A.F. Bastani, Z. Ahmadi, D. Damircheli, *A radial basis collocation method for pricing American options under regime-switching jump-diffusion models*, J. Comput. Appl. Math. **65** (2013) 79-90.

[2] S.D. Bates, *Jumps and stochastic volatility, exchange rate processes implicit in Deutsche Mark options*, Rev. Financ. Stud. **9**(1) (1996) 69-107.

[3] R. Campagna, S. Cuomo, S. De Marchi, E. Perracchione, G. Severino, *A stable meshfree PDE solver for source-type flows in Porous media*, Appl. Numer. Math. **149** (2020) 30-42.

[4] G. Cortazar, E. S. Schwartz, *Implementing a Real Option Model for Valuing an Undeveloped Oil Field*, Int. Trans. Oper Res. **4** (1997) 125-137.

[5] G. Cortazara, E. S. Schwartz, *Implementing a stochastic model for oil futures prices*, Energy. Econ. **25** (2003) 215-238.

[6] S. De Marchi, M. Mandara, A. Viero, *Meshfree approximation for multi-asset European and American option problems*, Ed. Aracne, Roma, 2010.

[7] S. De Marchi, A. Martinez, E. Perracchione, M. Rossini, *RBF-based partition of unity methods for elliptic PDEs: Adaptivity and stability issue via variable scaled kernels*, J. Sci. Comput. **79**(2) (2019) 321-324.

[8] S. De Marchi, R. Schaback, *Stability of kernel-based interpolation*, Adv. Comput. Math. **32** (2010) 155-161.

[9] G.E. Fasshauer, *Meshfree Approximation Methods with MATLAB*, World Scientific, 2007.

[10] G.E. Fasshauer, A.Q.M. Khaliq, D.A. Voss, *Using meshfree approximation for multiasset American options*, J. Chin. Inst. Eng. **27**(4) (2004) 563-571.

[11] R. Gibson, E. S. Schwartz, *Stochastic convenience yield and the pricing of oil contingent claims*, J. Finance. **45**(3) (1990) 959-976.

[12] A.Q.M. Khaliq, D.A. Voss, S.H.K. Kazmi, *Adaptive $\theta$-methods for pricing American options*, J. Comput. Appl. Math. **22** (2008) 210-227.

[13] S.G. Kou, *A jump-diffusion model for option pricing*, J. Manag. Sci. **48** (2002) 1086-1101.

[14] E. Larsson, K. Ahlander, A. Hall, *Multi-dimensional option pricing using radial basis functions and the generalized Fourier transform*, J. Comput. Appl. Math. **22** (2008) 175-192.

[15] E. Larsson, B. Fornberg, *A numerical study of some Radial Basis Function Based Solution Methods for Eliptic PDEs*, Comput. Math. Appl. **46** (2003) 891-902.

[16] R. C. Merton, *Option pricing when underlying stock returns are discontinuous*, J. Financ. Econ. **3**(1) (1976) 125-144.

[17] R. Mohamadinejad , J. Biazar , A. Neisy, *Spread option pricing under two jump-diffusion interest rates*, U. Politech. Buch. Ser. A. **82**(1) (2020) 171-182.

[18] A. Neisy, K. Salmani, *An inverse finance problem for estimation of the volatility*, Comput. Math. Math. Phys. **53**(1) (2013) 63-77.

[19] R. Paschke, M. Prokopczuk, *Commodity derivatives valuation with autoregressive and moving average components in the price dynamics*, J. Bank. Financ. **34**(11) (2010) 2742-2752.

[20] U. Pettersson, E. Larsson, G. Marcusson, J. Persson, *Improved radial basis function methods for multi-dimensional option pricing*, J. Comput. Appl. Math. **222** (2008) 82-93.

[21] M. Safaei, A. Neisy, N. Nematollahi, *Generalized component wise splitting scheme For option pricing under the Heston-Cox-Ingersoll-Ross model*, J. Stat. Theory Appl. **18**(4) (2019) 425-438.

[22] E.S. Schwartz, *The stochastic behavior of commodity prices: implications for valuation and hedging*, J. Finance. **52**(3) (1997) 923-973.

[23] A.J.A. Unger, *Pricing index-based catastrophe bonds: Part 1: formulation and discretization issues using a numerical PDE approach*, Comput. Geosci-uk. **36**(2) (2010) 139-149.