Journal of Mathematical Modeling
Vol. 6, No. 1, 2018, pp. 1-26

JMM

Influence of awareness programs by media in the typhoid fever: a study based on mathematical modeling

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Abstract. In this paper, we propose and analyze a mathematical model describing the effect of awareness programs by public media on the prevalence of Typhoid fever. A threshold quantity R_0, similar to the basic reproduction number is derived. We investigate the biologically meaningful equilibrium points and their local stability analysis. The global stability analysis has been performed with respect to the disease free equilibrium (DFE) E_0 by considering suitable Lyapunov function. We derive the stability condition of the DFE point E_0 and the interior steady-state E^* with respect to the basic reproduction number R_0. We perform the analysis of Hopf-bifurcation with respect to the rate of executing awareness programs which has a substantial role on the dynamics of the model system. We investigate extensive numerical simulations to validate our analytical findings.

Keywords: Typhoid fever, awareness program, Hopf-bifurcation, basic reproduction number, stability analysis.
AMS Subject Classification: 34G20, 37G10, 37G15, 65P30, 65P40..

1 Introduction

Typhoid fever is one of the major public health problems in the developing country. A gram-negative bacterium Salmonella enterica serovar Typhi

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Received: 9 June 2017 / Revised: 30 November 2017 / Accepted: 7 December 2017.
DOI 10.22124/jmm.2018.2760

(S. Typhi), that is a human restricted bacteria transmitted via faecal infectivity of food and water, causes typhoid fever which is one of the most dangerous human infectious diseases. For the urban and peri-urban population typhoid fever is a major cause of morbidity and mortality. Annually, 21.6 million people affected and approximately 2, 00, 000 people die due to typhoid fever throughout the world. In Asia almost 80 % cases are death and 20 % in Latin America and Africa. In India, the disease happen with an occurrence ranging from 102 to 2,219 per 100,000 of the population [25, 2]. This disease is transmitted by urine-oral route or feco-oral route, either by urine or carriers or directly through hands soiled with feces or indirectly by ingestion of contaminated water, milk, food or through flies [23]. When we intake unhygienic food or water, temporarily the Salmonella bacteria attack the small intestine and enter the bloodstream. The bacteria are carried by spleen, bone marrow and white blood cells to the liver. The incubation period for typhoid fever is eight to fourteen days and the duration of illness is about four to six weeks [1]. The signs and symptoms of the typhoid fever are as follows: vomiting, constipation, slight abdominal pain, headache, nausea and appearance of a rash (rose rash) on the abdomen. In South Asia, children carry one of the highest typhoid difficulties in the world [3, 20]. In another studies have publicized in areas of endemicity and outbreaks, between 6% to 21% are under two years of age and about 25% to 33% of pediatric typhoid fever cases are under five years of age [16]. Typhoid fever is known in the pediatric age group from different appearance, for instance diarrhoea in infants, septicemia in neonates and as lower respiratory tract infections in older children [19, 8, 21].

The most important source regarding health information for the common civilization is the media awareness programs; which are mainly play an important accountability in providing an influence for the population to convey their knowledge of diseases [9, 5, 10]. Media also makes people aware with the requisite preventive practices such as wearing protective masks, social distancing, vaccination etc. Aware people accept these practices and attempt to reduce their chances of becoming infected. Mass media are generally used to spread information to vast audiences through the use of media, such as Radio, Newspapers, Television, Posters, Internet, Books, Journals and Billboards. They could play a pivotal role in communicating typhoid fever-associated health information to the general public so as to raise awareness on typhoid fever-associated matter and troubles. A mathematical model presented an evaluation of the impact on case finding of a mass media health education campaign for TB control in Cali, Colombia is studied by Jaramillo [6]. Huo and Wang [5] studied a mathematical model with the influence of awareness programs but they did not change the drinker's behavior which focused on the community environment around the nondrinker. Many researchers have already studied theoretically by using media awareness effect in their model and they got some interesting results and some unexplore dynamics of typhoid fever [22, 17, 18].

In our article, we introduce the effect of awareness programs motivated by the works of Adtunde et al. [1]. Due to introduction of awareness programs the behavior of the susceptible population has been changed and reduced the contact rate of susceptible individuals with infective individuals. We also introduce a divide class $C(t)$ of those who are conscious of risk and renounce from infection by avoiding contact with the infected individuals. Moreover, we see that the cumulative density of awareness programs $M(t)$ increases at a rate proportional to the number of infected individuals. We establish the sufficient conditions for the global stability of the equilibrium points and give some numerical simulations to explain our main outcomes. Our result shows that the awareness programs are an effective measurement in reducing the infections.

This paper is organized in the following order. In the next section, we formulate a mathematical model for the treatment of typhoid fever by considering role of the awareness programs affected by media. The qualitative analysis of the model including the positivity of the solutions have been discussed in the Section 3. The global stability of the disease free equilibrium point E_0 and the local stability of the endemic equilibrium point E^* is studied in the Section 4. In Section 5, based on the normal form theory and center manifold theorem, we establish explicit expressions to determine the direction of Hopf–bifurcation. We perform extensive numerical simulations for suitable choices of parameter values in Section 6. The paper ends with a brief discussion and conclusions.

2 The Mathematical Model

In our model, $S(t)$ represents the susceptible individuals who can be infected but have not yet contracted with Salmonella typhi but may contract with it if exposed to any mode of its transmission. The number of infected individuals $I(t)$ who have contracted with the Salmonella typhi and they are active to capable of transmitting the diseases. The number of individuals who, although apparently healthy themselves, harbor infection which can be transmitted to others is denoted by carriers $C(t)$. The number of individuals who are recovered after treatment or media awareness programs and are the immune to the diseases is described by $R(t)$, whereas $M(t)$ represents the cumulative density of awareness programs driven by the media. It is considered that, due to awareness programs, uninfected population from a different class and avoid contacting with the infected population. Also we consider the interaction from awareness programs between aware individuals and uninfected population. The total number of population $N(t)$ at any time t is given by

$$N(t) = S(t) + I(t) + C(t) + R(t). \tag{1}$$

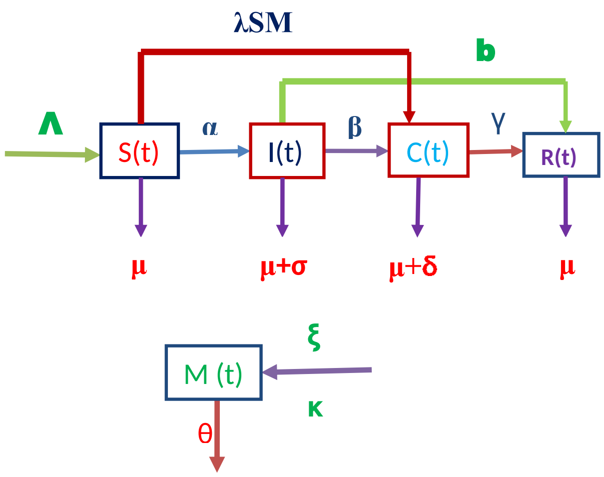

Figure 1: *The figure shows the schematic representation depicting the dynamics of the Typhoid fever.*

The schematic diagram (Figure 1) leads to the following system of coupled nonlinear ordinary differential equations

$$\begin{aligned}
\frac{dS}{dt} &= \Lambda - \alpha SI - \mu S - \lambda SM, \\
\frac{dI}{dt} &= \alpha SI - \mu I - \sigma I - \beta I - bI, \\
\frac{dC}{dt} &= \beta I - \mu C - \delta C - \gamma C, \\
\frac{dR}{dt} &= bI + \gamma C - \mu R, \\
\frac{dM}{dt} &= \kappa \xi I - \theta M,
\end{aligned} \tag{2}$$

where Λ is the recruitment rate of individuals into the community by birth or migration (susceptible) and μ is the per capita mortality rate of susceptible individuals. Dissemination rate of awareness among susceptible due to which they from a class is represent λ. The typhoid fever-indicated mortality rate is σ. The rate of infection is α. The rate of progression from infective to carrier is β. The rate of recovery from carrier stage is γ. The carrier-induced mortality rate is δ. The rate of recovery from the infectious stage is b. Moreover, the awareness programs increase with increase in disease related death rate ξ. Here κ is the proportionality constant which governs the implementation and θ represents the depletion rate of these programs due to ineffectiveness.

3 Qualitative analysis of the model

3.1 Invariant region

Lemma 1. *All feasible regions Ω defined by*

$$\begin{aligned}\Omega = \{(S(t), I(t), C(t), R(t), M(t)) \in \mathbb{R}^5_+ :\\ S(t) + I(t) + C(t) + R(t) \leq \tfrac{\Lambda}{\mu}; M(t) \leq \tfrac{\kappa\xi}{\theta}\}\end{aligned}$$

with initial conditions $S(0) \geq 0$, $I(0) \geq 0$, $C(0) \geq 0$, $R(0) \geq 0$ and $M(0) \geq 0$ are positively invariant for equation (2).

Proof. Adding the first four equations of equation(2), we have

$$\frac{dN}{dt} \leq \frac{\Lambda}{\mu} - \mu N.$$

It follows that

$$N(t) \leq \frac{\Lambda}{\mu} - N(0)\exp^{-\mu t},$$

where, $N(0)$ is the initial total number of people. Thus,

$$\limsup_{t\to\infty} N(t) \leq \frac{\Lambda}{\mu}.$$

Then, $0 \leq S(t) + I(t) + C(t) + R(t) \leq \frac{\Lambda}{\mu}$. Thus, since the equation (2) monitors human population, it is plausible to assume that all its state variables and parameters are nonnegative for all $t \geq 0$. Further, from the last equation of equation(2), we have,

$$\frac{dM}{dt} = \kappa\xi I - \theta M \leq \kappa\xi - \theta M.$$

It follows that

$$0 \leq M(t) \leq \frac{\kappa\xi}{\theta} + M(0)\exp^{-\theta t},$$

where $M(0)$ represents the initial value of cumulative density of awareness programs. Thus,

$$\limsup_{t\to\infty} M(t) \leq \frac{\kappa\xi}{\theta}.$$

It implies that the region

$$\begin{aligned}\Omega = \{&(S(t), I(t), C(t), R(t), M(t)) \in \mathbb{R}^5_+ :\\ &S(t) + I(t) + C(t) + R(t) \leq \tfrac{\Lambda}{\mu}; M(t) \leq \tfrac{\kappa\xi}{\theta}\},\end{aligned}$$

is a positively invariant set for equation(2). □

So we consider dynamics of equation (2) on the set Ω in this article.

3.2 Positivity of solutions

To ensure that the solutions of the equation (2) with positive initial conditions remain positive for all $t > 0$, it is necessary to prove that all the state variables are nonnegative. In order to do that, we study following lemma.

Lemma 2. *If $S(0) \geq 0$, $I(0) \geq 0$, $C(0) \geq 0$, $R(0) \geq 0$ and $M(0) \geq 0$, then the solutions $S(t)$, $I(t)$, $C(t)$, $R(t)$ and $M(t)$ of equation (2) are positive for all $t \geq 0$.*

Proof. Under the given initial conditions, it is easy to prove that the solutions of the equation (2) are positive; if not, we assume a contradiction that there exists a first time t_1 such that

$$\begin{array}{lll} S(0) > 0, & S(t_1) = 0, & S'(t_1) \leq 0,\\ C(t) > 0, & I(t) > 0, & R(t) > 0,\\ M(t) > 0, & 0 \leq t < t_1. & \end{array}$$

In that case, from the first equation of equation (2), we have

$$S'(t_1) = \Lambda > 0,$$

which is a contradiction meaning that $S(t) > 0$, $t > 0$. Or there exists a t_2 such that

$$\begin{array}{lll} C(0) > 0, & C(t_2) = 0, & C'(t_2) \leq 0,\\ S(t) > 0, & I(t) > 0, & R(t) > 0,\\ M(t) > 0, & 0 \leq t < t_2. & \end{array}$$

In that case, from the second equation of equation (2), we have

$$C'(t_2) = \lambda SM > 0,$$

which is a contradiction meaning that $C(t) > 0$, $t \geq 0$. Or there exists a t_3 such that

$$\begin{array}{lll} I(0) > 0, & I(t_3) \leq 0, & I'(t_3) = 0, \\ S(t) > 0, & C(t) > 0, & R(t) > 0, \\ M(t) > 0, & 0 \leq t < t_3. & \end{array}$$

In that case, from the third equation of equation (2), we have

$$I'(t_3) = 0,$$

which is a contradiction meaning that $I(t) = 0$, $t > 0$.

Similarly, it can be shown that $R(t) > 0$ and $M(t) > 0$ for all $t > 0$. Thus, the solutions $S(0) \geq 0$, $I(0) \geq 0$, $C(0) \geq 0$, $R(0) \geq 0$ and $M(0) \geq 0$ of equation (2) remain positive for all $t > 0$. □

4 Analysis of the Model

There are one disease-free equilibrium point E_0 and endemic equilibrium point E^* for equation (2).

4.1 Disease-free Equilibrium and the Basic Reproduction Number

The model has disease-free equilibrium given by $E_0(\Lambda, 0, 0, 0, 0)$. In the following, the basic reproduction number of equation (2) will be obtained by the next generation matrix method formulated in [24].

Let $x = (I, R, M, S, C)^T$ then equation (2) can be written as

$$\frac{dx}{dt} = \Phi(x) - \Psi(x),$$

where

$$\Phi(x) = \begin{pmatrix} \alpha SI \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix},$$

$$\Psi(x) = \begin{pmatrix} \mu I + \sigma I + \beta I + bI \\ -bI - \gamma C + \mu R \\ -\kappa \xi I + \theta M \\ -\Lambda + \alpha SI + \mu S + \lambda SM \\ -\beta I + \mu C + \delta C + \gamma C - \lambda SM \end{pmatrix}.$$

The Jacobian matrices of $\Phi(x)$ and $\Psi(x)$ at the disease-free equilibrium E_0 are, respectively,

$$D\Phi(E_0) = \begin{pmatrix} F_{2\times 2} & 0 \\ 0 & 0 \end{pmatrix},$$

$$D\Psi(E_0) = \begin{pmatrix} V_{2\times 2} & & 0 & 0 & 0 \\ -\kappa\xi & 0 & \theta & 0 & 0 \\ \frac{\Lambda\alpha}{\mu} & 0 & \frac{\Lambda\lambda}{\mu} & \mu & 0 \\ 0 & 0 & -\frac{\Lambda\lambda}{\mu} & 0 & \mu+\delta+\gamma \end{pmatrix},$$

where

$$F = \begin{pmatrix} \frac{\Lambda\alpha}{\mu} & 0 \\ 0 & 0 \end{pmatrix}, \quad V = \begin{pmatrix} \mu+\delta+\beta+b & 0 \\ -b & \mu \end{pmatrix}.$$

The model reproduction number denoted by R_0 is thus given by

$$R_0 = \frac{\Lambda\alpha}{\mu(\mu+\delta+\beta+b)}.$$

4.2 Global Stability of E_0

Theorem 1. *For equation (2), the disease-free equilibrium E_0 is globally asymptotically stable if $R_0 \leq 1$.*

Proof. We introduce the following Lyapunov function [12, 15]:

$$U(S(t), I(t), C(c), R(t), M(t)) = S - \ln S + C + I + \frac{1}{\mu}R.$$

The derivative of U is given by

$$\begin{aligned} \dot{U} &= \dot{S} - \tfrac{\dot{S}}{S} + \dot{I} + \dot{C} + \tfrac{1}{\mu}\dot{R} \\ &= \Lambda - \alpha SI - \mu S - \lambda SM - \tfrac{1}{S}(\Lambda - \alpha SI - \mu S - \lambda SM) \\ &\quad + \alpha SI - \mu I - \sigma I - \beta I - bI + \beta I - \mu C - \delta C - \gamma C \\ &\quad + \tfrac{1}{\mu}(bI + \gamma C - \mu R) + \kappa\xi I - \theta M \\ &= \Lambda(1 - \tfrac{1}{S}) - \mu S - bR - (\mu+\delta)C - \gamma(1 - \tfrac{1}{\mu})C \\ &\quad + (\alpha + \beta + \tfrac{b}{\mu} - \tfrac{\Lambda\alpha}{\mu})I + \tfrac{\Lambda\alpha}{\mu}(1 - \tfrac{1}{R_0})I. \end{aligned}$$

If $R_0 \leq 1$, then $\frac{\Lambda\alpha}{\mu}(1 - \frac{1}{R_0}) \leq 0$. As we know, $1 - \frac{1}{S} \leq 0$, $\mu < 1$ and $\frac{(\alpha+\beta)\mu+b}{\alpha} < \Lambda$, so we obtain $\dot{U} \leq 0$. Furthermore, $\dot{U} = 0$ only if $S = 1$ or $R_0 = 1$. The maximum invariant set in $\{(S, I, C, R, M) : \dot{U} = 0\}$ is the singleton E_0. By LaSalle's Invariance Principle [13], E_0 is globally asymptotically stable in Ω. □

4.3 Endemic equilibrium

4.3.1 Existence of the endemic equilibrium

Theorem 2. *If $R_0 > 1$, equation (2) has a endemic equilibrium $E^*(S^*, I^*, C^*, R^*, M^*)$, where*

$$S^* = \frac{\mu+\sigma+\beta+b}{\alpha},$$
$$C^* = \frac{\lambda\kappa\xi(\mu+\sigma+\beta+b)+\alpha\theta\beta}{\theta\alpha(\mu+\delta+\gamma)} I^*,$$
$$R^* = \frac{bI^*+\gamma C^*}{\mu},$$
$$M^* = \frac{\kappa\xi}{\theta} I^*.$$

Proof. From equation (2) it follows that

$$\begin{aligned} &\Lambda - \alpha SI - \mu S - \lambda SM = 0, \\ &\alpha SI - \mu I - \sigma I - \beta I - bI = 0, \\ &\beta I - \mu C - \delta C - \gamma C = 0, \\ &bI + \gamma C - \mu R = 0, \\ &\kappa\xi I - \theta M = 0. \end{aligned} \tag{27}$$

Now from the fourth and fifth equations of (27), we obtain

$$R = \frac{bI + \gamma C}{\mu}, \tag{28}$$

$$M = \frac{\kappa\xi}{\theta} I. \tag{29}$$

and from the second equation of (27), we obtain

$$S = \frac{\mu + \sigma + \beta + b}{\alpha}. \tag{30}$$

Then substituting (29) and (30) in to the third equation of (27), we obtain

$$C = \frac{\lambda\kappa\xi(\mu + \sigma + \beta + b) + \alpha\theta\beta}{\theta\alpha(\mu + \delta + \gamma)} I.$$

For $I \neq 0$ substituting equations (29)and (30) into the first equation of (27), we get

$$I = \frac{\mu\theta(R_0 - 1)}{\lambda\kappa\xi + \theta}.$$

Therefore, there exists a unique positive root in the interval $(0, 1)$ when $R_0 > 1$; there is no positive root in the interval $[0, 1]$ when $R_0 \leq 1$. For $R_0 > 1$, we carry out the simulation and give the following conjecture.

□

Conjecture: If $R_0 > 1$, the endemic equilibrium E^* of equation(2) is globally asymptotically stable.

Remark: Since the global stability of equation(2) is a hard problem, we only carry out simulation. How to prove the global stability of the endemic equilibrium E^* of equation(2) and give the conditions based on model parameters is still an open problem.

5 Analysis of Hopf-bifurcation

The variational matrix around $E^*(S^*, I^*, C^*, R^*, M^*)$ is

$$\begin{pmatrix} -A_{11} & -\alpha S^* & 0 & 0 & -\lambda S^* \\ \alpha I^* & -A_{22} & 0 & 0 & 0 \\ \lambda M^* & \beta & -A_{33} & 0 & 0 \\ 0 & b & \gamma & -\mu & 0 \\ 0 & \kappa\xi & 0 & 0 & -\theta \end{pmatrix}$$

with $A_{11} = \alpha I^* + \lambda M^* + \mu, A_{22} = \mu + \sigma + \beta + b$ and $A_{33} = \mu + \delta + \gamma$. Therefore the corresponding characteristic equation is

$$\omega^5 + A_1\omega^4 + A_2\omega^3 + A_3\omega^2 + A_4\omega + A_5 = 0, \tag{33}$$

where

$$\begin{aligned} A_1 &= A_{11} + A_{22} + A_{33} + \mu + \theta, \\ A_2 &= A_{11}A_{22} + \theta(A_{11} + A_{22}) + A_{33}\mu + (A_{33} + \mu)(A_{11} + A_{22} + \theta) \\ &\quad + \alpha^2 S^* I^*, \\ A_3 &= A_{11}A_{22}\theta + \alpha^2\theta S^* I^* - \lambda\alpha\kappa\xi S^* I^* + A_{33}\mu(A_{11} + A_{22} + \theta) \\ &\quad + (A_{33} + \mu)(A_{11}A_{22} + \theta(A_{11} + A_{22}) + \alpha^2 S^* I^*), \\ A_4 &= (A_{33} + \mu)(A_{11}A_{22}\theta + \alpha^2\theta S^* I^* - \lambda\alpha\kappa\xi S^* I^*) \\ &\quad + A_{33}\mu(A_{11}A_{22} + \theta(A_{11} + A_{22}) + \alpha^2 S^* I^*), \\ A_5 &= A_{33}\mu(A_{11}A_{22}\theta + \alpha^2\theta S^* I^* - \lambda\alpha\kappa\xi S^* I^*). \end{aligned}$$

Now, we will study the Hopf-bifurcation [11, 12, 4] of the above system, taking λ as the bifurcation parameter. The necessary and sufficient condition for the existence of the Hopf-bifurcation, if it exists is $\lambda = \lambda_0$ such that

(i) $A_i > 0, i = 1, 2, 3, 4, 5$,
(ii) $A_1(\lambda_0)A_2(\lambda_0) > A_3(\lambda_0)$,
(iii) $A_1(\lambda_0)A_2(\lambda_0)A_3(\lambda_0) > (A_3(\lambda_0))^2 - (A_1(\lambda_0))^2 A_4(\lambda_0)$,
(iv) $(A_3(\lambda_0)A_4(\lambda_0) - A_2(\lambda_0)A_5(\lambda_0))(A_1(\lambda_0)A_2(\lambda_0) - A_3(\lambda_0))$

$-(A_1(\lambda_0)A_4(\lambda_0)-A_5(\lambda_0))^2=0,$
and (v) if we consider the eigenvalues of the characteristic equation (33) is of the form, $\omega_i=\nu_i+i\eta_i$, then $\frac{d\nu_i}{d\lambda}\neq 0, i=1,2,3,4,5$. The condition

$$(A_3(\lambda_0)A_4(\lambda_0)-A_2(\lambda_0)A_5(\lambda_0))(A_1(\lambda_0)A_2(\lambda_0)-A_3(\lambda_0)) \\ -(A_1(\lambda_0)A_4(\lambda_0)-A_5(\lambda_0))^2=0.$$

Therefore, one pair of eigenvalues of the characteristic equation (33) at $\lambda=\lambda_0$ are of the form $\omega_{1,2}=\pm i\eta$ where η is positive real number.

Now, we will verify the Hopf-bifurcation condition (v), putting $\omega=\nu+i\eta$ in (33) and separating real and imaginary parts, we have

$$\nu^5+A_1\nu^4+(A_2-10\eta^2)\nu^3+(A_3-6A_1\eta^2)\nu^2 \\ +(5\eta^4-3A_2\eta^2+A_4)\nu+(A_1\eta^4-A_3\eta^2+A_5)=0, \tag{35}$$

$$(\eta^2)^2-(10\nu^2+4A_1\nu+A_2)\eta^2 \\ +(5\nu^4+4A_1\nu^3+3A_2\nu^2+2A_3\nu+A_4)=0. \tag{36}$$

Substituting the value of η^2 from (36) in (35), we get

$$\nu^5+A_1\nu^4+(A_2-10h(\nu))\nu^3+(A_3-6A_1h(\nu))\nu^2 \\ +(5(h(\nu))^2-3A_2h(\nu)+A_4)\nu+(A_1(h(\nu))^2-A_3h(\nu)+A_5)=0, \tag{37}$$

where

$$h(\nu)=\frac{1}{2}((10\nu^2+4A_1\nu+A_2)-F),$$

and

$$F=\sqrt{(10\nu^2+4A_1\nu+A_2)^2-4(5\nu^4+4A_1\nu^3+3A_2\nu^2+2A_3\nu+A_4)}.$$

Differentiating with respect to λ and putting $\lambda=\lambda_0$, we have

$$\left[\frac{d\nu}{d\lambda}\right]_{\lambda=\lambda_0}=\frac{h(0)\frac{dA_3}{d\lambda_0}-(h(0))^2\frac{dA_1}{d\lambda_0}-\frac{dA_5}{d\lambda_0}}{5(h(0))^2+2A_1h(0)h'(0)-3A_2h(0)-A_3h'(0)+A_4}\neq 0.$$

Since $h(0)\frac{dA_3}{d\lambda_0}-(h(0))^2\frac{dA_1}{d\lambda_0}-\frac{dA_5}{d\lambda_0}=S^*I^*[h(\nu)(2\lambda_0\theta+2\mu\lambda_0+2A_{33}\lambda_0-\lambda\kappa\xi)-2A_{33}\mu\theta\lambda_0]\neq 0$, this ensures that the above system has a Hopf-bifurcation around the interior equilibrium E^*. Hence as the recruitment rate of individuals into the community by birth or migration (susceptible) λ crosses its threshold value, $\lambda=\lambda_0$, then all population starts oscillating around the interior equilibrium point.

This ensures that the above system has a Hopf-bifurcation around the interior equilibrium E^*. Now, we further reduce the set of differential equation (2) into the normal form in order to determine the direction and stability criterion of the bifurcating periodic solution. Here, the Poincare's method is used to put equation (2) into the normal form following the procedure outlined by Hassard et al. [4]. For the sake of simplicity, introducing the new variables.

The equation (2) can be written as

$$\dot{x} = f(x, \lambda), \quad \text{where} \quad x = (S\ I\ C\ R\ M)^T. \tag{41}$$

Now we calculate right eigenvectors u_1, u_3, u_4 and u_5 of the Jacobian matrix J at E^* corresponding to the eigenvalues $\omega_1 = i\eta, \omega_3, \omega_4$ and ω_5 respectively, at $\lambda = \lambda_0$:

$$u_1 = \begin{pmatrix} R_{11} - iR_{12} \\ R_{21} - iR_{22} \\ R_{31} - iR_{32} \\ R_{41} - iR_{42} \\ R_{51} - iR_{52} \end{pmatrix}, u_3 = \begin{pmatrix} R_{13} \\ R_{23} \\ R_{33} \\ R_{43} \\ R_{53} \end{pmatrix}, u_4 = \begin{pmatrix} R_{14} \\ R_{24} \\ R_{34} \\ R_{44} \\ R_{54} \end{pmatrix}, u_5 = \begin{pmatrix} R_{15} \\ R_{25} \\ R_{35} \\ R_{45} \\ R_{55} \end{pmatrix}.$$

For equation (41), define

$$R = (Re(u_1), -Im(u_1), u_3, u_4, u_5)$$

$$= \begin{pmatrix} R_{11} & R_{12} & R_{13} & R_{14} & R_{15} \\ R_{21} & R_{22} & R_{23} & R_{24} & R_{25} \\ R_{31} & R_{32} & R_{33} & R_{34} & R_{35} \\ R_{41} & R_{42} & R_{43} & R_{44} & R_{45} \\ 1 & 0 & 1 & 1 & 1 \end{pmatrix},$$

where

$$R_{11} = \frac{A_{22}\theta - \eta^2}{\kappa\xi\alpha I^*}, \quad R_{12} = \frac{A_{22}\eta + \theta\eta}{\kappa\xi\alpha I^*}, \quad R_{21} = \frac{\theta}{\kappa\xi}, \quad R_{22} = \frac{\eta}{\kappa\xi},$$

$$R_{31} = \frac{(\lambda A_{22}M^* + \alpha\beta I^*)(\theta A_{33} + \eta^2) + \lambda\eta^2 M^*(\theta - A_{33})}{(A_{33}^2 + \eta^2)\kappa\xi\alpha I^*},$$

$$R_{32} = \frac{\lambda\eta M^*(\theta A_{33} + \eta^2) + \eta(A_{33} - \theta)(\lambda A_{22}M^* + \alpha\beta I^*)}{(A_{33}^2 + \eta^2)\kappa\xi\alpha I^*},$$

$$R_{41} = \frac{b(\theta\mu - \eta^2)}{\kappa\xi(\mu^2 + \eta^2)} + \frac{\gamma(\lambda A_{22}M^* + \alpha\beta I^*)(\theta A_{33} + \eta^2) + \lambda\eta^2 M^*(\theta - A_{33})}{(A_{33}^2 + \eta^2)\kappa\xi\alpha I^*},$$

$$R_{13} = \frac{(A_{22} + \omega_3)(\theta + \omega_3)}{\kappa\xi\alpha I^*}, \quad R_{23} = \frac{A_{22}\theta + \omega_3(\theta + A_{22}) + \omega_3^2}{\kappa\xi\alpha I^*},$$

$$R_{42} = \frac{\lambda\eta\gamma M^*(\theta A_{33} + \eta^2) + \eta\gamma(A_{33} - \theta)(\lambda A_{22}M^* + \alpha\beta I^*)}{(A_{33}^2 + \eta^2)\kappa\xi\alpha I^*} + \frac{\mu(\eta + \mu)}{\kappa\xi(\mu^2 + \eta^2)},$$

$$R_{33} = \frac{\lambda M^*(A_{22} + \omega_3)(\theta + \omega_3) + \alpha\beta I^*(\theta + \omega_3)}{(A_{33} + \omega_3)\alpha\kappa\xi I^*},$$

$$R_{43} = \frac{(\theta + \omega_3)(\alpha b I^*(A_{33} + \omega_3) + \lambda M^*(A_{33} + \omega_3) + \alpha\beta I^*)}{(A_{33} + \omega_3)(\mu + \omega_3)\alpha\kappa\xi I^*},$$

$$R_{14} = \frac{(A_{22} + \omega_4)(\theta + \omega_4)}{\kappa\xi\alpha I^*}, \quad R_{24} = \frac{A_{22}\theta + \omega_4(\theta + A_{22}) + \omega_4^2}{\kappa\xi\alpha I^*},$$

$$R_{34} = \frac{\lambda M^*(A_{22}+\omega_4)(\theta+\omega_4)+\alpha\beta I^*(\theta+\omega_4)}{(A_{33}+\omega_4)\alpha\kappa\xi I^*},$$
$$R_{44} = \frac{(\theta+\omega_4)(\alpha b I^*(A_{33}+\omega_4)+\lambda M^*(A_{33}+\omega_4)+\alpha\beta I^*)}{(A_{33}+\omega_4)(\mu+\omega_4)\alpha\kappa\xi I^*},$$
$$R_{15} = \frac{(A_{22}+\omega_5)(\theta+\omega_5)}{\kappa\xi\alpha I^*}, \quad R_{25} = \frac{A_{22}\theta+\omega_5(\theta+A_{22})+\omega_5^2}{\kappa\xi\alpha I^*},$$
$$R_{35} = \frac{\lambda M^*(A_{22}+\omega_5)(\theta+\omega_5)+\alpha\beta I^*(\theta+\omega_5)}{(A_{33}+\omega_5)\alpha\kappa\xi I^*},$$
$$R_{45} = \frac{(\theta+\omega_5)(\alpha b I^*(A_{33}+\omega_5)+\lambda M^*(A_{33}+\omega_5)+\alpha\beta I^*)}{(A_{33}+\omega_5)(\mu+\omega_5)\alpha\kappa\xi I^*},$$

Equation (41) is transformed such that $F(X) = R^{-1}f(E^* + RX)$, where $X = (x_1\ y_1\ z_1\ v_1\ w_1)^T$ and E^* is steady state. Thus we obtain

$$\dot{X} = F(X). \tag{43}$$

We now use the transformation

$$S = S^* + R_{11}x_1 + R_{12}y_1 + R_{13}z_1 + R_{14}v_1 + R_{15}w_1,$$
$$I = I^* + R_{21}x_1 + R_{22}y_1 + R_{23}z_1 + R_{24}v_1 + R_{25}w_1,$$
$$C = C^* + R_{31}x_1 + R_{32}y_1 + R_{33}z_1 + R_{34}v_1 + R_{35}w_1,$$
$$R = R^* + R_{41}x_1 + R_{42}y_1 + R_{43}z_1 + R_{44}v_1 + R_{45}w_1,$$
$$M = M^* + R_{51}x_1 + R_{52}y_1 + R_{53}z_1 + R_{54}v_1 + R_{55}w_1.$$

Using the above transformation, equation (2) reduces to

$$\frac{dx_1}{dt} = \frac{(a_{10}a_{33}-a_{30}a_{13})(a_{12}a_{23}-a_{22}a_{13})}{L} - \frac{(a_{10}a_{23}-a_{20}a_{13})(a_{12}a_{33}-a_{32}a_{13})}{L},$$
$$\frac{dy_1}{dt} = \frac{(a_{10}a_{33}-a_{30}a_{13})(a_{11}a_{23}-a_{21}a_{13})}{L} - \frac{(a_{10}a_{23}-a_{20}a_{13})(a_{11}a_{33}-a_{31}a_{13})}{L},$$
$$\frac{dz_1}{dt} = \frac{(a_{10}a_{32}-a_{30}a_{12})(a_{11}a_{22}-a_{21}a_{12})}{L} - \frac{(a_{10}a_{22}-a_{20}a_{12})(a_{11}a_{32}-a_{31}a_{12})}{L_1},$$
$$\frac{dv_1}{dt} = \frac{(a_{40}a_{61}-a_{60}a_{41})(a_{43}a_{51}-a_{53}a_{41})}{L} - \frac{(a_{40}a_{51}-a_{50}a_{41})(a_{43}a_{61}-a_{63}a_{41})}{L_2},$$
$$\frac{dw_1}{dt} = \frac{(a_{40}a_{51}-a_{50}a_{41})(a_{42}a_{61}-a_{62}a_{41})}{L} - \frac{(a_{40}a_{61}-a_{60}a_{41})(a_{42}a_{51}-a_{52}a_{41})}{L_2},$$

where

$$\begin{aligned} P_1 = \Lambda - \mu(S^* + R_{11}x_1 + R_{12}y_1 + R_{13}z_1 + R_{14}v_1 + R_{15}w_1) \\ -\alpha(S^* + R_{11}x_1 + R_{12}y_1 + R_{13}z_1 + R_{14}v_1 + R_{15}w_1) \\ +(I^* + R_{21}x_1 + R_{22}y_1 + R_{23}z_1 + R_{24}v_1 + R_{25}w_1) \\ -\lambda(S^* + R_{11}x_1 + R_{12}y_1 + R_{13}z_1 + R_{14}v_1 + R_{15}w_1) \\ +(M^* + R_{51}x_1 + R_{52}y_1 + R_{53}z_1 + R_{54}v_1 + R_{55}w_1), \end{aligned}$$

$$
\begin{aligned}
P_2 &= \alpha(S^* + R_{11}x_1 + R_{12}y_1 + R_{13}z_1 + R_{14}v_1 + R_{15}w_1)\\
&\quad +(I^* + R_{21}x_1 + R_{22}y_1 + R_{23}z_1 + R_{24}v_1 + R_{25}w_1)\\
&\quad -(\mu+\sigma+\beta+b)(I^* + R_{21}x_1 + R_{22}y_1 + R_{23}z_1 + R_{24}v_1 + R_{25}w_1),\\
P_3 &= \beta(I^* + R_{21}x_1 + R_{22}y_1 + R_{23}z_1 + R_{24}v_1 + R_{25}w_1)\\
&\quad -(\mu+\delta+\gamma)(C^* + R_{31}x_1 + R_{32}y_1 + R_{33}z_1 + R_{34}v_1 + R_{35}w_1),\\
P_4 &= b(I^* + R_{21}x_1 + R_{22}y_1 + R_{23}z_1 + R_{24}v_1 + R_{25}w_1)\\
&\quad +\gamma(C^* + R_{31}x_1 + R_{32}y_1 + R_{33}z_1 + R_{34}v_1 + R_{35}w_1)\\
&\quad -\mu(R^* + R_{41}x_1 + R_{42}y_1 + R_{43}z_1 + R_{44}v_1 + R_{45}w_1),\\
P_5 &= \kappa\xi(I^* + R_{21}x_1 + R_{22}y_1 + R_{23}z_1 + R_{24}v_1 + R_{25}w_1)\\
&\quad -\theta(M^* + R_{51}x_1 + R_{52}y_1 + R_{53}z_1 + R_{54}v_1 + R_{55}w_1),\\
L &= (a_{11}a_{23} - a_{21}a_{13})(a_{12}a_{33} - a_{32}a_{13})\\
&\quad -(a_{11}a_{33} - a_{31}a_{13})(a_{12}a_{23} - a_{22}a_{13}),\\
L_1 &= (a_{13}a_{22} - a_{23}a_{12})(a_{11}a_{32} - a_{31}a_{12})\\
&\quad -(a_{13}a_{32} - a_{33}a_{12})(a_{11}a_{22} - a_{21}a_{12}),\\
L_2 &= (a_{42}a_{51} - a_{52}a_{41})(a_{43}a_{61} - a_{63}a_{41})\\
&\quad -(a_{42}a_{61} - a_{62}a_{41})(a_{43}a_{51} - a_{53}a_{41}),\\
a_{10} &= (P_1R_{25} - P_2R_{15})(R_{14}R_{35} - R_{34}R_{15})\\
&\quad -(P_1R_{35} - P_3R_{15})(R_{14}R_{25} - R_{24}R_{15}),\\
a_{11} &= (R_{11}R_{25} - R_{21}R_{15})(R_{14}R_{35} - R_{34}R_{15})\\
&\quad -(R_{11}R_{35} - R_{31}R_{15})(R_{14}R_{25} - R_{24}R_{15}),\\
a_{12} &= (R_{12}R_{25} - R_{15}R_{22})(R_{14}R_{35} - R_{34}R_{15})\\
&\quad -(R_{12}R_{35} - R_{32}R_{15})(R_{14}R_{25} - R_{24}R_{15}),\\
a_{13} &= (R_{13}R_{25} - R_{23}R_{15})(R_{14}R_{35} - R_{34}R_{15})\\
&\quad -(R_{13}R_{35} - R_{33}R_{15})(R_{14}R_{25} - R_{24}R_{15}),\\
a_{20} &= (P_1R_{25} - P_2R_{15})(R_{14}R_{45} - R_{44}R_{15})\\
&\quad -(P_1R_{45} - P_4R_{15})(R_{14}R_{25} - R_{24}R_{15}),\\
a_{21} &= (R_{11}R_{25} - R_{21}R_{15})(R_{14}R_{45} - R_{44}R_{15})\\
&\quad -(R_{11}R_{45} - R_{41}R_{15})(R_{14}R_{25} - R_{24}R_{15}),\\
a_{22} &= (R_{12}R_{25} - R_{15}R_{22})(R_{14}R_{45} - R_{44}R_{15})\\
&\quad -(R_{12}R_{45} - R_{42}R_{15})(R_{14}R_{25} - R_{24}R_{15}),\\
a_{23} &= (R_{13}R_{25} - R_{23}R_{15})(R_{14}R_{45} - R_{44}R_{15})\\
&\quad -(R_{13}R_{25} - R_{23}R_{15})(R_{14}R_{25} - R_{24}R_{15})\\
a_{30} &= (P_1R_{25} - P_2R_{15})(R_{14}R_{55} - R_{54}R_{15})\\
&\quad -(P_1R_{55} - P_5R_{15})(R_{14}R_{25} - R_{24}R_{15}),\\
a_{31} &= (R_{11}R_{25} - R_{21}R_{15})(R_{14}R_{55} - R_{54}R_{15})\\
&\quad -(R_{11}R_{55} - R_{51}R_{15})(R_{14}R_{25} - R_{24}R_{15}),\\
a_{32} &= (R_{12}R_{25} - R_{15}R_{22})(R_{14}R_{55} - R_{54}R_{15})\\
&\quad -(R_{12}R_{55} - R_{52}R_{15})(R_{14}R_{25} - R_{24}R_{15}),\\
a_{33} &= (R_{13}R_{25} - R_{23}R_{15})(R_{14}R_{55} - R_{54}R_{15})\\
&\quad -(R_{13}R_{55} - R_{53}R_{15})(R_{14}R_{25} - R_{24}R_{15}),
\end{aligned}
$$

$$
\begin{aligned}
a_{40} &= (P_1R_{21} - P_2R_{11})(R_{12}R_{31} - R_{32}R_{11})\\
&\quad -(P_1R_{31} - P_3R_{11})(R_{12}R_{21} - R_{22}R_{11}),\\
a_{41} &= (R_{13}R_{21} - R_{23}R_{11})(R_{12}R_{31} - R_{32}R_{11})\\
&\quad -(R_{13}R_{31} - R_{33}R_{11})(R_{12}R_{21} - R_{22}R_{11})\\
a_{42} &= (R_{14}R_{21} - R_{24}R_{11})(R_{12}R_{41} - R_{32}R_{11})\\
&\quad -(R_{14}R_{31} - R_{34}R_{11})(R_{12}R_{21} - R_{22}R_{11})\\
a_{43} &= (R_{15}R_{21} - R_{25}R_{11})(R_{12}R_{31} - R_{32}R_{11})\\
&\quad -(R_{15}R_{31} - R_{35}R_{11})(R_{12}R_{21} - R_{22}R_{11}),\\
a_{50} &= (P_1R_{21} - P_2R_{11})(R_{12}R_{41} - R_{42}R_{11})\\
&\quad -(P_1R_{41} - P_4R_{11})(R_{12}R_{21} - R_{22}R_{11}),\\
a_{51} &= (R_{13}R_{21} - R_{23}R_{11})(R_{12}R_{41} - R_{42}R_{11})\\
&\quad -(R_{13}R_{41} - R_{43}R_{11})(R_{12}R_{21} - R_{22}R_{11}),\\
a_{52} &= (R_{14}R_{21} - R_{24}R_{11})(R_{12}R_{41} - R_{42}R_{11})\\
&\quad -(R_{14}R_{41} - R_{44}R_{11})(R_{12}R_{21} - R_{22}R_{11}),\\
a_{53} &= (R_{15}R_{21} - R_{25}R_{11})(R_{12}R_{41} - R_{42}R_{11})\\
&\quad -(R_{15}R_{41} - R_{44}R_{11})(R_{12}R_{21} - R_{22}R_{11}),\\
a_{60} &= (P_1R_{21} - P_2R_{11})(R_{12}R_{51} - R_{52}R_{11}) - (P_1R_{51}\\
&\quad -P_5R_{11})(R_{12}R_{21} - R_{22}R_{11}),\\
a_{61} &= (R_{13}R_{21} - R_{23}R_{11})(R_{12}R_{51} - R_{52}R_{11})\\
&\quad -(R_{13}R_{51} - R_{53}R_{11})(R_{12}R_{21} - R_{22}R_{11}),\\
a_{62} &= (R_{14}R_{21} - R_{24}R_{11})(R_{21}R_{51} - R_{52}R_{11})\\
&\quad -(R_{14}R_{51} - R_{54}R_{11})(R_{12}R_{21} - R_{22}R_{11}),\\
a_{63} &= (R_{15}R_{21} - R_{25}R_{11})(R_{12}R_{51} - R_{52}R_{11})\\
&\quad -(R_{15}R_{51} - R_{55}R_{11})(R_{12}R_{21} - R_{22}R_{11}).
\end{aligned}
$$

Therefore, equation (43) is the normal form of equation(41) from which the stability and direction of the Hopf-bifurcation can be computed. In equation(41), on the right hand side of the first term is linear and the second is non-linear in $Y's$. For evaluating the direction of periodic solution, we can evaluate the following quantities at $\lambda = \lambda_0$ and origin:

$$
\begin{aligned}
g_{11} &= \frac{1}{4}\left[\frac{\partial^2 F^1}{\partial y_1^2} + \frac{\partial^2 F^1}{\partial y_2^2} + i\left(\frac{\partial^2 F^2}{\partial y_1^2} + \frac{\partial^2 F^2}{\partial y_2^2}\right)\right],\\
g_{02} &= \frac{1}{4}\left[\frac{\partial^2 F^1}{\partial y_1^2} - \frac{\partial^2 F^1}{\partial y_2^2} - 2\frac{\partial^2 F^2}{\partial y_1 \partial y_2} + i\left(\frac{\partial^2 F^2}{\partial y_1^2} - \frac{\partial^2 F^2}{\partial y_2^2} + 2\frac{\partial^2 F^2}{\partial y_1 \partial y_2}\right)\right],\\
g_{20} &= \frac{1}{4}\left[\frac{\partial^2 F^1}{\partial y_1^2} - \frac{\partial^2 F^1}{\partial y_2^2} + 2\frac{\partial^2 F^2}{\partial y_1 \partial y_2} + i\left(\frac{\partial^2 F^2}{\partial y_1^2} - \frac{\partial^2 F^2}{\partial y_2^2} - 2\frac{\partial^2 F^2}{\partial y_1 \partial y_2}\right)\right],\\
G_{21} &= \frac{1}{8}\left[\frac{\partial^3 F^1}{\partial y_1^3} + \frac{\partial^3 F^1}{\partial y_1 \partial y_2^2} + \frac{\partial^3 F^2}{\partial y_1^2 \partial y_2} + \frac{\partial^3 F^2}{\partial y_2^3}\right.\\
&\quad \left. + i\left(\frac{\partial^3 F^2}{\partial y_1^3} + \frac{\partial^3 F^2}{\partial y_1 \partial y_2^2} - \frac{\partial^3 F^1}{\partial y_1^2 \partial y_2} - \frac{\partial^3 F^1}{\partial y_2^3}\right)\right],\\
G_{110}^j &= \frac{1}{2}\left[\frac{\partial^2 F^1}{\partial y_1 \partial y_j} + \frac{\partial^2 F^2}{\partial y_2 \partial y_j} + i\left(\frac{\partial^2 F^2}{\partial y_1 \partial y_j} - \frac{\partial^2 F^1}{\partial y_2 \partial y_j}\right)\right],
\end{aligned}
$$

$$
\begin{aligned}
&G^j_{101} = \frac{1}{2}\left[\frac{\partial^2 F^1}{\partial y_1 \partial y_j} - \frac{\partial^2 F^2}{\partial y_2 \partial y_j} + i\left(\frac{\partial^2 F^2}{\partial y_1 \partial y_j} + \frac{\partial^2 F^1}{\partial y_2 \partial y_j}\right)\right],\\
&h^j_{11} = \frac{1}{4}\left[\frac{\partial^2 F^j}{\partial y_1^2} + \frac{\partial^2 F^j}{\partial y_2^2}\right], \quad h^j_{20} = \frac{1}{4}\left[\frac{\partial^2 F^j}{\partial y_1^2} - \frac{\partial^2 F^j}{\partial y_2^2} - 2i\frac{\partial^2 F^j}{\partial y_1 \partial y_2}\right],\\
&w^j_{11} = \frac{h^j_{11}}{p_j}, \quad w^j_{20} = \frac{h^j_{20}}{p_j + 2i\eta}, \quad j = 1, 2, 3,
\end{aligned}
$$

and

$$
g_{11} = G_{21} + \sum_{j=1}^{3}\left(2G^j_{110}\eta^j_{11} + G^j_{101}\eta^j_{20}\right).
$$

Based on the above analysis, we can see that each g_{ij} can be determined by the parameters. Thus we can compute the following quantities:

$$
\begin{aligned}
&C_1(0) = \frac{i}{2\eta}\left(g_{11}g_{20} - 2|g_{11}|^2 - \frac{|g_{02}|^2}{3}\right) + \frac{g_{21}}{2},\\
&\mu_2 = -\frac{Re\{C_1(0)\}}{Re\{E'(\lambda_0)\}},\\
&\beta_2 = -2Re\{C_1(0)\},\\
&T_2 = -\frac{Im\{C_1(0)\} + \mu_2 Im\{E'(\lambda_0)\}}{\eta}.
\end{aligned}
$$

μ_2 determines the direction of the Hopf-bifurcation: if $\mu_2 > 0(\mu_2 < 0)$, then the Hopf-bifurcation is supercritical (subcritical) and the bifurcating periodic solutions exist for $\lambda > \lambda_0(\lambda < \lambda_0)$, β_2 determines the stability of bifurcating periodic solutions. The bifurcating periodic solutions are orbitally asymptotically stable (unstable) if $\beta_2 < 0(\beta_2 > 0)$ and T_2 determines the period of the bifurcating periodic solutions, the period increases (decreases) if $T_2 > 0(T_2 < 0)$.

6 Numerical Simulation

In this section, we carry out extensive numerical simulations to check the feasibility of our analytical findings regarding stability and bifurcation of the model equation (2) around the interior equilibrium point $E^*(S^*, I^*, C^*, R^*, M^*)$. We perform the model simulation by using the standard MATLAB differential equations integrator for the Runge-Kutta method and the set of parameter values are specified in Table 1. For the set of parameter values in Table 1, it may be verified that the model equation (2) has two equilibrium points namely, the disease-free equilibrium point $E_0(8.04189 \times 10^6, 0, 0, 0, 0)$ and the interior equilibrium point $E^*(1220, 5872.91, 189.245, 40784.5, 587.291)$. The eigenvalues corresponding to the variational matrix for the equilibrium point E_0 of the model equation (2) are 1608.13, - 0.931, -0.5, - 0.018, -0.018. Form the eigenvalues it is clear that E_0 is unstable in

Table 1: Description of parameters with their values.

Parameter	Definition	Default Value (day^{-1})
Λ	The recruitment rate of individuals into the community by birth or migration(susceptible)	144754 [1]
μ	The per capita mortality rate	0.018 [1]
β	The rate of progression from infective to carrier	0.03,0.04,0.05 [1]
α	The rate of infection	0.0002 Estimated
b	The rate of recovery from the infectious stage	0.096 [1]
δ	The carrier-induced mortality rate	0.013 [1]
λ	Dissemination rate of awareness among susceptible	0.2 [5]
γ	The rate of recovery from carrier stage	0.9 [5] 0.15
σ	The typhoid fever-indicated mortality mortality rate	0.1,0.9,1 [1]
ξ	The awareness programs increase with increase in disease related death rate	0.5 [5]
κ	The proportionality constant which governs the implementation	0.1 [22]
θ	The depletion rate of these programs due to ineffectiveness	0.5 [5]

nature. The eigenvalues of the Jacobian matrix corresponding to the equilibrium point E^* are given by - 118.659, - 0.246097 $\pm$ 1.07159 i, - 0.931, - 0.018. It can be noticed that the two eigenvalues of the Jacobian matrix are complex conjugate and the real part is negative and the other eigenvalues are negative. Therefore, the interior equilibrium point E^* ia locally asymptotically stable.

Figure 2 shows that the impact of susceptible individuals for different values of α namely, 0.02, 0.002 and 0.0002. From the Figure 2 it is clear that for the lower value of α the susceptible individuals shows maximum variation of the dynamics. From the parameter values in Table 1, the computer generated figure of infected individual $I(t)$ versus cumulative density of awareness programs $M(t)$ has been drawn in Figure 3 which designates the stable limit cycle oscillations around the interior equilibrium point E^*

with $\beta = 5$.

The bifurcation diagram of the model equation (2) has been shown in Figures (4–8) with respect to the parameter λ. From the bifurcation diagram it is clear that for the lower value of λ the model equation (2) is locally asymptotically stable, but above the critical value of λ $(= \lambda^c = 0.0296)$ the model equation (2) losses its stability and periodic solution arises through Hopf - bifurcation. Above the critical value of λ $(= \lambda^c = 0.0296)$, the unique positive endemic equilibrium point E^* is globally asymptotically stable.

From the Figures (9 - 10), we can observe that for the lower threshold value of $\lambda = 0.02 < 0.0296 = \lambda^c$ the model equation (2) is locally asymptotically stable. Also, from the Figures (11 - 12), we can observe that for the upper threshold value of $\lambda = 0.035 > 0.0296 = \lambda^c$ the model equation (2) leads to oscillations with same amplitude. With respect to the parameter values in Table 1, the computer generated figure of the cumulative density of awareness programs $M(t)$ versus the carrier individuals has been shown in Figure 13 with different initial values. From Figure 13 it can be observed that all the trajectories initiating inside the region of attraction approach towards the equilibrium value (M^*, C^*). This indicates the nonlinear stability analysis of (M^*, C^*) in $M - C$ plane. The phase portrait diagram generated in the three dimensional system which have been shown in Figure (14 - 15). The Figure 14 shows the stable limit cycle of the sub-system of the model equation (2) for the lower threshold value of $\lambda = 0.02 < 0.0296 = \lambda^c$. The Figure 15 shows the oscillations of the sub-system of the model equation (2) for the upper threshold value of $\lambda = 0.035 > 0.0296 = \lambda^c$.

7 Discussion and conclusions

In this article we investigate a mathematical model for Typhoid fever incorporating the influence of awareness programs. To control the infectious diseases such as Typhoid fever, the media is widely acknowledged as a pivotal tool in influencing people consciousness towards the disease to devise proper strategies. The qualitative analysis of the model, the existence of disease-free and endemic equilibrium points and their stabilities is analyzed. The positivity and boundedness of solutions indicate that the model equation (2) is biologically meaningful. The disease-free equilibrium point is globally asymptotically stable has been proved by constructing suitable Lyapunov function. We perform the analysis of Hopf- bifurcation with respect to the parameter λ, the dissemination rate of awareness among sus-

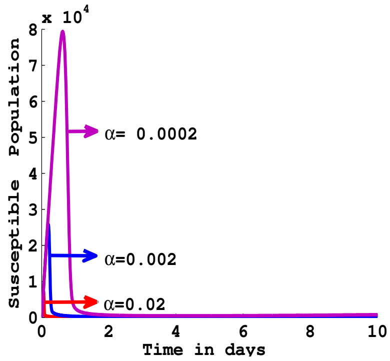

Figure 2: The figure shows the impact of susceptible individuals for different values of α.

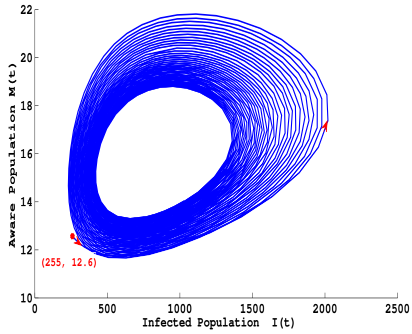

Figure 3: There is a stable limit cycle oscillations of the model equation (2) with $\beta = 5$ in the $I(t) - M(t)$ plane and the other parameters are in Table 1.

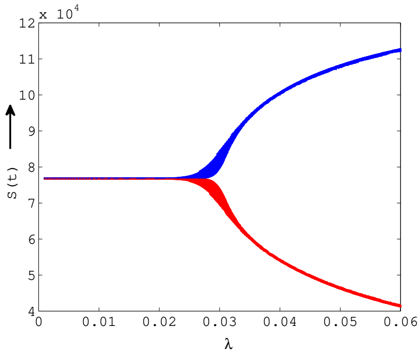

Figure 4: Bifurcation diagram of the susceptible individuals with respect to parameter λ.

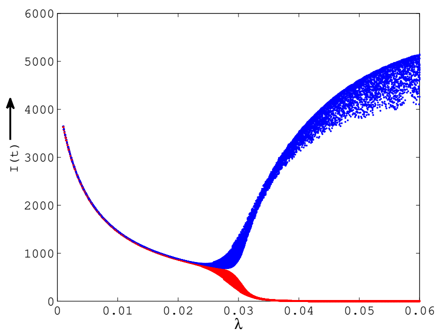

Figure 5: Bifurcation diagram of the infected individuals with respect to parameter λ.

ceptible due to awareness programs. Also, we investigate the direction and stability of Hopf- bifurcation.

Awareness programs through public media make people aware about the disease to take different consciousness (considering preventive medicine, social distancing, vaccination etc.) to decrease their chances of being infected. Awareness among the human population alters the pattern of disease spread and reduce the rate of infection [7, 14]. Our study indicates that if we increase the dissemination rate (λ) of awareness among susceptible due to awareness programs then the lower threshold value of λ, that is,

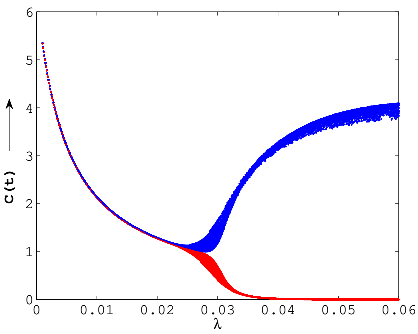

Figure 6: Bifurcation diagram of the carriers individuals with respect to parameter λ.

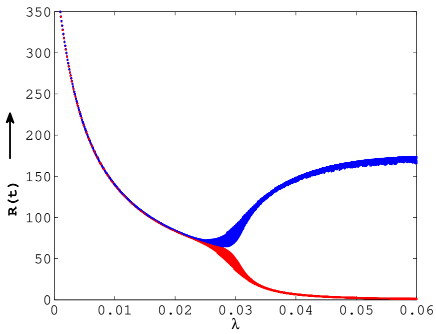

Figure 7: Bifurcation diagram of the recovery individuals with respect to parameter λ.

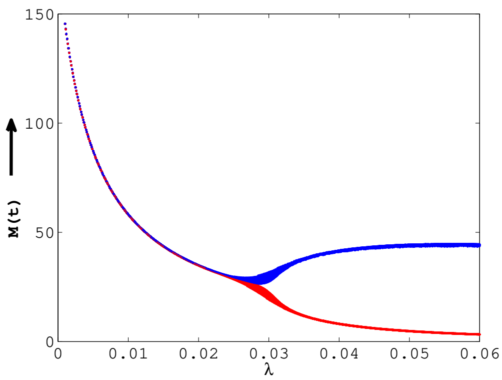

Figure 8: Bifurcation diagram of the awarded individuals with respect to parameter λ.

$\lambda^c < 0.0296$ the model system is locally asymptotically stable and beyond the threshold value λ^c the model system become unstable. The threshold value of the crucial parameter λ gives an idea how to prevent the disease by various precautions like vaccination, preventive medicine etc. Another important parameter α (rate of infection from susceptible to infections) shows an impact of the dynamics of the model considered by us.

In most of the theoretical works on awareness programs, the basic assumption is that the awareness could alter the pattern of disease spread and reduce the rate of infection. The pattern of disease outbreak for various diseases are likely to be different and the consciousness are taken by

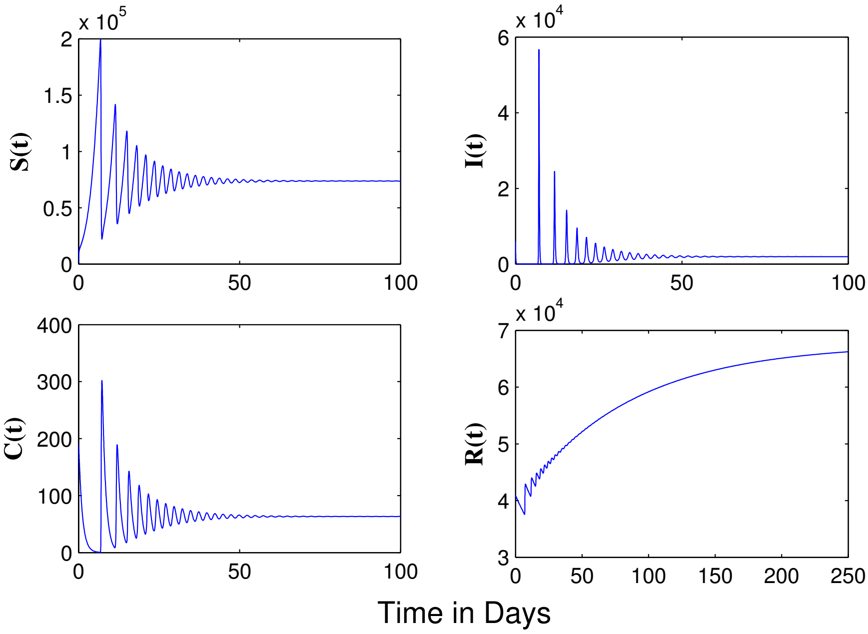

Figure 9: The time series solutions shows that the interior equilibrium point E^* is locally asymptotically stable for $\lambda = 0.02$ and the other parameters are specified in Table 1.

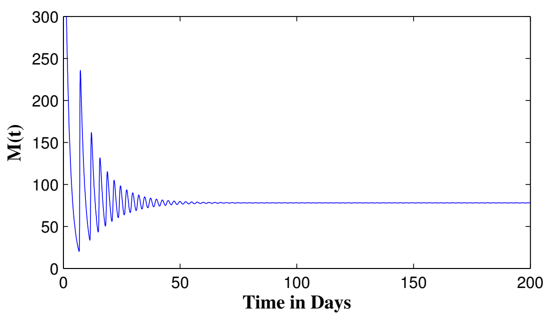

Figure 10: The time series solutions shows that the interior equilibrium point E^* is locally asymptotically stable for $\lambda = 0.02$ and the other parameters are specified in Table 1.

aware people according to the said disease. In our study, we did not take into account the optimization of the problem, that is, we did not minimize the infection as well as we did not maximize the susceptible individuals

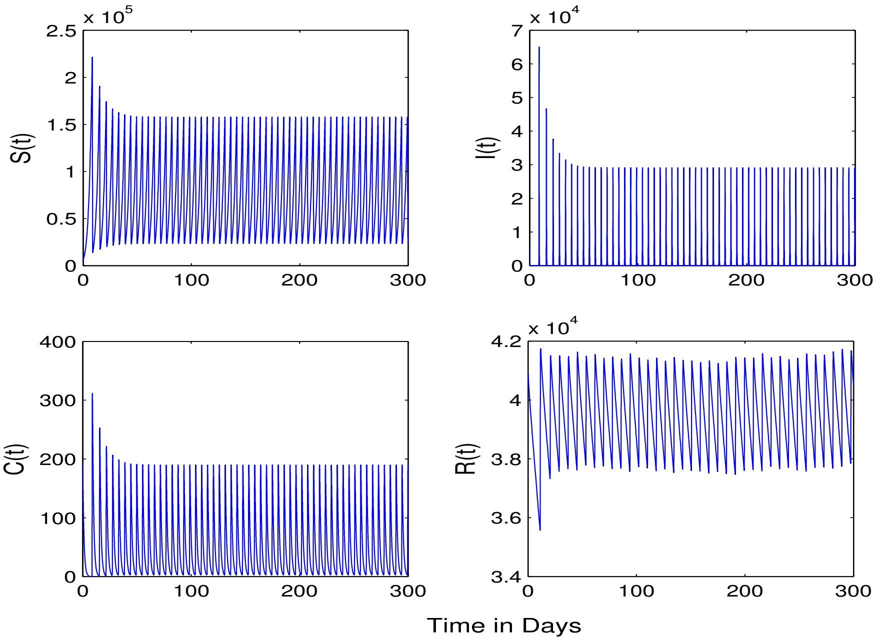

Figure 11: The trajectories of individuals shows the same magnitude of oscillations with $\lambda = 0.035$ and the other parameters are specified in Table 1.

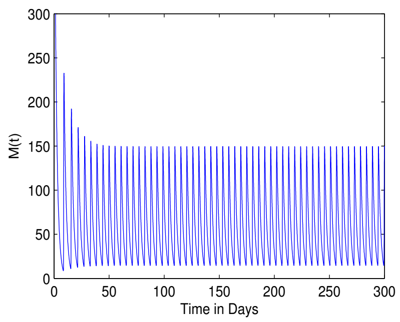

Figure 12: The trajectories of individuals shows the same magnitude of oscillations with $\lambda = 0.035$ and the other parameters are specified in Table 1.

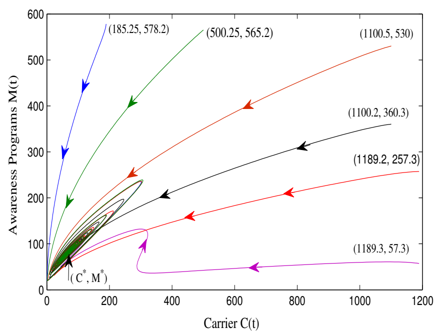

Figure 13: The figure shows the nonlinear stability of $M(t) - C(t)$ plane for different initial values.

and awareness programs. We leave them for future research. We conclude

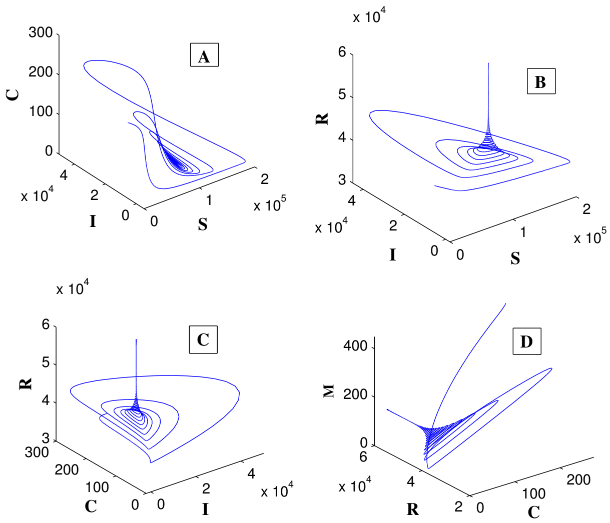

Figure 14: The figure shows the nonlinear stability of A) S(t) - I(t) - C(t); B) S(t) - I(t) - R(t); C) I(t) - C(t) - R(t); D) C(t) - R(t) - M(t); in the 3D phase portrait diagram with $\lambda = 0.02$. Other parameters are presented in Table 1.

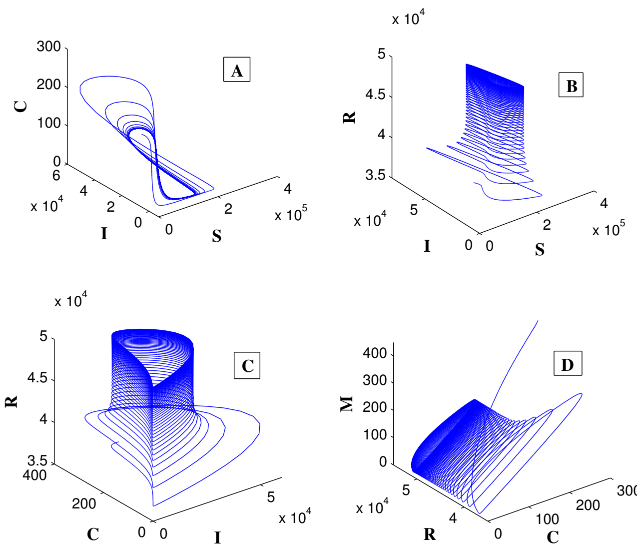

Figure 15: The figure shows the nonlinear stability of A) S(t) - I(t) - C(t); B) S(t) - I(t) - R(t); C) I(t) - C(t) - R(t); D) C(t) - R(t) - M(t); in the 3D phase portrait diagram with $\lambda = 0.035$. Other parameters are presented in Table 1.

by saying that the analysis and results are presented in this paper may be helpful to the future researchers and shed some light on the cumulative density of awareness programs.

Acknowledgements

I would like to acknowledge my respected Professor Roy, my family, present University and Barasat College for giving me an opportunity to work in the Computer Laboratory.

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