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Entropy generation due to unsteady hydromagnetic Couette flow and heat transfer with asymmetric convective cooling in a rotating system

Sanatan Das[†], Rabindranath Jana[‡] and Ali J. Chamkha[§*]

[†]*Department of Mathematics, University of Gour Banga*
Malda 732 103, West Bengal, India
email: tutusanasd@yahoo.co.in

[‡]*Department of Applied Mathematics, Vidyasagar University*
Midnapore 721 102, West Bengal, India
email: jana261171yahoo.co.in

[§]*Mechanical Engineering Department, Prince Mohammad Bin Fahd University (PMU), Al-Khobar 31952, Kingdom of Saudi Arabia*
email: achamkha@pmu.edu.sa

Abstract. Entropy generation in an unsteady hydromagnetic Couette flow of a viscous incompressible electrically conducting fluid between two infinite horizontal parallel plates in a rotating system have been analyzed. Both the lower and upper plates of the channel are subjected to asymmetric convective heat exchange with the ambient following the Newton's law of cooling. A numerical solution for governing equations is developed. The influences of the pertinent parameters on the fluid velocity components, temperature, entropy generation and Bejan number are discussed graphically.

Keywords: Couette flow, convective cooling, entropy generation and Bejan number.
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1 Introduction

The flow of an electrically conducting fluid between parallel plates in the

[*]Corresponding author.
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presence of a magnetic field is of a special technical significance because of its frequent occurrence in many industrial applications such as magnetohydrodynamic power generators, pumps, cooling of nuclear reactors, geothermal systems,thermal insulators, nuclear waste disposal, petroleum and polymer technology, heat exchangers and others, Moreau [1]. Use of an external magnetic field is of considerable importance in many industrial applications, particularly as a control mechanism in material manufacturing. Homogeneity and quality of single crystals grown from doped semiconductor melts is of interest to manufacturers of electronic chips. One of the main purposes of electromagnetic control is to stabilize the flow and suppress oscillatory instabilities, which degrades the resulting crystal. The magnetic field strength is one of the most important factors for crystal formation. The scientific treatment of the problems of irrigation, soil erosion and tile drainage are the present focus of the development of porous channel flow. The magnetohydrodynamic channel flow with heat transfer has attracted the attention of many researchers due to its numerous engineering and industrial applications. Such flows finds applications in thermofluid transport modeling in magnetic geosystems, meteorology, turbo machinery, solidification process in metallurgy and in some astrophysical problems. The rotating flows of electrically conducting fluid in the presence of a magnetic field is encountered in cosmical and geophysical fluid dynamics.

The rapid depletion of energy resources worldwide has prompted almost every country in the world to focus attention on energy conservation and improving existing energy systems to minimize the energy waste. The scientific community has responded to the challenge by developing new techniques of analysis and design so that the available work destruction is either eliminated or minimized. Unlike the old approach which relied solely on the first of thermodynamics, this approach uses a combination of the first and second laws of thermodynamics. This effort has given birth to the new discipline of entropy generation minimization. Calculations of the efficiency of thermal systems are now routinely performed using the concept of second law efficiency, defined as the ratio of actual thermal efficiency to reversible thermal efficiency under the same conditions. The popular approach is to quantify the effect of the irreversible processes occurring in the systems in terms of the entropy generation rate. Entropy generation analysis has also proved an effective tool in enhancing the second law based performance of existing systems. The tool relies on determining the distribution of the entropy generation within the flow field due to momentum and energy interactions. The study of entropy generation in thermal systems, such as heat exchangers, nuclear reactors, energy storage devices, and electronic cooling, has grown exponentially during the last three decades, largely as a result of the pioneering contributions of Bejan [2, 3]. Salas et al. [4] analytically showed a way of applying entropy generation analysis for modelling and optimization of magnetohydrodynamic induction devices. They restricted their analysis to only Hartmann model flow in a channel. Thermodynamics analysis of mixed convection in a channel with transverse hydromagnetic effect has been investigated by Mahmud et al. [5].

The entropy generation during fluid flow between two parallel plates with moving bottom plate has been analyzed by Latife et al. [6]. Ibanez et al. [7] have examined the minimization of entropy generation by asymmetric convective cooling. The entropy generation inside a porous channel with viscous dissipation have been investigated by Mahmud [8]. The heat transfer and entropy generation during compressible fluid flow in a channel partially filled with porous medium have been analyzed by Chauhan and Kumar [9]. Tasnim et al. [10] have studied the entropy generation in a porous channel with hydromagnetic effects. Eegunjobi and Makinde [11] have studied the combined effect of buoyancy force and Navier slip on entropy generation in a vertical porous channel. The second law analysis of laminar flow in a channel filled with saturated porous media has been studied by Makinde and Osalusi [12]. Makinde and Maserumule [13] has presented the thermal criticality and entropy analysis for Couette flow with variable viscosity . Makinde and Osalusi [14] have investigated the entropy generation in a liquid film falling along an incline porous heated plate. Second law analysis for a variable viscosity plane Poiseuille flow with asymmetric convective cooling has been presented by Makinde and Aziz [15]. Cimpean and Pop [16, 17] have presented the parametric analysis of entropy generation in a channel. The effect of an external oriented magnetic field on entropy generation in natural convection has been investigated by Jery et al. [18]. Dwivedi et al. [19] have made an analysis on the incompressible viscous laminar flow through a channel filled with porous media. Numerical study of unsteady hydromagnetic Generalized Couette flow of a reactive third grade fluid with asymmetric convective cooling has been carried out by Makindea and Chinyoka [20]. Analysis of entropy generation rate in an unsteady porous channel flow with Navier slip and convective cooling has been presented by Chinyoka and Makinde [21]. Chinyoka and Makinde [22] have presented the transient Generalized Couette flow of a reactive variable viscosity third-grade liquid with asymmetric convective cooling.

The objective of the present study is to investigate the entropy generation in an unsteady MHD Couette flow of a viscous incompressible electrically conducting fluid in the presence of a transverse magnetic field in a rotating system. The upper plate is moving with a constant velocity while the lower plate is kept stationary. The flow is subjected to an external uniform magnetic field perpendicular to the plates. The induced magnetic field is neglected by assuming a very small magnetic Reynolds number [23]. The two plates are cooling asymmetrically. The Joule and viscous dissipations are taken into consideration in the energy equation. The governing momentum and energy equations are solved numerically using MATLAB PDE solver. A parametric study is carried out to see how the pertinent parameters of the problem affect the flow field, temperature field and the entropy generation. This model is an acceptable representation for some practical engineering problems such as those involving flows through pipes, nuclear reactors, pumps and heat exchangers.

2 Mathematical formulation and its solution

Consider the unsteady flow of a viscous incompressible electrically conducting fluid between two infinite parallel plates when the fluid and the plates rotate in unison with uniform angular velocity Ω about an axis normal to the plates. Let d be the distance between the two plates, where d is small in comparison with the characteristic length of the plates. The upper plate moves with a uniform velocity U in its own plane in the x-direction, where the x-axis is taken along the lower stationary plate. The z-axis is taken normal to the x-axis and the y-axis is taken normal to the zx-plane, lying in the plane of the lower plate, and it is also assumed that the flow is fully developed. The top and bottom plates are cooled by convection. The coolant temperature T_a is the same for both plates but the convection heat transfer coefficients h_0 and h_1 are different, thus providing an asymmetric cooling effect. A uniform transverse magnetic field B_0 is applied perpendicular to the channel plates. Since the magnetic Reynolds number which is the ratio of the fluid flux to the magnetic diffusivity and is one of the more important parameters in MHD, is very small for most fluid used in industrial applications, we assume that the induced magnetic field is negligible. Further, there is no applied pressure gradient as the flow is due to the motion of the upper plate. Since the plates are infinitely long along the x- and y-directions, all physical quantities will be functions of z and t only.

Denoting the velocity components u and v along the x- and y-directions respectively, the Navier-Stokes equations of motion in a rotating frame of reference are

$$\frac{\partial u}{\partial t} - 2\Omega\, v \quad = \quad \nu \frac{\partial^2 u}{\partial z^2} - \frac{\sigma\, B_0^2}{\rho}\, u, \tag{1}$$

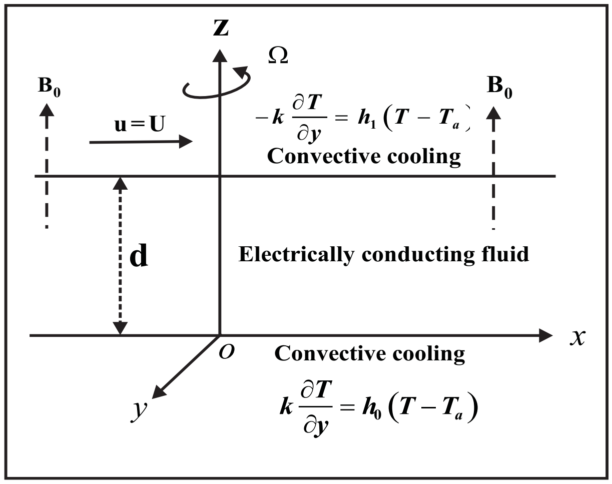

Figure 1: Geometry of the problem

$$\frac{\partial v}{\partial t} + 2\Omega\, u \quad = \quad \nu\frac{\partial^2 v}{\partial z^2} - \frac{\sigma\, B_0^2}{\rho}\, v, \tag{2}$$

where ρ is the fluid density, ν the kinematic viscosity and σ the electrical conductivity of the fluid.

The energy equation is

$$\rho\, c_p\frac{\partial T}{\partial t} \quad = \quad k\frac{\partial^2 T}{\partial z^2} + \mu\left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right] + \sigma\, B_0^2(u^2+v^2), \tag{3}$$

where T is the fluid temperature, μ the coefficient of viscosity, k the thermal conductivity of the fluid, c_p the specific heat at constant pressure. The second and third terms on the right-hand side represent the Joule and viscous dissipations respectively.

The initial and boundary conditions for velocity and temperature distributions are

$$\begin{aligned} &u = 0,\;\; v = 0,\;\; T = T_0 \;\text{ for }\; 0 \le y \le d \;\text{ and }\; t \le 0,\\ &u = 0,\;\; v = 0,\;\; k\frac{\partial T}{\partial y} = h_0(T - T_a) \;\text{ at }\; y = 0 \;\text{ for }\; t > 0,\\ &u = 0,\;\; v = U,\;\; -k\frac{\partial T}{\partial y} = h_1(T - T_a) \;\text{ at }\; y = d \;\text{ for }\; t > 0. \end{aligned} \tag{4}$$

where T_0 is the initial fluid temperature, T_a the ambient temperature, h_0 the heat transfer coefficient at the lower plate and h_1 the heat transfer coefficient at the upper plate.

Introducing the non-dimensional variables

$$\eta = \frac{y}{h}, \quad \tau = \frac{\nu t}{d^2}, \quad u_1 = \frac{u}{U}, \quad v_1 = \frac{v}{U}, \quad \theta = \frac{T - T_0}{T_a - T_0}, \tag{5}$$

equations (1) and (2) become

$$\begin{aligned} \frac{\partial u_1}{\partial \tau} &= \frac{\partial^2 u_1}{\partial \eta^2} + 2K^2 v_1 - M^2 u_1, & (6)\\ \frac{\partial v_1}{\partial \tau} &= \frac{\partial^2 v_1}{\partial \eta^2} - 2K^2 u_1 - M^2 v_1, & (7)\\ \Pr \frac{\partial \theta}{\partial \tau} &= \frac{\partial^2 \theta}{\partial \eta^2} + \mathrm{Br}\left[\left(\frac{\partial u_1}{\partial \eta}\right)^2 + \left(\frac{\partial v_1}{\partial \eta}\right)^2 + M^2(u_1^2 + v_1^2)\right], & (8) \end{aligned}$$

where $M^2 = \frac{\sigma B^2 d^2}{\rho \nu}$ is the magnetic parameter, $K^2 = \frac{\Omega d^2}{\nu}$ the rotation parameter, $\mathrm{Br} = \frac{\mu U^2}{k(T_a - T_0)}$ the Brinkman number and $\Pr = \frac{\rho \nu c_p}{k}$ the Prandlt number.

The initial and boundary conditions for velocity and temperature distributions are

$$\begin{aligned} & u_1 = 0, \quad v_1 = 0, \quad \theta = 0 \ \text{ for } \ 0 \le \eta \le 1 \ \text{ and } \ \tau \le 0,\\ & u_1 = 0, \quad v_1 = 0, \quad \frac{\partial \theta}{\partial \eta} = \mathrm{Bi}_0(\theta - 1) \ \text{ at } \ \eta = 0 \ \text{ for } \ \tau > 0, \qquad (9)\\ & u_1 = 0, \quad v_1 = 1, \quad \frac{\partial \theta}{\partial \eta} = \mathrm{Bi}_1(1 - \theta) \ \text{ at } \ \eta = 1 \ \text{ for } \ \tau > 0. \end{aligned}$$

where $\mathrm{Bi}_0 = \frac{h_0 d}{k}$ and $\mathrm{Bi}_1 = \frac{h_1 d}{k}$ are the Biot numbers at both the lower and the upper plates, respectively.

3 Results and discussion

The numerical computations are done by a written program which used a symbolic and computational computer language MATLAB. The entire procedure is implemented on MATLAB. To study the effects of magnetic field and Coriolis force on the velocity field we have presented the non-dimensional velocity u_1 against η in Figures 2-5 for several values of magnetic parameter M^2, rotation parameter K^2 and time τ. It is seen from Figure 2 that both the primary velocity u_1 and secondary velocity v_1. An

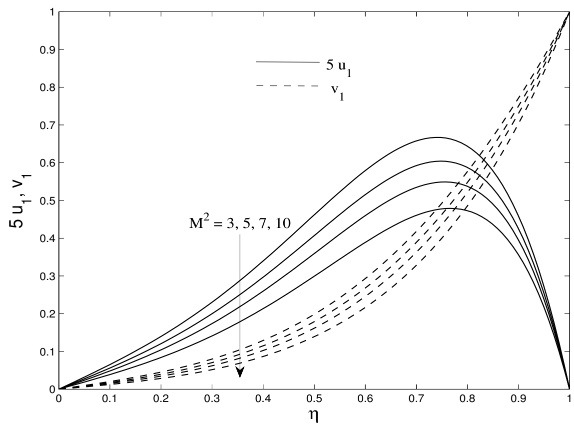

Figure 2: Primary and secondary velocities for different M^2 when $\tau = 0.2$ and $K^2 = 4$.

increase in the magnetic parameter M^2 leads to corresponding increases in damping magnetic properties of the fluid. These forces result in increased resistance to flow and thus explain the reduction in fluid velocities with an increase in magnetic parameter M^2. Consequently, the electrically conducting fluid receives a push from the magnetic force-the mechanism by which the magnetic field has the potential to manipulate an electrically conducting fluid in the micro scale system. Figure 3 shows that the primary velocity u_1 increases whereas the secondary velocity v_1 decreases with an increase in rotation parameter K^2. The effect of the Coriolis force is found to be significant as compared to the inertial and viscous forces in the equation of motion. The Coriolis and the electromagnetic forces are of comparable magnitude. The Coriolis force exerts a strong influence on the hydromagnetic flow in the earth's liquid core which plays an important role in the mean geomagnetic field. The primary velocity u_1 as well as secondary velocity v_1 increase when time progresses shown in Figure 4.

We have plotted the temperature distribution θ against η in Figures 5-11 for several values of magnetic parameter M^2, rotation parameter K^2, Prandtl number Pr, Brinkman number Br, Biot numbers Bi_1, Bi_2 and time τ. It is seen from Figure 5 that the fluid temperature θ increases with an increase in magnetic parameter M^2. This is due to the fact that the applied transverse magnetic field produces a body force such as Lorentz force, which opposes the motion. Hence, the resistance offered by this body force to the flow is the cause of enhancing the fluid temperature. At the moving upper plate ($\eta = 1$), the temperature boundary condition is of the Dirichlet

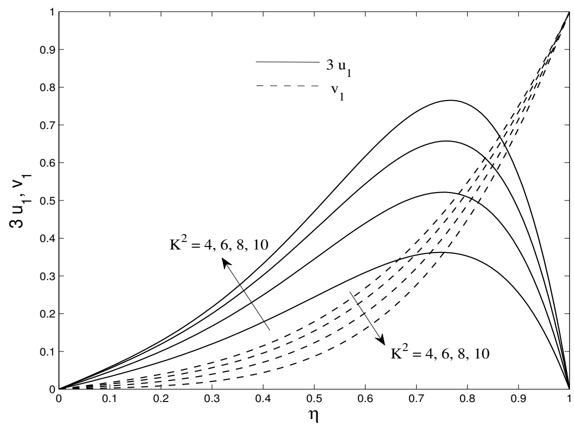

Figure 3: Primary and secondary velocities for different K^2 when $M^2 = 5$ and $\tau = 0.2$.

type, the gradient is the steepest and the heat transfer is maximum as the physics dictates. Fig.6 illustrates that the fluid temperature θ increases for increasing values of K^2. It is observed that just like in case of magnetic field, the highest temperature gradient is recorded in regions very close to the upper plate ($\eta = 1$). Larger values of the Prandtl number Pr correspondingly decrease the strength of the source terms in the temperature equation and hence in turn reduce the overall fluid temperature θ as clearly illustrated in Figure 7. Figure 8 shows how the Brinkman number Br influences the velocity distribution in a symmetrically cooled channel. As the Brinkman number increases, it implies higher viscous dissipation in the flow, which in turn implies steeper temperature gradient and consequently the fluid temperature rises.

The effect of asymmetrical cooling of the channel is depicted in Figures 9-10. The fluid temperature θ increases with an increase in Biot numbers Bi_0 and Bi_1. Biot number is the ratio of the hot fluid side convection resistance to the cold fluid side convection resistance on a surface. As the convection at the moving plate ($\eta = 1$) becomes dominant, the temperature gradient is high at the upper plate and hence a proportionately larger amount of heat is removed through the upper plate. The temperature profiles maintain their asymmetry about the centreline of the channel in view of the asymmetrical cooling conditions imposed at the plates. Figure 11 reveals that the fluid temperature θ increases when time progresses. It is noted that the heat transfer at the upper plate is greatly affected by variation in pertinent parameters while that at the lower plate is marginal.

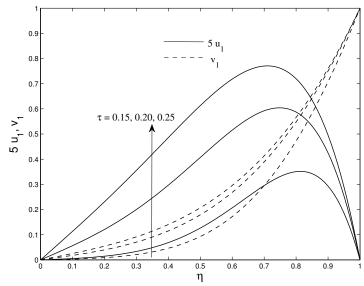

Figure 4: Primary and secondary velocities for different time τ when $M^2 = 5$ and $K^2 = 4$.

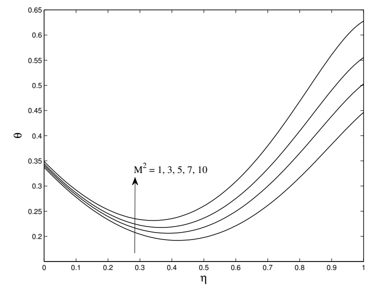

Figure 5: Temperature profiles for different M^2 when $\tau = 0.2$, $K^2 = 4$, $\mathrm{Pr} = 0.72$, $\mathrm{Bi}_0 = 0.1$, $\mathrm{Bi}_1 = 0.1$ and $\mathrm{Br} = 1$.

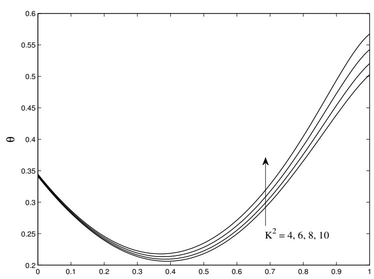

Figure 6: Temperature profiles for different K^2 when $M^2 = 5$, $\tau = 0.2$, $\mathrm{Pr} = 0.72$, $\mathrm{Bi}_0 = 0.1$, $\mathrm{Bi}_1 = 0.1$ and $\mathrm{Br} = 1$.

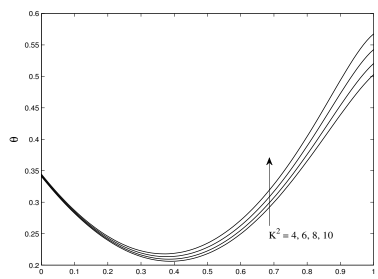

Figure 7: Temperature profiles for different Pr when $M^2 = 5$, $K^2 = 4$, $\mathrm{Br} = 1$, $\mathrm{Bi}_0 = 0.1$, $\mathrm{Bi}_1 = 0.1$ and $\tau = 0.2$.

4 Entropy generation

In many engineering and industrial processes, entropy production destroys the available energy in the system. It is therefore imperative to determine the rate of entropy generation in a system, in order to optimize energy in the system for efficient operation in the system. The convection process in a channel is inherently irreversible and this causes continuous entropy generation. Woods [24] gave the local volumetric rate of entropy generation for a viscous incompressible conducting fluid in the presence of magnetic field as follows:

$$E_G = \frac{k}{T_0^2}\left(\frac{\partial T}{\partial z}\right)^2 + \frac{\mu}{T_0}\left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right] + \frac{\sigma B_0^2}{T_0}(u^2 + v^2). \quad (10)$$

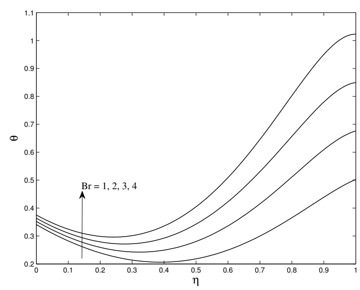

Figure 8: Temperature profiles for different Br when $M^2 = 5$, $K^2 = 4$, $\mathrm{Pr} = 0.72$, $\mathrm{Bi}_0 = 0.1$, $\mathrm{Bi}_1 = 0.1$ and $\tau = 0.2$.

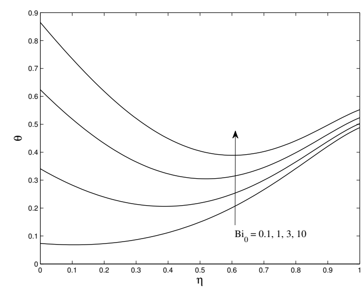

Figure 9: Temperature profiles for different Bi_0 when $M^2 = 5$, $K^2 = 4$, $\mathrm{Pr} = 0.72$, $\tau = 0.2$, $\mathrm{Bi}_1 = 0.1$ and $\mathrm{Br} = 1$.

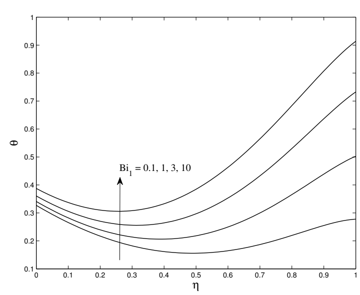

Figure 10: Temperature profiles for different Bi_1 when $M^2 = 5$, $K^2 = 4$, $\mathrm{Pr} = 0.72$, $\mathrm{Bi}_0 = 0.1$, $\tau = 0.2$ and $\mathrm{Br} = 1$.

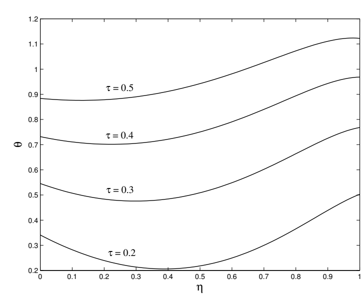

Figure 11: Temperature profiles for different τ when $M^2 = 5$, $K^2 = 4$, $\mathrm{Pr} = 0.72$, $\mathrm{Bi}_0 = 0.1$, $\mathrm{Bi}_1 = 0.1$ and $\mathrm{Br} = 1$.

The entropy generation equation (10) consists of three terms, the first term is the irreversibility due to the heat transfer, the second term is entropy generation due to viscous dissipation, while the third term is local entropy generation due to the effect of magnetic field (Joule heating or Ohmic heating).

The dimensionless entropy generation number may be defined by the following relationship:

$$\mathrm{N_S} = \frac{T_0^2 d^2 E_G}{k(T_a - T_0)^2}. \tag{11}$$

In terms of the dimensionless velocity and temperature, the entropy generation number becomes

$$\mathrm{N_S} = \left(\frac{\partial\theta}{\partial\eta}\right)^2 + \frac{\mathrm{Br}}{\Omega}\left[\left\{\left(\frac{\partial u_1}{\partial\eta}\right)^2 + \left(\frac{\partial v_1}{\partial\eta}\right)^2\right\} + M^2(u_1^2 + v_1^2)\right], \quad (12)$$

where $\Omega = \frac{T_a - T_0}{T_0}$ is the non-dimensional temperature difference.

The entropy generation number $\mathrm{N_S}$ can be written as a summation of the entropy generation due to heat transfer denoted by N_1 and the entropy generation due to fluid friction with magnetic field denoted by N_2 given as

$$\begin{aligned} N_1 &= \left(\frac{\partial\theta}{\partial\eta}\right)^2, \\ N_2 &= \frac{\mathrm{Br}}{\Omega}\left[\left\{\left(\frac{\partial u_1}{\partial\eta}\right)^2 + \left(\frac{\partial v_1}{\partial\eta}\right)^2\right\} + M^2(u_1^2 + v_1^2)\right]. \end{aligned} \quad (13)$$

An alternative irreversibility distribution parameter is the Bejan number, which gives an idea whether the fluid friction irreversibility dominates over heat transfer irreversibility or the heat transfer irreversibility dominates over fluid friction irreversibility ([25]). It is simply the ratio of entropy generation due to heat transfer to the total entropy generation:

$$\mathrm{Be} = \frac{\text{Entropy generation due to heat transfer}}{\text{Total entropy generation}} = \frac{N_1}{\mathrm{N_S}} = \frac{1}{1+\Phi}, \quad (14)$$

where Φ is the irreversibility distribution ratio which is given by:

$$\begin{aligned} \Phi &= \frac{\text{Fluid friction irreversibility + Magnetic field irreversibility}}{\text{Heat transfer irreversibility}} \\ &= \frac{N_2}{N_1} \end{aligned} \quad (15)$$

As the Bejan number ranges from 0 to 1, it approaches zero when the entropy generation due to the combined effects of fluid friction and magnetic field is dominant ([26]). Similarly, $\mathrm{Be} > 0.5$ indicates that the irreversibility due to heat transfer dominates, with $\mathrm{Be} = 1$ as the limit at which the irreversibility is solely due to heat transfer. Consequently, $0 \leq \Phi < 1$ indicates that the irreversibility is primarily due to the heat transfer irreversibility, whereas for $\Phi > 1$ it is due to the sum of the fluid friction and magnetic field irreversibility.

The influences of the different governing parameters on entropy generation within the channel are presented in Figures 12-23. The effect of magnetic parameter M^2 on the entropy generation number is displayed in Figure 12. This figure shows that the entropy generation increases with an increase in magnetic parameter M^2. It is indicated that an increase in magnetic parameter M^2 tends to increase the entropy generation number since the magnetic parameter has an increasing effect on all friction, heat transfer and magnetic irreversibilities. Figure 13 show that the entropy generation number N_S increases with an increase in rotation parameter K^2. Figure 14 shows that the entropy generation number N_S decreases with an increase in group parameter $\mathrm{Br}\Omega^{-1}$. This is attributed to decrease in fluid friction irreversibility (N_2) with an increase in $\mathrm{Br}\Omega^{-1}$. The influence of the Biot numbers on the entropy generation number N_S is shown in Figures 15-16. As the Biot numbers Bi_0 and Bi_1, the entropy generation number increases. Figure 17 reveals that the entropy generation number N_S decreases as time τ increase. In Figures 12-17, the entropy generation is expectedly maximum near the moving plate $(\eta = 1)$ where fluid velocity and temperature gradients are highest.

It is seen from Figure 18-19 that the Bejan number Be increases with an increase in either magnetic parameter M^2 or rotation parameter K^2. It is observed that the Bejan number decreases with an increase in magnetic parameter, due to its significant effect on friction irreversibilities. Figure 20 reveals that the Bejan number Be increases with an increase in group parameter $\mathrm{Br}\Omega^{-1}$. An increase in the values of the group parameter $\mathrm{Br}\Omega^{-1}$ due to the combined effects of viscous heating and temperature difference yields a higher entropy generation number. The group parameter is an important dimensionless number for irreversibility analysis. It determines the relative importance of viscous effects to that of temperature gradient entropy generation. Generally, it is observed that an increase in group parameter strengthens the effect of fluid friction irreversibility, but heat transfer irreversibility dominates over fluid friction irreversibility. Because of asymmetric cooling, the entropy generation profiles are asymmetrical about the centreline of the channel $(\eta = 0.5)$ shown in Figures 12-20. Figures 21-22 show that the Bejan number Be increases with an increase in Biot numbers Bi_0 and Bi_1. An increase in the values of the Biot number results in an increase in the dominant effect of heat transfer irreversibility at the plate surface. This means that the plate surface acts as a strong source of irreversibility. The Bejan number is asymmetric about the centerline of the channel due to the asymmetric temperature distribution. From Figure 23, it is noted that the Bejan number increases when time increases.

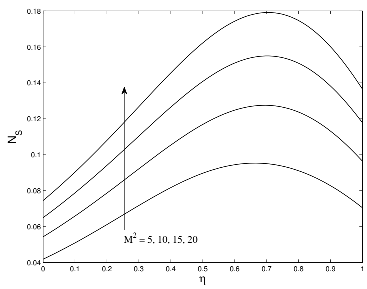

Figure 12: $\mathrm{N_S}$ for different M^2 when $\tau = 0.2$, $K^2 = 4$, $\mathrm{Pr} = 0.71$, $\mathrm{Bi}_0 = 1$, $\mathrm{Bi}_1 = 1$ and $\mathrm{Br}\Omega^{-1} = 1$.

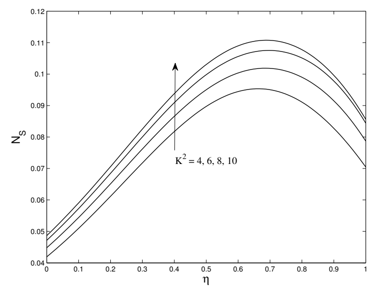

Figure 13: $\mathrm{N_S}$ for different K^2 when $M^2 = 5$, $\tau = 0.2$, $\mathrm{Pr} = 0.71$, $\mathrm{Bi}_0 = 1$, $\mathrm{Bi}_1 = 1$ and $\mathrm{Br}\Omega^{-1} = 1$.

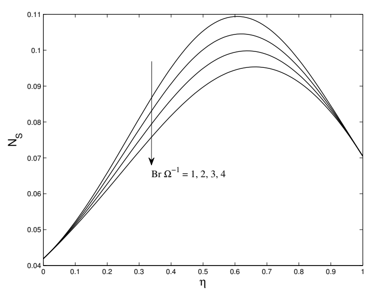

Figure 14: $\mathrm{N_S}$ for different $\mathrm{Br}\Omega^{-1}$ when $M^2 = 5$, $K^2 = 4$, $\mathrm{Pr} = 0.71$, $\mathrm{Bi}_0 = 1$, $\mathrm{Bi}_1 = 1$ and $\tau = 0.2$.

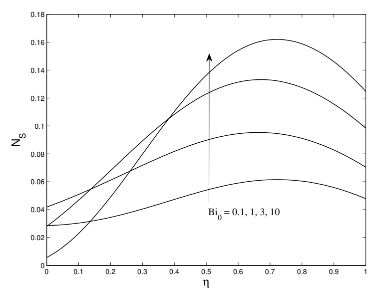

Figure 15: $\mathrm{N_S}$ for different Bi_0 when $M^2 = 5$, $K^2 = 4$, $\mathrm{Pr} = 0.71$, $\tau = 0.2$, $\mathrm{Bi}_1 = 1$ and $\mathrm{Br}\Omega^{-1} = 1$.

5 Conclusion

A numerical study of unsteady hydromagnetic Couette flow and heat transfer of of viscous incompressible electrically conducting fluid between parallel insulated plates with asymmetric convective cooling in the presence of a transverse applied magnetic field has been performed. The model also accommodates the presence of viscous and Joule dissipations. The velocity and temperature profiles are used to evaluate the entropy generation profiles in the flow field. The study leads to the following conclusions.

- The velocities decrease with the increased magnetic field whereas the temperature is noticed to increase under these conditions.
- The fluid temperature decreases for increasing values of Prandtl number.
- The fluid temperature increases for increasing values of Biot numbers.

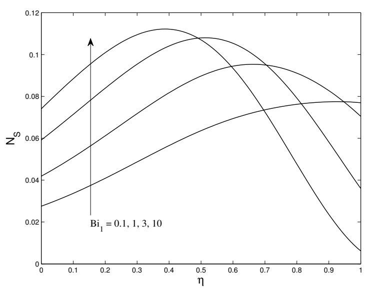

Figure 16: $\mathrm{N_S}$ for different $\mathrm{Bi_1}$ when $M^2 = 5$, $K^2 = 4$, $\mathrm{Pr} = 0.71$, $\mathrm{Bi_0} = 1$, $\tau = 0.2$ and $\mathrm{Br}\Omega^{-1} = 1$.

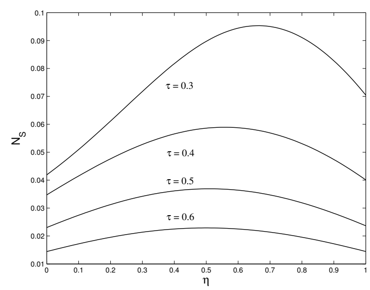

Figure 17: $\mathrm{N_S}$ for different τ when $M^2 = 5$, $K^2 = 4$, $\mathrm{Pr} = 0.71$, $\mathrm{Bi_0} = 1$, $\mathrm{Bi_1} = 1$ and $\mathrm{Br}\Omega^{-1} = 1$.

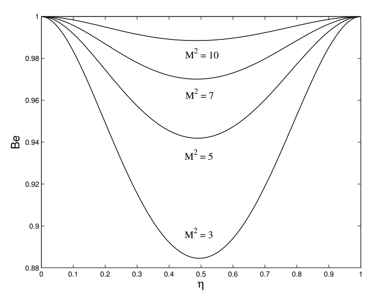

Figure 18: Bejan number for different M^2 when $\tau = 0.2$, $K^2 = 4$, $\mathrm{Pr} = 0.71$, $\mathrm{Bi_0} = 1$, $\mathrm{Bi_1} = 1$ and $\mathrm{Br}\Omega^{-1} = 1$.

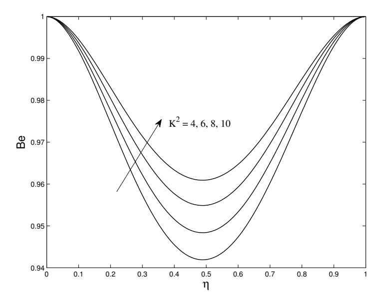

Figure 19: Bejan number for different K^2 when $M^2 = 5$, $\tau = 0.2$, $\mathrm{Pr} = 0.71$, $\mathrm{Bi_0} = 1$, $\mathrm{Bi_1} = 1$ and $\mathrm{Br}\Omega^{-1} = 1$.

- It is noted that the entropy generation number decreases for increasing values of magnetic parameter.
- Slip parameters controls the entropy generation.
- With the use of asymmetric cooling of the plates, it is possible to operate the system with reduced entropy generation rates.
- The plate surfaces act as strong source of entropy and heat transfer irreversibility.
- The results show that heat transfer irreversibility dominates over fluid friction irreversibility and viscous dissipation has no effect on the entropy generation rate at the centerline of the channel.
- The results of the study provide valuable fundamental information on the physics of the simultaneously developing transient laminar convection in a parallel plate channel with moving bottom plate to

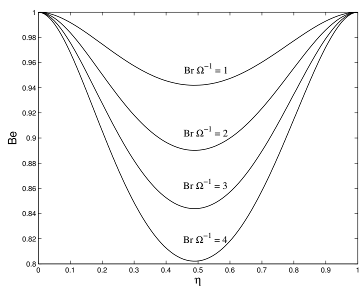

Figure 20: Bejan number for different $\mathrm{Br}\Omega^{-1}$ when $M^2 = 5$, $K^2 = 4$, $\mathrm{Pr} = 0.71$, $\mathrm{Bi}_0 = 1$, $\mathrm{Bi}_1 = 1$ and $\tau = 0.2$.

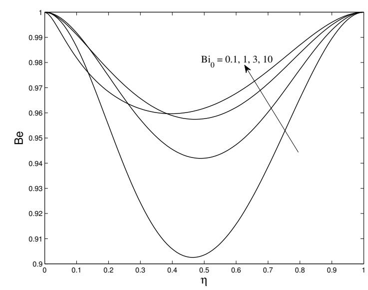

Figure 21: Bejan number for different Bi_0 when $M^2 = 5$, $K^2 = 4$, $\mathrm{Pr} = 0.71$, $\mathrm{Bi}_0 = 1$, $\tau = 0.2$ and $\mathrm{Br}\Omega^{-1} = 1$.

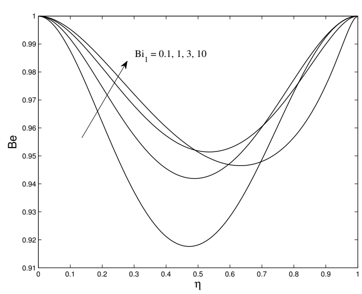

Figure 22: Bejan number for different Bi_1 when $M^2 = 5$, $K^2 = 4$, $\mathrm{Pr} = 0.71$, $\mathrm{Bi}_0 = 1$, $\tau = 0.2$ and $\mathrm{Br}\Omega^{-1} = 1$.

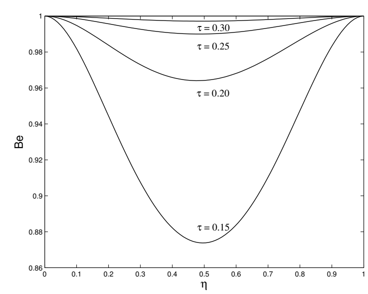

Figure 23: Bejan number for different τ when $M^2 = 5$, $K^2 = 4$,$\mathrm{Pr} = 0.71$, $\mathrm{Bi}_0 = 1$, $\mathrm{Bi}_1 = 1$ and $\mathrm{Br}\Omega^{-1} = 1$.

improve the corresponding engineering applications. The designers under the responsibilities for the design and optimization of corresponding thermal systems can employ the results given about the entropy generation to reduce the loss of available work.

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