

On intuitionistic L-fuzzy PMS-ideals in PMS-algebras

Beza Lamesgin Derseh*

*Department of Mathematics, College of Natural and Computational Science, Debre Markos
University, Debre Markos, Ethiopia
Email: beza_lamesgin@dmu.edu.et*

Abstract. In this paper, we apply the concept of an intuitionistic L-fuzzy set to PMS-ideals of PMS-algebras. The notion of the intuitionistic L-fuzzy PMS-ideal of a PMS-algebra is introduced and several associated properties are investigated. A condition for an intuitionistic L-fuzzy set in a PMS-algebra to be an intuitionistic L-fuzzy PMS-ideal is established. Characterizations of intuitionistic L-fuzzy PMS-ideals of a PMS-algebra in terms of their level subsets are given. The algebraic nature of intuitionistic L-fuzzy PMS-ideals of a PMS-algebra under homomorphism is discussed. Moreover, the Cartesian product of the intuitionistic L-fuzzy PMS-ideals of a PMS-algebra is also studied and some interesting results are obtained. Finally, the relationship between the strongest intuitionistic L-fuzzy relations in a PMS-algebra and intuitionistic L-fuzzy PMS-ideals of a PMS-algebra are explored.

Keywords: PMS-algebra, PMS-ideal, Intuitionistic L-fuzzy PMS-ideal, Homomorphism and Cartesian product.

AMS Subject Classification 2010: 06F35, 03G25, 08A72, 03B52.

1 Introduction

In 1965, Zadeh [27] introduced the concept of fuzzy sets as a generalization of classical sets to deal with vagueness and imprecision in real-life situations. Since then this concept has been applied to various algebraic structures. Rosenfield [22] applied the notion of fuzzy sets in group theory to develop fuzzy groups and studied them under homomorphism. Choudhury [7] also introduced the notion of a fuzzy homomorphism and studied its effect on fuzzy subgroups. Priya and Ramachandran [21] discussed fuzzy PS-ideals and fuzzy PS-subalgebras of PS-algebra under homomorphism and Cartesian products as well as the strongest fuzzy relations on PS-algebra.

After introducing the concept of fuzzy sets by Zadeh, many generalizations of this fundamental concept have been given by several researchers. Atanassov [3–5] extended the concept

*Corresponding author

Received: 20 August 2024/ Revised: 14 May 2025/ Accepted: 12 May 2026

DOI: [10.22124/JART.2026.28223.1701](https://doi.org/10.22124/JART.2026.28223.1701)

of a fuzzy set to an intuitionistic fuzzy set to better deal with uncertainties and imprecision by introducing an additional function, which he called a nonmembership function with some natural conditions on membership and nonmembership functions to each element of a set. Zarandi and Saeid [28] studied the intuitionistic fuzzy BG-ideals of the BG-algebra. Panigrahi and Nanda [20] studied the idea of intuitionistic fuzzy relations over intuitionistic fuzzy subsets and found several interesting properties of intuitionistic fuzzy relations in intuitionistic fuzzy subsets. Anitha and Arjunan [2] studied the strongest intuitionistic fuzzy relations on intuitionistic fuzzy ideals of Hemirings and obtained some interesting results

In 1967, Goguen [13] introduced the notion of L -fuzzy sets as another extension of zadeh's fuzzy set where L is a complete lattice with least element 0 and greatest element 1. Jefferson et al. [8,9] introduced the notion of L -Fuzzy BP-Algebras and explored some properties of L -Fuzzy BP-subalgebras and L -Fuzzy BP ideals. In 1984, Atanassov and Stoeva [5] further generalized the notion of L -fuzzy subset and intuitionistic fuzzy subset to L -intuitionistic fuzzy subset. Atanassov and Stoeva have defined both membership and nonmembership functions from the nonempty set X to L , where L is any complete lattice with a complete order reversing involution N . Basing with this motivation, many mathematicians started to review various concepts in intuitionistic L -fuzzy structures. Naganathan et al. [18,19] introduced the intuitionistic L -fuzzy subgroups and Level subsets of intuitionistic L -fuzzy subgroups of a group and discussed their properties in detail. Muralikrishna and Chandramouleeswaran [16,17] discussed Intuitionistic L -fuzzy BF-ideals and the Cartesian product on Intuitionistic L -fuzzy ideals of BF-algebras. Chandramouleeswaran and Muralikrishna [6] also discussed the notions of Intuitionistic L -fuzzy subalgebras of a BG-algebra and some of their basic properties. Sowmiya and Jeyalakshmi [25] introduced the notions of intuitionistic L -fuzzy Z -ideals in Z -algebras and discussed some of their properties. Moreover, they explored the Cartesian product of intuitionistic L -fuzzy Z -ideals in Z -algebras.

The notion of BCK-algebras was introduced by Iseki and Tanaka [14] and Iseki [15] also introduced the notion of a BCI-algebra as a generalization of a BCK-algebra. In 2016, Sithar Selvam and Nagalakshmi [24] introduced a new class of algebra called PMS-algebra as a generalization of several algebras such as BCK, BCI, TM and PS-algebras. In our earlier works, we studied intuitionistic fuzzy PMS-subalgebras and intuitionistic fuzzy PMS-ideals [11,26], and we looked at a number of findings. Moreover, Alaba et al. [1,12] discussed the notion of intuitionistic fuzzy PMS-subalgebras and intuitionistic fuzzy PMS-ideals under homomorphism and Cartesian product in PMS-algebras, and addressed several results. Several researchers have applied the concept of intuitionistic L -fuzzy set to different algebraic structures. However, as far as we know there is no any study on intuitionistic L -fuzzy PMS-ideals of a PMS-algebra. With this motivation we develop the intuitionistic L -fuzzy PMS-ideals in a PMS-subalgebra.

In this article, we apply the concept of intuitionistic L -fuzzy set to PMS-ideals of PMS-algebras. We introduce the notion of the intuitionistic L -fuzzy PMS-ideal of a PMS-algebra and investigate several results. We establish a condition for an intuitionistic L -fuzzy set in a PMS-algebra to be an intuitionistic L -fuzzy PMS-ideal. We provide some characterizations of intuitionistic L -fuzzy PMS-ideals of a PMS-algebra in terms of their level subsets. We discuss the algebraic nature of intuitionistic L -fuzzy PMS-ideals of a PMS-algebra under homomorphism and obtain some important results. Moreover, we study the Cartesian product of the intuitionistic L -fuzzy PMS-ideals of a PMS-algebra and investigate some interesting results. Finally, we discuss

the relationship between the strongest intuitionistic L-fuzzy relations in a PMS-algebra and intuitionistic L-fuzzy PMS-ideals of a PMS-algebra.

2 Preliminaries

In this section, we recall some basic definitions that are required for the development of this work.

Definition 1 ([24]). *A nonempty set X with a constant element 0 and a binary operation $*$ is called a PMS-algebra if it satisfies the following axioms;*

- (i). $0 * x = x$,
- (ii). $(y * x) * (z * x) = z * y$, for all $x, y, z \in X$.

A binary relation " $\leq$ " on X can be defined as $x \leq y$ if and only if $x * y = 0$.

Definition 2 ([24]). *A nonempty subset S of a PMS-algebra X is said to be a PMS-subalgebra if $x * y \in S$, for all $x, y \in S$.*

Definition 3 ([24]). *A nonempty subset I of a PMS-algebra X is said to be a PMS-ideal of X if it satisfies the following conditions:*

- (i). $0 \in I$,
- (ii). $z * y, z * x \in I \Rightarrow y * x \in I$, for all $x, y, z \in X$.

Proposition 1 ([24]). *In any PMS-algebra X the following properties hold for all $x, y, z \in X$.*

- (i). $x * x = 0$,
- (ii). $(y * x) * x = y$,
- (iii). $x * (y * x) = y * 0$,
- (iv). $(y * x) * z = (z * x) * y$,
- (v). $(x * y) * 0 = y * x = (0 * y) * (0 * x)$.

Definition 4 ([24]). *Let X and Y be two PMS-algebras. A mapping $f : X \rightarrow Y$ is said to be a homomorphism of PMS-algebras if $f(x * y) = f(x) * f(y)$ for all $x, y \in X$. f is called an epimorphism of PMS-algebras if it is onto. If $f : X \rightarrow Y$ is a homomorphism, then $f(0) = 0$.*

Definition 5 ([13]). *Let $(L, \leq, \wedge, \vee)$ be a complete lattice with least element 0 and greatest element 1 . An L-fuzzy subset A of any non-empty set X is defined as a membership function $\zeta_A : X \rightarrow L$.*

Definition 6 ([5]). Let $(L, \leq, \wedge, \vee)$ be a complete lattice with least element 0 and greatest element 1 and an involutive order reversing operation $N: L \rightarrow L$. Then Intuitionistic L-Fuzzy Subset (ILFS) A of a nonempty set X is an object having the form $A = \{\langle x, \zeta_A(x), \xi_A(x) \rangle | x \in X\}$, where $\zeta_A: X \rightarrow L$ is the degree of membership and $\xi_A: X \rightarrow L$ is the degree of nonmembership of the element $x \in X$ satisfying $\zeta_A(x) \leq N(\xi_A(x))$. For simplicity we write $A = (\zeta_A, \xi_A)$ for $A = \{\langle x, \zeta_A(x), \xi_A(x) \rangle | x \in X\}$.

Lemma 1 ([10]). Let $A = (\zeta_A, \xi_A)$ be an intuitionistic L-fuzzy set in X . Then the following statements hold for any $x, y \in X$.

- (1). $1 - [\zeta_A(x) \vee \zeta_A(y)] = [1 - \zeta_A(x)] \wedge [1 - \zeta_A(y)]$.
- (2). $1 - [\zeta_A(x) \wedge \zeta_A(y)] = [1 - \zeta_A(x)] \vee [1 - \zeta_A(y)]$.
- (3). $1 - [\xi_A(x) \vee \xi_A(y)] = [1 - \xi_A(x)] \wedge [1 - \xi_A(y)]$.
- (4). $1 - [\xi_A(x) \wedge \xi_A(y)] = [1 - \xi_A(x)] \vee [1 - \xi_A(y)]$.

Definition 7 ([10]). An ILFS $A = (\zeta_A, \xi_A)$ of a PMS-algebra X is called an Intuitionistic L-fuzzy PMS-subalgebra (ILF PMS-SA) of X if it satisfies the following conditions;

- (i). $\zeta_A(x * y) \geq \zeta_A(x) \wedge \zeta_A(y)$,
- (ii). $\xi_A(x * y) \leq \xi_A(x) \vee \xi_A(y)$, for all $x, y \in X$.

Definition 8 ([5]). Let $A = (\zeta_A, \xi_A)$ and $B = (\zeta_B, \xi_B)$ be any two intuitionistic L-fuzzy subset of a nonempty set X . Then we have

- (1). $\bar{A} = \{\langle x, \xi_A(x), \zeta_A(x) \rangle | x \in X\}$,
- (2). $A \cap B = \{\langle x, \zeta_A(x) \wedge \zeta_B(x), \xi_A(x) \vee \xi_B(x) \rangle | x \in X\}$,
- (3). $A \cup B = \{\langle x, \zeta_A(x) \vee \zeta_B(x), \xi_A(x) \wedge \xi_B(x) \rangle | x \in X\}$,
- (4). $\Box A = \{\langle x, \zeta_A(x), 1 - \zeta_A(x) \rangle | x \in X\}$,
- (5). $\Diamond A = \{\langle x, 1 - \xi_A(x), \xi_A(x) \rangle | x \in X\}$,
- (6). $\cap A_i = \{\langle x, \wedge \zeta(A_i)(x), \vee \xi(A_i)(x) \rangle | x \in X\}$, where $\{A_i | i \in I\}$ is an arbitrary family of Intuitionistic L-fuzzy subsets of a nonempty set X .

Definition 9 ([2, 23]). Let $A = (\zeta_A, \xi_A)$ be ILFS of Y and $f: X \rightarrow Y$ be a mapping. Then the inverse image of A under f , denoted by $f^{-1}(A)(x) = \{\langle x, \zeta_{f^{-1}(B)}(x), \xi_{f^{-1}(A)}(x) \rangle | x \in X\}$, is an intuitionistic L-fuzzy set in Y defined by $\zeta_{f^{-1}(A)}(x) = \zeta_A(f(x))$ and $\xi_{f^{-1}(A)}(x) = \xi_A(f(x))$, for all $x \in X$.

Definition 10 ([5]). Let $A = (\zeta_A, \xi_A)$ and $B = (\zeta_B, \xi_B)$ be any two intuitionistic L-fuzzy subsets (ILFSs) of a set X . The Cartesian product $A \times B = (\zeta_{A \times B}, \xi_{A \times B})$ whose membership function $\zeta_{A \times B}: X \times X \rightarrow L$ and nonmembership function $\xi_{A \times B}: X \times X \rightarrow L$ are defined by $\zeta_{A \times B}(x, y) = \zeta_A(x) \wedge \zeta_B(y)$ and $\xi_{A \times B}(x, y) = \xi_A(x) \vee \xi_B(y)$, for all $x, y \in X$.

Definition 11 ([2, 20]). Let $B = (\zeta_B, \xi_B)$ be an intuitionistic L-fuzzy set on a set X and $A = (\zeta_A, \xi_A)$ is an intuitionistic L-fuzzy relation on a set X . Then the strongest intuitionistic L-fuzzy relation A_B on X , whose membership function $\zeta_{A_B} : X \times X \rightarrow L$ and whose nonmembership function $\xi_{A_B} : X \times X \rightarrow L$ are given by $\zeta_{A_B}(x, y) = \zeta_B(x) \wedge \zeta_B(y)$ and $\xi_{A_B}(x, y) = \xi_B(x) \vee \xi_B(y)$.

3 Intuitionistic L-fuzzy PMS-ideals in PMS-algebras

In this section, we introduce the notion of intuitionistic L-fuzzy PMS-ideal of a PMS-algebra and find some interesting results. Throughout this and the subsequent sections, X and Y denote the PMS-algebras.

Definition 12. An ILFS $A = (\zeta_A, \xi_A)$ of a PMS-algebra X is called an intuitionistic L-fuzzy PMS-ideal (ILF PMS-Id) of X if it satisfies the following conditions;

- (i). $\zeta_A(0) \geq \zeta_A(x)$ and $\xi_A(0) \leq \xi_A(x)$,
- (ii). $\zeta_A(y * x) \geq \zeta_A(z * y) \wedge \zeta_A(z * x)$,
- (iii). $\xi_A(y * x) \leq \xi_A(z * y) \vee \xi_A(z * x)$, for all $x, y, z \in X$.

Example 1. Let $L = [0, 1]$ be a complete lattice and $\mathbb{Z}$ be the ring of integers. For all $x, y \in \mathbb{Z}$, the binary operation $*$ is defined by $x * y = y - x$, where $' - '$ is the usual subtraction of integers gives that $(\mathbb{Z}, *, 0)$ is a PMS-algebra. Then one can easily verify that the ILFS $A = (\zeta_A, \xi_A)$ of $\mathbb{Z}$ defined by

$$\zeta_A(x) = \begin{cases} 0.6 & \text{if } x \in \langle 4 \rangle \\ 0.4 & \text{if } x \in \langle 2 \rangle - \langle 4 \rangle \\ 0 & \text{otherwise} \end{cases} \text{ and } \xi_A(x) = \begin{cases} 0.2 & \text{if } x \in \langle 4 \rangle \\ 0.5 & \text{if } x \in \langle 2 \rangle - \langle 4 \rangle \\ 1 & \text{otherwise} \end{cases} \text{ is an ILF PMS-ideal}$$

of $\mathbb{Z}$.

Theorem 1. Let $A = (\zeta_A, \xi_A)$ be an ILF PMS-Id of a PMS-algebra X with $x * y = 0$, for all $x, y \in X$. Then $\zeta_A(x) = \zeta_A(y)$ and $\xi_A(x) = \xi_A(y)$.

Proof. Let $x, y \in X$ such that $x * y = 0$. Since A is an ILF PMS-Id of X , by Proposition 1 (ii) and Definition 1(i), $\zeta_A(x) = \zeta_A((x * y) * y) = \zeta_A(0 * y) = \zeta_A(y)$ and $\xi_A(x) = \xi_A((x * y) * y) = \xi_A(0 * y) = \xi_A(y)$. Hence, $\zeta_A(x) = \zeta_A(y)$ and $\xi_A(x) = \xi_A(y)$, for all $x, y \in X$. $\square$

Theorem 2. Every ILF PMS-Id $A = (\zeta_A, \xi_A)$ of X is an ILF PMS-SA of X .

Theorem 3. Let $A = (\zeta_A, \xi_A)$ be an ILF PMS-Id of X such that $\zeta_A(x * y) \geq \zeta_A(0)$ and $\xi_A(x * y) \leq \xi_A(0)$. Then $\zeta_A(x) = \zeta_A(y)$ and $\xi_A(x) = \xi_A(y)$ for all $x, y \in X$.

Proof. Since $\zeta_A(0) \geq \zeta_A(x * y)$ and $\xi_A(0) \leq \xi_A(x * y)$ by Definition 12(i), we get $\zeta_A(0) = \zeta_A(x * y)$ and $\xi_A(0) = \xi_A(x * y)$. Again by Proposition 1(ii) and Theorem 2, we have

$$\zeta_A(x) = \zeta_A((x * y) * y) \geq \zeta_A(x * y) \wedge \zeta_A(y) = \zeta_A(0) \wedge \zeta_A(y) = \zeta_A(y),$$

and

$$\xi_A(x) = \xi_A((x * y) * y) \leq \xi_A(x * y) \wedge \xi_A(y) = \xi_A(0) \wedge \xi_A(y) = \xi_A(y).$$

Therefore, $\zeta_A(x) \geq \zeta_A(y)$ and $\xi_A(x) \leq \xi_A(y)$.

Conversely, $\zeta_A(y) = \zeta_A((y * x) * x) \geq \zeta_A(y * x) \wedge \zeta_A(x) = \zeta_A(0) \wedge \zeta_A(x) = \zeta_A(x)$ and $\xi_A(y) = \xi_A((y * x) * x) \leq \xi_A(y * x) \wedge \xi_A(x) = \xi_A(0) \wedge \xi_A(x) = \xi_A(x)$. Hence, $\zeta_A(y) \geq \zeta_A(x)$ and $\xi_A(y) \leq \xi_A(x)$. Thus, $\zeta_A(x) = \zeta_A(y)$ and $\xi_A(x) = \xi_A(y)$ for all $x, y \in X$. $\square$

Corollary 1. Let $A = (\zeta_A, \xi_A)$ be an ILF PMS-Id of X such that $\zeta_A(x * y) = 1$ and $\xi_A(x * y) = 0$, for all $x, y \in X$. Then $\zeta_A(y) = \zeta_A(x)$ and $\xi_A(x) = \xi_A(y)$.

Proof. By Proposition 1(ii, v) and Theorem 2, it follows that $\zeta_A(x) = \zeta_A(y)$ and $\xi_A(x) = \xi_A(y)$. $\square$

Theorem 4. If $A = (\zeta_A, \xi_A)$ and $B = (\zeta_B, \xi_B)$ are two ILF PMS-Ids of a PMS-algebra X , then $A \cap B$ is again an ILF PMS-Id of X .

Proof. Let A and B be ILF PMS-Ids of X . Then for all $x, y, z \in X$, $\zeta_{A \cap B}(0) = \zeta_A(0) \wedge \zeta_B(0) \geq \zeta_A(x) \wedge \zeta_B(x) = \zeta_{A \cap B}(x)$ and $\xi_{A \cap B}(0) = \xi_A(0) \vee \xi_B(0) \leq \xi_A(x) \vee \xi_B(x) = \xi_{A \cap B}(x)$. Therefore, $\zeta_{A \cap B}(0) \geq \zeta_{A \cap B}(x)$ and $\xi_{A \cap B}(0) \leq \xi_{A \cap B}(x)$.

$$\begin{aligned} \zeta_{A \cap B}(y * x) &= \zeta_A(y * x) \wedge \zeta_B(y * x) \\ &\geq [\zeta_A(z * y) \wedge \zeta_A(z * x)] \wedge [\zeta_B(z * y) \wedge \zeta_B(z * x)] \\ &= [\zeta_A(z * y) \wedge \zeta_B(z * y)] \wedge [\zeta_A(z * x) \wedge \zeta_B(z * x)] \\ &= \zeta_{A \cap B}(z * y) \wedge \zeta_{A \cap B}(z * x) \end{aligned}$$

and also,

$$\begin{aligned} \xi_{A \cap B}(y * x) &= \xi_A(y * x) \vee \xi_B(y * x) \\ &\leq [\xi_A(z * y) \vee \xi_A(z * x)] \vee [\xi_B(z * y) \vee \xi_B(z * x)] \\ &= [\xi_A(z * y) \vee \xi_B(z * y)] \vee [\xi_A(z * x) \vee \xi_B(z * x)] \\ &= \xi_{A \cap B}(z * y) \vee \xi_{A \cap B}(z * x) \end{aligned}$$

Therefore, $A \cap B$ is an ILF PMS-Id of X . $\square$

We can generalize the above theorem to any family of ILF PMS-Ids in a PMS-algebra of as follows.

Corollary 2. Let $\{A_i \mid i \in \mathbb{N}\}$ be a family of ILF PMS-Ids of X . Then $\bigcap_{i \in \mathbb{N}} A_i$ is an ILF PMS-Id of X .

Theorem 5. An ILFS $A = (\zeta_A, \xi_A)$ of X is an ILF PMS-Id of X if and only if the L -fuzzy subsets ζ_A and $\bar{\xi}_A$ are L -fuzzy PMS-ideals of X .

Proof. Let $A = (\zeta_A, \xi_A)$ be an ILF PMS-Id of X . Since A is an ILF PMS-Id of X , it follows that the L-fuzzy subset ζ_A is an L-fuzzy PMS-ideal of X . Now, it remains to show that $\bar{\xi}_A$ is a fuzzy PMS-ideal of X . Let $x, y, z \in X$. Then we have $\bar{\xi}_A(0) = 1 - \xi_A(0) \geq 1 - \xi_A(x) = \bar{\xi}_A(x)$ and also

$$\begin{aligned}\bar{\xi}_A(y * x) &= 1 - \xi_A(y * x) \geq 1 - [\xi_A(z * y) \vee \xi_A(z * x)] \\ &= [1 - \xi_A(z * y)] \wedge [1 - \xi_A(z * x)] \text{ (By lemma 1(3))} \\ &= \bar{\xi}_A(z * y) \wedge \bar{\xi}_A(z * x)\end{aligned}$$

Hence, $\bar{\xi}_A$ is an L-fuzzy PMS-ideal of X .

Conversely, assume that ζ_A and $\bar{\xi}_A$ are L-fuzzy PMS-ideals of X . Then for every $x, y, z \in X$, we get $\zeta_A(0) \geq \zeta_A(x)$ and $\bar{\xi}_A(0) \geq \bar{\xi}_A(x)$. Now,

$$\bar{\xi}_A(0) \geq \bar{\xi}_A(x) \implies 1 - \xi_A(0) \geq 1 - \xi_A(x) \implies \xi_A(0) \leq \xi_A(x).$$

Also, $\zeta_A(y * x) \geq \zeta_A(z * y) \wedge \zeta_A(z * x)$ and $\bar{\xi}_A(y * x) \geq \bar{\xi}_A(z * y) \wedge \bar{\xi}_A(z * x)$, and

$$\begin{aligned}1 - \xi_A(y * x) &= \bar{\xi}_A(y * x) \geq \bar{\xi}_A(z * y) \wedge \bar{\xi}_A(z * x) \\ &= [1 - \xi_A(z * y)] \wedge [1 - \xi_A(z * x)] \\ &= 1 - [\xi_A(z * y) \vee \xi_A(z * x)] \text{ (By lemma 1 (3)).}\end{aligned}$$

This implies that $\xi_A(y * x) \leq \xi_A(z * y) \vee \xi_A(z * x)$. Hence, $A = (\zeta_A, \xi_A)$ is an ILF PMS-Id of X . $\square$

Theorem 6. If $A = (\zeta_A, \xi_A)$ is an ILF PMS-ideal of a PMS-algebra X , then $\square A = (\zeta_A, \bar{\zeta}_A)$ is an ILF PMS-Id of X .

Proof. Let $A = (\zeta_A, \xi_A)$ be an ILF PMS-Id of X . So, by Definition 12, $\zeta_A(0) \geq \zeta_A(x)$ and $\zeta_A(y * x) \geq \zeta_A(z * y) \wedge \zeta_A(z * x)$, for all $x, y, z \in X$. Now, we have to show that $\bar{\zeta}_A(0) \leq \bar{\zeta}_A(x)$ and $\bar{\zeta}_A(y * x) \leq \bar{\zeta}_A(z * y) \vee \bar{\zeta}_A(z * x)$, for all $x, y, z \in X$. Now,

$$\bar{\zeta}_A(0) = 1 - \zeta_A(0) \leq 1 - \zeta_A(x) = \bar{\zeta}_A(x) \implies \bar{\zeta}_A(0) \leq \bar{\zeta}_A(x).$$

Also,

$$\begin{aligned}\bar{\zeta}_A(y * x) &= 1 - \zeta_A(y * x) \leq 1 - [\zeta_A(z * y) \wedge \zeta_A(z * x)] \\ &= [1 - \zeta_A(z * y)] \vee [1 - \zeta_A(z * x)] \text{ (By lemma 1 (2))} \\ &= \bar{\zeta}_A(z * y) \vee \bar{\zeta}_A(z * x)\end{aligned}$$

Therefore, $\square A = (\zeta_A, \bar{\zeta}_A)$ is an ILF PMS-Id of X . $\square$

Remark 1. The converse of Theorem 6 is not necessarily true. This fact is shown by the following example.

Example 2. Let $\mathbb{Z}$ be the set of integers. Suppose $*$ is a binary operation in $\mathbb{Z}$ as defined in Example 1. Clearly, $(\mathbb{Z}, *, 0)$ is a PMS-algebra. Let $A = (\zeta_A, \xi_A)$ be an ILFS defined by

$$\zeta_A(x) = \begin{cases} 0.6 & \text{if } x \text{ is even} \\ 0.4 & \text{if } x \text{ is odd} \end{cases} \quad \text{and} \quad \xi_A(x) = \begin{cases} 0.4 & \text{if } x \text{ is even} \\ 0.2 & \text{if } x \text{ is odd} \end{cases}.$$

Observe that $\xi_A(0) = 0.4 > 0.2 = \xi_A(1)$. This contradicts Definition 12(i). Therefore, $A = (\zeta_A, \xi_A)$ is not an ILF PMS-Id of $\mathbb{Z}$. Now,

$$\bar{\zeta}_A(x) = 1 - \zeta_A(x) = \begin{cases} 0.4 & \text{if } x \text{ is even} \\ 0.6 & \text{if } x \text{ is odd} \end{cases}.$$

By routine calculation we can show that $\square A = (\zeta_A, \bar{\zeta}_A)$ is an ILF PMS-Id of $\mathbb{Z}$. So, this example verifies that the converse of the above Theorem is not necessarily true.

Theorem 7. If $A = (\zeta_A, \xi_A)$ is an ILF PMS-Id of X , then $\diamond A = (\bar{\xi}_A, \xi_A)$ is an ILF PMS-Id of X .

Proof. Suppose $A = (\zeta_A, \xi_A)$ is an ILF PMS-Id of X . Then by Definition 12, we have, $\xi_A(0) \leq \xi_A(x)$ and $\xi_A(y * x) \leq \xi_A(z * y) \vee \xi_A(z * x)$, for all $x, y, z \in X$. So it suffices to show that $\bar{\xi}_A(0) \geq \bar{\xi}_A(x)$ and $\bar{\xi}_A(y * x) \geq \bar{\xi}_A(z * y) \wedge \bar{\xi}_A(z * x)$, for all $x, y, z \in X$. Now, $\bar{\xi}_A(0) = 1 - \zeta_A(0) \geq 1 - \xi_A(x) = \bar{\xi}_A(x) \implies \bar{\xi}_A(0) \geq \bar{\xi}_A(x)$, and

$$\begin{aligned} \bar{\xi}_A(y * x) &= 1 - \zeta_A(y * x) \\ &\geq 1 - [\xi_A(z * y) \vee \xi_A(z * x)] \\ &= [1 - \xi_A(z * y)] \wedge [1 - \xi_A(z * x)] \quad (\text{By lemma 1(3)}) \\ &= \bar{\xi}_A(z * y) \wedge \bar{\xi}_A(z * x) \end{aligned}$$

Hence, $\diamond A = (\bar{\xi}_A, \xi_A)$ is an ILF PMS-Id of X . □

Remark 2. The converse of the Theorem 7 is not true. It is shown by the following example.

Example 3. Consider the PMS-algebra $(\mathbb{Z}, *, 0)$ as defined in Example 2 and define an ILFS $A = (\zeta_A, \xi_A)$ by

$$\zeta_A(x) = \begin{cases} 0.3 & \text{if } x \text{ is even} \\ 0.5 & \text{if } x \text{ is odd} \end{cases} \quad \text{and} \quad \xi_A(x) = \begin{cases} 0.4 & \text{if } x \text{ is even} \\ 0.5 & \text{if } x \text{ is odd} \end{cases}.$$

We observe that $\zeta_A(0) = 0.3 < 0.5 = \zeta_A(3)$. This contradicts Definition 12. Therefore, $A = (\zeta_A, \xi_A)$ is not an ILF PMS-Id of $\mathbb{Z}$. Also,

$$\bar{\xi}_A(x) = 1 - \xi_A(x) = \begin{cases} 0.6 & \text{if } x \text{ is even} \\ 0.5 & \text{if } x \text{ is odd} \end{cases}.$$

Clearly, $\Diamond A = (\bar{\xi}_A, \xi_A)$ is an ILF PMS-Id of $\mathbb{Z}$. So, this example also verifies that the converse of the above Theorem is not necessarily true.

The next corollary gives the generalization of Theorem 6 and Theorem 7.

Corollary 3. *An ILFS $A = (\zeta_A, \xi_A)$ of a PMS-algebra X is an ILF PMS-Id of X if and only if $\Box A = (\zeta_A, \bar{\zeta}_A)$ and $\Diamond A = (\bar{\xi}_A, \xi_A)$ are ILF PMS-Ids of X .*

Theorem 8. *If $A = (\zeta_A, \xi_A)$ is an ILF PMS-Id of X , then $\bar{A}^c$ is also an ILF PMS-Id of X .*

Proof. Let A be an ILF PMS-Id of a PMS-algebra X . Then $A^c = \{\langle x, \zeta_{A^c}(x), \xi_{A^c}(x) \rangle | x \in X\}$, where $\zeta_{A^c}(x) = 1 - \zeta_A(x)$ and $\xi_{A^c}(x) = 1 - \xi_A(x)$. Therefore, $\bar{A}^c = \{\langle x, \xi_{A^c}(x), \zeta_{A^c}(x) \rangle | x \in X\}$. Let $x, y, z \in X$, then we have $\xi_{A^c}(0) = 1 - \xi_A(0) \geq 1 - \xi_A(x) = \xi_{A^c}(x)$ and $\zeta_{A^c}(0) = 1 - \zeta_A(0) \leq 1 - \zeta_A(x) = \zeta_{A^c}(x)$.

Also,

$$\begin{aligned} \xi_{A^c}(y * x) &= 1 - \xi_A(y * x) \\ &\geq 1 - [\xi_A(z * y) \vee \xi_A(z * x)] \\ &= [1 - \xi_A(z * y)] \wedge [1 - \xi_A(z * x)] \text{ (By Lemma 1(1))} \\ &= \xi_{A^c}(z * y) \wedge \xi_{A^c}(z * x), \end{aligned}$$

and,

$$\begin{aligned} \zeta_{A^c}(y * x) &= 1 - \zeta_A(y * x) \\ &\leq 1 - [\zeta_A(z * y) \wedge \zeta_A(z * x)] \\ &= [1 - \zeta_A(z * y)] \vee [1 - \zeta_A(z * x)] \text{ (By lemma 1(2))} \\ &= \zeta_{A^c}(z * y) \vee \zeta_{A^c}(z * x). \end{aligned}$$

Therefore, $\bar{A}^c$ is an ILF PMS-Id of X . □

Definition 13. *Let $A = (\zeta_A, \xi_A)$ be an ILFS of X . Then the intuitionistic L-fuzzy level subset of A is the subset $A_{s,t}$ of X given by $A_{s,t} = \{x \in X : \zeta_A(x) \geq s \text{ and } \xi_A(x) \leq t\}$, for $s, t \in L$.*

Theorem 9. *Let $A = (\zeta_A, \xi_A)$ be an ILFS of X . Then $A = (\zeta_A, \xi_A)$ is an ILF PMS-Id of X if and only if the nonempty subset $A_{s,t}$ of X is a PMS-Id of X for all $s, t \in L$.*

Proof. Suppose that A is an ILF PMS-Id of X . As $A_{s,t} \neq \emptyset$, there exist $x \in X$ such that $x \in A_{s,t}$. Then $\zeta_A(x) \geq s$ and $\xi_A(x) \leq t$. Since $A = (\zeta_A, \xi_A)$ is an ILF PMS-Id of X , $\zeta_A(0) \geq \zeta_A(x) \geq s$ and $\xi_A(0) \leq \xi_A(x) \leq t$ for all $x \in X$. Therefore, $0 \in A_{s,t}$.

Let $x, y, z \in X$ such that $z * y, z * x \in A_{s,t}$. Then $\zeta_A(z * y) \geq s$, $\zeta_A(z * x) \geq s$ and $\xi_A(z * y) \leq t$, $\xi_A(z * x) \leq t$. Since $A = (\zeta_A, \xi_A)$ is an ILF PMS-Id of X , $\zeta_A(y * x) \geq \zeta_A(z * y) \wedge \zeta_A(z * x) \geq s$ and $\xi_A(y * x) \leq \xi_A(z * y) \vee \xi_A(z * x) \leq t$. Thus, $y * x \in A_{s,t}$. Therefore, $A_{s,t}$ is a PMS-Id of X .

Conversely, $A_{s,t}$ is a PMS-Id of X . Let $x \in X$ such that $\zeta_A(x) = s$ and $\xi_A(x) = t$. As, $A_{s,t}$ is a PMS-Id of X , we have $0 \in A_{s,t}$. This implies that $\zeta_A(0) \geq s = \zeta_A(x)$ and $\xi_A(0) \leq t =$

$\xi_A(x)$. Therefore, $\zeta_A(0) \geq \zeta_A(x)$ and $\xi_A(0) \leq \xi_A(x)$, for all $x \in X$. Let $x, y, z \in X$ such that $s = \zeta_A(z * y) \wedge \zeta_A(z * x)$ and $t = \xi_A(z * y) \vee \xi_A(z * x)$. Then $\zeta_A(z * y) \geq s$, $\zeta_A(z * x) \geq s$ and $\xi_A(z * y) \leq t$, $\xi_A(z * x) \leq t$. Thus, $z * y, z * x \in A_{s,t}$. Since $A_{s,t}$ is a PMS-Id of X , it implies that $y * x \in A_{s,t}$. Therefore, $\zeta_A(y * x) \geq s = \zeta_A(z * y) \wedge \zeta_A(z * x)$ and $\xi_A(y * x) \leq t = \xi_A(z * y) \vee \xi_A(z * x)$. Therefore, $A = (\zeta_A, \xi_A)$ is an ILF PMS-Id of X . $\square$

As a consequence Theorem 9, we have the following Theorem.

Theorem 10. *Any PMS-Id of X can be realized as level PMS-ideal of some ILF PMS-Id of X .*

Proof. Let I be a PMS-Id of X and $A = (\zeta_A, \xi_A)$ be an ILFS in X defined by

$$\zeta_A(x) = \begin{cases} s & \text{if } x \in I \\ s_0 & \text{otherwise} \end{cases}, \text{ and } \xi_A(x) = \begin{cases} t & \text{if } x \in I \\ t_0 & \text{otherwise} \end{cases}$$

for all $x \in X$, where $s, s_0, t, t_0 \in L$ with $s \geq s_0$ and $t \leq t_0$. Clearly, $A_{s,t} = I$. Since $0 \in I$, $\zeta_A(0) = s \geq \zeta_A(x)$ and $\xi_A(0) = t \leq \xi_A(x)$. Hence, $\zeta_A(0) \geq \zeta_A(x)$ and $\xi_A(0) \leq \xi_A(x)$, for all $x \in X$. Also, consider the following cases for all $x, y, z \in X$.

Case(i). If $z * x, z * y \in I$, then $y * x \in I$. So, $\zeta_A(y * x) = s = \zeta_A(z * y) \wedge \zeta_A(z * x)$ and $\xi_A(y * x) = t = \xi_A(z * y) \vee \xi_A(z * x)$.

Case(ii). If $z * x \notin I$ or $z * y \notin I$, then $\zeta_A(y * x) \geq s_0 = \zeta_A(z * y) \wedge \zeta_A(z * x)$ and $\xi_A(y * x) \leq t_0 = \xi_A(z * y) \vee \xi_A(z * x)$.

Hence, $A = (\zeta_A, \xi_A)$ is an ILF PMS-ideal of X . Therefore, I is the level PMS-ideal an ILF PMS-ideal A of X . $\square$

4 ILF PMS-ideals of a PMS-algebra under homomorphism

In this section, we prove that the effect of homomorphism on ILF PMS-ideals and obtain interesting results.

Theorem 11. *Let $f : X \rightarrow Y$ be a homomorphism of PMS-algebras. If $A = (\zeta_A, \xi_A)$ is an ILF PMS-Id of Y , then the inverse image $f^{-1}(A) = (\zeta_{f^{-1}(A)}, \xi_{f^{-1}(A)})$ of A under f in X is an ILF PMS-Id of X .*

Proof. For all $x \in X$, $\zeta_{f^{-1}(A)}(0) = \zeta_A(f(0)) \geq \zeta_A(f(x)) = \zeta_{f^{-1}(A)}(x)$ and $\xi_{f^{-1}(A)}(0) = \xi_A(f(0)) \leq \xi_A(f(x)) = \xi_{f^{-1}(A)}(x)$. Also, Let $x, y, z \in X$. Then,

$$\begin{aligned} \zeta_{f^{-1}(A)}(y * x) &= \zeta_A(f(y * x)) = \zeta_A(f(y) * f(x)) \\ &\geq \zeta_A(f(z) * f(y)) \wedge \zeta_A(f(z) * f(x)) \\ &= \zeta_A(f(z * y)) \wedge \zeta_A(f(z * x)) \\ &= \zeta_{f^{-1}(A)}(z * y) \wedge \zeta_{f^{-1}(A)}(z * x), \end{aligned}$$

and,

$$\xi_{f^{-1}(A)}(y * x) = \xi_A(f(y * x)) = \xi_A(f(y) * f(x))$$

$$\begin{aligned}
&\leq \xi_A(f(z) * f(y)) \vee \xi_A(f(z) * f(x)) \\
&= \xi_A(f(z * y)) \vee \xi_A(f(z * x)) \\
&= \xi_{f^{-1}(A)}(z * y) \vee \xi_{f^{-1}(A)}(z * x).
\end{aligned}$$

Therefore, $f^{-1}(A) = (\zeta_{f^{-1}(A)}, \xi_{f^{-1}(A)})$ is an ILF PMS-Id of X . $\square$

Theorem 12. Let $f : X \rightarrow Y$ be an epimorphism of PMS-algebras. If $f^{-1}(A) = (\zeta_{f^{-1}(A)}, \xi_{f^{-1}(A)})$ is an ILF PMS-Id of X . Then A is an ILF PMS-Id of Y .

Proof. Since f is an epimorphism of PMS-algebras for any $x \in Y$, there exists $a \in X$ such that $f(a) = x$. Then, $\zeta_A(0) = \zeta_A(f(0)) = \zeta_{f^{-1}(A)}(0) \geq \zeta_{f^{-1}(A)}(a) = \zeta_A(f(a)) = \zeta_A(x)$ and $\xi_A(0) = \xi_A(f(0)) = \xi_{f^{-1}(A)}(0) \leq \xi_{f^{-1}(A)}(a) = \xi_A(f(a)) = \xi_A(x)$. Also, let $x, y, z \in Y$. Then, $f(a) = x$, $f(b) = y$ and $f(c) = z$, for some $a, b, c \in X$. Thus,

$$\begin{aligned}
\zeta_A(y * x) &= \zeta_A(f(b) * f(a)) = \zeta_A(f(b * a)) \\
&= \zeta_{f^{-1}(A)}(b * a) \\
&\geq \zeta_{f^{-1}(A)}(c * b) \wedge \zeta_{f^{-1}(A)}(c * a) \\
&= \zeta_A(f(c * b)) \wedge \zeta_A(f(c * a)) \\
&= \zeta_A(f(c) * f(b)) \wedge \zeta_A(f(c) * f(a)) \\
&= \zeta_A(z * y) \wedge \zeta_A(z * x),
\end{aligned}$$

and,

$$\begin{aligned}
\xi_A(y * x) &= \xi_A(f(b) * f(a)) = \xi_A(f(b * a)) \\
&= \xi_{f^{-1}(A)}(b * a) \\
&\leq \xi_{f^{-1}(A)}(c * b) \vee \xi_{f^{-1}(A)}(c * a) \\
&= \xi_A(f(c * b)) \vee \xi_A(f(c * a)) \\
&= \xi_A(f(c) * f(b)) \vee \xi_A(f(c) * f(a)) \\
&= \xi_A(z * y) \vee \xi_A(z * x).
\end{aligned}$$

Therefore, $A = (\zeta_A, \xi_A)$ is an ILF PMS-Id of Y . $\square$

Definition 14. Let $f : X \rightarrow Y$ be a homomorphism of PMS-algebras. Then, for any ILFS $A = (\zeta_A, \xi_A)$ of Y , we define an ILFS $A^f = (\zeta_A^f, \xi_A^f)$ of X by $\zeta_A^f(x) = \zeta_A(f(x))$ and $\xi_A^f(x) = \xi_A(f(x))$, for all $x \in X$.

Theorem 13. Let $f : X \rightarrow Y$ be a homomorphism of PMS-algebras. If an ILFS $A = (\zeta_A, \xi_A)$ of Y is an ILF PMS-ideal of Y , then an IFS $A^f = (\zeta_A^f, \xi_A^f)$ in X is an ILF PMS-ideal of X .

Proof. Suppose $f : X \rightarrow Y$ be a homomorphism of PMS-algebras and $A = (\zeta_A, \xi_A)$ is an ILF PMS-ideal of Y . Then we have $\zeta_A^f(0) = \zeta_A(f(0)) = \zeta_A(0) \geq \zeta_A(f(x)) = \zeta_A^f(x)$ and $\xi_A^f(0) = \xi_A(f(0)) = \xi_A(0) \leq \xi_A(f(x)) = \xi_A^f(x)$, for all $x \in X$. Also, let $x, y, z \in X$. Then,

$$\begin{aligned}
\zeta_A^f(y * x) &= \zeta_A(f(y * x)) = \zeta_A(f(y) * f(x)) \\
&\geq \zeta_A(f(z) * f(y)) \wedge \zeta_A(f(z) * f(x)) \\
&= \zeta_A(f(z * y) \wedge \zeta_A(f(z * x)) \\
&= \zeta_A^f(z * y) \wedge \zeta_A^f(z * x),
\end{aligned}$$

and,

$$\begin{aligned}
\xi_A^f(y * x) &= \xi_A(f(y * x)) = \xi_A(f(y) * f(x)) \\
&\leq \xi_A(f(z) * f(y)) \vee \xi_A(f(z) * f(x)) \\
&= \xi_A(f(z * y) \vee \xi_A(f(z * x)) \\
&= \xi_A^f(z * y) \vee \xi_A^f(z * x).
\end{aligned}$$

Hence, $A^f = (\zeta_A^f, \xi_A^f)$ is an ILF PMS-ideal of X . $\square$

Theorem 14. Let $f : X \rightarrow Y$ be an epimorphism of PMS-algebras and let $A = (\zeta_A, \xi_A)$ be an ILFS in Y . If $A^f = (\zeta_A^f, \xi_A^f)$ is an ILF PMS-Id of X , then $A = (\zeta_A, \xi_A)$ is an ILF PMS-Id of Y .

Proof. Let $A^f = (\zeta_A^f, \xi_A^f)$ be an ILF PMS-Id of X . Since f is an epimorphism of PMS-algebras, for any $x, y, z \in Y$, there exist $a, b, c \in X$ such that $f(a) = x, f(b) = y$ and $f(c) = z$. Then we have, $\zeta_A(0) = \zeta_A(f(0)) = \zeta_A^f(0) \geq \zeta_A^f(a) = \zeta_A(f(a)) = \zeta_A(x)$ and $\xi_A(0) = \xi_A(f(0)) = \xi_A^f(0) \leq \xi_A^f(a) = \xi_A(f(a)) = \xi_A(x)$. Also,

$$\begin{aligned}
\zeta_A(y * x) &= \zeta_A(f(b) * f(a)) = \zeta_A(f(b * a)) = \zeta_A^f(b * a) \\
&\geq \zeta_A^f(c * b) \wedge \zeta_A^f(c * a) \\
&= \zeta_A(f(c * b)) \wedge \zeta_A(f(c * a)) \\
&= \zeta_A(f(c) * f(b)) \wedge \zeta_A(f(c) * f(a)) \\
&= \zeta_A(z * y) \wedge \zeta_A(z * x),
\end{aligned}$$

and,

$$\begin{aligned}
\xi_A(y * x) &= \xi_A(f(b) * f(a)) = \xi_A(f(b * a)) = \xi_A^f(b * a) \\
&\leq \xi_A^f(c * b) \vee \xi_A^f(c * a) \\
&= \xi_A(f(c * b)) \vee \xi_A(f(c * a)) \\
&= \xi_A(f(c) * f(b)) \vee \xi_A(f(c) * f(a)) \\
&= \xi_A(z * y) \vee \xi_A(z * x).
\end{aligned}$$

Therefore, $A = (\zeta_A, \xi_A)$ is an ILF PMS-Id of Y . $\square$

Theorem 15. *Let A be an ILF PMS-Id of Y and $f : X \rightarrow Y$ be an isomorphism of PMS-algebras. Then $A \circ f$ is an ILF PMS-Id of X , where $\circ$ is the operation of composition of functions.*

Proof. Let $x, y, z \in X$ and $A = (\zeta_A, \xi_A)$ be an ILF PMS-Id of Y . Then $(\zeta_A \circ f)(0) = \zeta_A(f(0)) \geq \zeta_A(f(x)) = (\zeta_A \circ f)(x)$ and $(\xi_A \circ f)(0) = \xi_A(f(0)) \leq \xi_A(f(x)) = (\xi_A \circ f)(x)$.

$$\begin{aligned} (\zeta_A \circ f)(y * x) &= \zeta_A(f(y * x)) = \zeta_A(f(y) * f(x)) \\ &\geq \zeta_A(f(z) * f(y)) \wedge \zeta_A(f(z) * f(x)) \\ &= (\zeta_A \circ f)(z * y) \wedge (\zeta_A \circ f)(z * x), \end{aligned}$$

and,

$$\begin{aligned} (\xi_A \circ f)(y * x) &= \xi_A(f(y * x)) = \xi_A(f(y) * f(x)) \\ &\leq \xi_A(f(z) * f(y)) \vee \xi_A(f(z) * f(x)) \\ &= (\xi_A \circ f)(z * y) \vee (\xi_A \circ f)(z * x). \end{aligned}$$

Hence, $A \circ f$ is an ILF PMS-Id of X . □

5 ILF PMS-ideals of a PMS-algebra under cartesian products

In this section, we discuss the Cartesian products and the strongest fuzzy relation on ILF PMS-ideals of a PMS-algebra and obtain some interesting results.

Theorem 16. *Let $A = (\zeta_A, \xi_A)$ and $B = (\zeta_B, \xi_B)$ be any two ILF PMS-Ids of X . Then $A \times B$ is an ILF PMS-Id of $X \times X$.*

Proof. Suppose $A = (\zeta_A, \xi_A)$ and $B = (\zeta_B, \xi_B)$ be any two ILF PMS-Ids of X . Then for each $(x, y) \in X \times X$, we have $\zeta_{A \times B}(0, 0) = \zeta_A(0) \wedge \zeta_B(0) \geq \zeta_A(x) \wedge \zeta_B(y) = \zeta_{A \times B}(x, y)$ and $\xi_{A \times B}(0, 0) = \xi_A(0) \vee \xi_B(0) \geq \xi_A(x) \vee \xi_B(y) = \xi_{A \times B}(x, y)$. Also, let $(x_1, x_2), (y_1, y_2), (z_1, z_2) \in X \times X$. Then,

$$\begin{aligned} \zeta_{A \times B}[(y_1, y_2) * (x_1, x_2)] &= \zeta_{A \times B}[(y_1 * x_1, y_2 * x_2)] \\ &= \zeta_A(y_1 * x_1) \wedge \zeta_B(y_2 * x_2) \\ &\geq [\zeta_A(z_1 * y_1) \wedge \zeta_A(z_1 * x_1)] \wedge [\zeta_B(z_2 * y_2) \wedge \zeta_B(z_2 * x_2)] \\ &= [\zeta_A(z_1 * y_1) \wedge \zeta_B(z_2 * y_2)] \wedge \zeta_A(z_1 * x_1) \wedge \zeta_B(z_2 * x_2) \\ &= \zeta_{A \times B}(z_1 * y_1, z_2 * y_2) \wedge \zeta_{A \times B}(z_1 * x_1, z_2 * x_2) \\ &= \zeta_{A \times B}[(z_1, z_2) * (y_1, y_2)] \wedge \zeta_{A \times B}[(z_1, z_2) * (x_1, x_2)] \end{aligned}$$

and,

$$\xi_{A \times B}[(y_1, y_2) * (x_1, x_2)] = \xi_{A \times B}[(y_1 * x_1, y_2 * x_2)]$$

$$\begin{aligned}
&= \xi_A(y_1 * x_1) \vee \xi_B(y_2 * (y_2)) \\
&\leq [\xi_A(z_1 * y_1) \vee \xi_A(z_1 * x_1)] \vee [\xi_B(z_2 * y_2) \vee \xi_B(z_2 * x_2)] \\
&= [\xi_A(z_1 * y_1) \vee \xi_B(z_2 * y_2)] \vee [\xi_A(z_1 * x_1) \vee \xi_B(z_2 * x_2)] \\
&= \xi_{A \times B}(z_1 * y_1, z_2 * y_2) \vee \xi_{A \times B}(z_1 * x_1, z_2 * x_2) \\
&= \xi_{A \times B}[(z_1, z_2) * (y_1, y_2)] \vee \xi_{A \times B}[(z_1, z_2) * (x_1, x_2)].
\end{aligned}$$

Therefore, $A \times B$ is an ILF PMS-Id of $X \times X$. $\square$

Theorem 17. Let $A = (\zeta_A, \xi_A)$ and $B = (\zeta_B, \xi_B)$ be any two ILFSs of X such that $A \times B$ is an ILF PMS-Id of $X \times X$. Then either A or B is an ILF PMS-Id of X .

Proof. Let $A = (\zeta_A, \xi_A)$ and $B = (\zeta_B, \xi_B)$ be ILFSs of X such that $A \times B$ is an ILF PMS-Id of $X \times X$. Then, $\zeta_{A \times B}(0, 0) \geq \zeta_{A \times B}(x, y)$ and $\xi_{A \times B}(0, 0) \leq \xi_{A \times B}(x, y)$, for all $(x, y) \in X \times X$. Assume that $\zeta_A(x) > \zeta_A(0)$ and $\zeta_B(y) > \zeta_B(0)$, for some $x, y \in X$. Then $\zeta_{A \times B}(x, y) = \zeta_A(x) \wedge \zeta_B(y) > \zeta_A(0) \wedge \zeta_B(0) = \zeta_{A \times B}(0, 0)$, which is a contradiction. Therefore, $\zeta_A(0) \geq \zeta_A(x)$ or $\zeta_B(0) \geq \zeta_B(y)$, for all $x, y \in X$.

Similarly, Assume that $\xi_A(x) < \xi_A(0)$ and $\xi_B(y) < \xi_B(0)$, for some $x, y \in X$. Then, $\xi_{A \times B}(x, y) = \xi_A(x) \wedge \xi_B(y) < \xi_A(0) \wedge \xi_B(0) = \xi_{A \times B}(0, 0)$, which is a contradiction. Therefore, $\xi_A(0) \leq \xi_A(x)$ or $\xi_B(0) \leq \xi_B(y)$, for all $x, y \in X$. Let $(x_1, x_2), (y_1, y_2), (z_1, z_2) \in X \times X$. Then By Theorem 16, we have

$$\begin{aligned}
\zeta_{A \times B}((y_1, y_2) * (x_1, x_2)) &\geq \zeta_{A \times B}((z_1, z_2) * (y_1, y_2)) \wedge \zeta_{A \times B}((z_1, z_2) * (x_1, x_2)) \\
&= \zeta_{A \times B}(z_1 * y_1, z_2 * y_2) \wedge \zeta_{A \times B}(z_1 * x_1, z_2 * x_2) \\
&= (\zeta_A(z_1 * y_1) \wedge \zeta_B(z_2 * y_2)) \wedge (\zeta_A(z_1 * x_1) \wedge \zeta_B(z_2 * x_2)).
\end{aligned}$$

As $\zeta_{A \times B}((y_1, y_2) * (x_1, x_2)) = \zeta_{A \times B}(y_1 * x_1, y_2 * x_2)$, we have $\zeta_{A \times B}(y_1 * x_1, y_2 * x_2) \geq (\zeta_A(z_1 * y_1) \wedge \zeta_B(z_2 * y_2)) \wedge (\zeta_A(z_1 * x_1) \wedge \zeta_B(z_2 * x_2))$. So, $\zeta_A(y_1 * x_1) \wedge \zeta_B(y_2 * x_2) \geq (\zeta_A(z_1 * y_1) \wedge \zeta_B(z_2 * y_2)) \wedge (\zeta_A(z_1 * x_1) \wedge \zeta_B(z_2 * x_2)) = (\zeta_A(z_1 * y_1) \wedge \zeta_A(z_1 * x_1)) \wedge (\zeta_B(z_2 * y_2) \wedge \zeta_B(z_2 * x_2))$. Hence, either $\zeta_A(y_1 * x_1) \geq \zeta_A(z_1 * y_1) \wedge \zeta_A(z_1 * x_1)$ or $\zeta_B(y_2 * x_2) \geq \zeta_B(z_2 * y_2) \wedge \zeta_B(z_2 * x_2)$.

Similarly, we can also show that either $\xi_A(y_1 * x_1) \leq \xi_A(z_1 * y_1) \vee \xi_A(z_1 * x_1)$ or $\xi_B(y_2 * x_2) \leq \xi_B(z_2 * y_2) \vee \xi_B(z_2 * x_2)$. Therefore, either A or B is an ILF PMS-Id of X . $\square$

Theorem 18. If $A = (\zeta_A, \xi_A)$ and $B = (\zeta_B, \xi_B)$ are ILF PMS-Ids of X , then $\square(A \times B) = (\zeta_{A \times B}, \bar{\zeta}_{A \times B})$ is an ILF PMS-Id of $X \times X$.

Proof. Since $A = (\zeta_A, \xi_A)$ and $B = (\zeta_B, \xi_B)$ are ILF PMS-Ids of X , By Theorem 16, $A \times B$ is an ILF PMS-Id of $X \times X$. Then for any $(x_1, x_2), (y_1, y_2), (z_1, z_2) \in X \times X$. We have $\zeta_{A \times B}(0, 0) \geq \zeta_{A \times B}(x_1, x_2)$ and $\zeta_{A \times B}((y_1, y_2) * (x_1, x_2)) \geq \zeta_{A \times B}((z_1, z_2) * (y_1, y_2)) \wedge \zeta_{A \times B}((z_1, z_2) * (x_1, x_2))$. So it suffices to show that $\bar{\zeta}_{A \times B}$ satisfies the condition $\bar{\zeta}_{A \times B}(0, 0) \leq \bar{\zeta}_{A \times B}(x_1, x_2)$ and $\bar{\zeta}_{A \times B}((y_1, y_2) * (x_1, x_2)) \leq \bar{\zeta}_{A \times B}((z_1, z_2) * (y_1, y_2)) \vee \bar{\zeta}_{A \times B}(z_1, z_2) * (x_1, x_2)$. Thus, for any $(x_1, x_2), (y_1, y_2), (z_1, z_2) \in X \times X$, we have $\bar{\zeta}_{A \times B}(0, 0) = 1 - \zeta_{A \times B}(0, 0) \leq 1 - \zeta_{A \times B}(x_1, x_2) = \bar{\zeta}_{A \times B}(x_1, x_2)$ and,

$$\bar{\zeta}_{A \times B}((y_1, y_2) * (x_1, x_2)) = 1 - \zeta_{A \times B}((y_1, y_2) * (x_1, x_2))$$

$$\begin{aligned}
&\leq 1 - [\zeta_{A \times B}((z_1, z_2) * (y_1, y_2)) \wedge \zeta_{A \times B}((z_1, z_2) * (x_1, x_2))] \\
&= [1 - \zeta_{A \times B}((z_1, z_2) * (y_1, y_2))] \vee [1 - \zeta_{A \times B}((z_1, z_2) * (x_1, x_2))] \\
&= \bar{\zeta}_{A \times B}((z_1, z_2) * (y_1, y_2)) \vee \bar{\zeta}_{A \times B}((z_1, z_2) * (x_1, x_2)).
\end{aligned}$$

Hence, $\square(A \times B) = (\zeta_{A \times B}, \bar{\zeta}_{A \times B})$ is an ILF PMS-Id of $X \times X$. $\square$

Theorem 19. If $A = (\zeta_A, \xi_A)$ and $B = (\zeta_B, \xi_B)$ are ILF PMS-Ids of X , then $\diamond(A \times B)$ is an ILF PMS-Id of $X \times X$.

Proof. Since A and B are ILF PMS-Ids of X , By Theorem 16, $A \times B$ is an ILF PMS-Id of $X \times X$. Then for any $(x_1, x_2), (y_1, y_2), (z_1, z_2) \in X \times X$, we have $\xi_{A \times B}(0, 0) \leq \xi_{A \times B}(x_1, x_2)$ and $\xi_{A \times B}((y_1, y_2) * (x_1, x_2)) \leq \xi_{A \times B}((z_1, z_2) * (y_1, y_2)) \wedge \xi_{A \times B}((z_1, z_2) * (x_1, x_2))$. So it is sufficient to show that $\bar{\zeta}_{A \times B}$ satisfies the conditions $\bar{\xi}_{A \times B}(0, 0) \leq \bar{\xi}_{A \times B}(x_1, x_2)$ and $\bar{\xi}_{A \times B}((y_1, y_2) * (x_1, x_2)) \leq \bar{\xi}_{A \times B}((z_1, z_2) * (y_1, y_2)) \vee \bar{\xi}_{A \times B}(z_1, z_2) * (x_1, x_2)$. Thus for any $(x_1, x_2), (y_1, y_2), (z_1, z_2) \in X \times X$, we have $\bar{\xi}_{A \times B}(0, 0) = 1 - \xi_{A \times B}(0, 0) \geq 1 - \xi_{A \times B}(x_1, x_2) = \bar{\xi}_{A \times B}(x_1, x_2)$ and,

$$\begin{aligned}
\bar{\xi}_{A \times B}((y_1, y_2) * (x_1, x_2)) &= 1 - \xi_{A \times B}((y_1, y_2) * (x_1, x_2)) \\
&\geq 1 - [\xi_{A \times B}((z_1, z_2) * (y_1, y_2)) \vee \xi_{A \times B}((z_1, z_2) * (x_1, x_2))] \\
&= [1 - \xi_{A \times B}((z_1, z_2) * (y_1, y_2))] \wedge [1 - \xi_{A \times B}((z_1, z_2) * (x_1, x_2))] \\
&= \bar{\zeta}_{A \times B}((z_1, z_2) * (y_1, y_2)) \wedge \bar{\zeta}_{A \times B}((z_1, z_2) * (x_1, x_2)).
\end{aligned}$$

Hence, $\diamond(A \times B) = (\bar{\xi}_{A \times B}, \xi_{A \times B})$ is an ILF PMS-Id of $X \times X$. $\square$

The following Theorem follows from the above two theorems

Theorem 20. The ILFSs $A = (\zeta_A, \xi_A)$ and $B = (\zeta_B, \xi_B)$ are ILF PMS-Ids of X if and only if $\square(A \times B) = (\zeta_{A \times B}, \bar{\zeta}_{A \times B})$ and $\diamond(A \times B) = (\bar{\xi}_{A \times B}, \xi_{A \times B})$ are an ILF PMS-Id of $X \times X$.

Proof. Follows from Theorems 18 and 19. $\square$

Definition 15. Let $A = (\zeta_A, \xi_A)$ and $B = (\zeta_B, \xi_B)$ be ILFSs of X . Then the intuitionistic L-fuzzy level subset of $A \times B$ is a subset $(A \times B)_{s,t}$ of $X \times X$ given by

$$(A \times B)_{s,t} = \{(x, y) \in X \times X : \zeta_A(x, y) \geq s \text{ and } \xi_A(x, y) \leq t\},$$

for $s, t \in L$.

Theorem 21. Let $A = (\zeta_A, \xi_A)$ and $B = (\zeta_B, \xi_B)$ be ILFSs of X . Then $A \times B$ is a ILF PMS-Id of $X \times X$ if and only if the nonempty subset $(A \times B)_{s,t}$ of $X \times X$ is a PMS-Id of $X \times X$ for all $s, t \in L$.

Proof. Since $(A \times B)_{s,t} \neq \emptyset$, there exist $(x, y) \in X \times X$ such that $(x, y) \in (A \times B)_{s,t}$. Then $\zeta_A(x, y) \geq s$ and $\xi_A(x, y) \leq t$. As $A \times B$ is a ILF PMS-Id of $X \times X$, $\zeta_{A \times B}(0, 0) \geq \zeta_{A \times B}(x, y) \geq s$ and $\xi_{A \times B}(0, 0) \leq \xi_{A \times B}(x, y) \leq t$. So that $(0, 0) \in (A \times B)_{s,t}$. Let $(x_1, x_2), (y_1, y_2), (z_1, z_2) \in X \times X$ such that $(z_1, z_2) * (y_1, y_2), (z_1, z_2) * (x_1, x_2) \in (A \times B)_{s,t}$ for $s, t \in L$. Then

$\zeta_{A \times B}((z_1, z_2) * (y_1, y_2)) \geq s$, $\zeta_{A \times B}((z_1, z_2) * (x_1, x_2)) \geq s$, $\xi_{A \times B}((z_1, z_2) * (y_1, y_2)) \leq t$, and $\xi_{A \times B}((z_1, z_2) * (x_1, x_2)) \leq t$. Since $A \times B$ is an ILF PMS-Id of $X \times X$, it follows that

$$\zeta_{A \times B}((y_1, y_2) * (x_1, x_2)) \geq \zeta_{A \times B}((z_1, z_2) * (y_1, y_2)) \wedge \zeta_{A \times B}((z_1, z_2) * (x_1, x_2)) \geq s$$

and,

$$\xi_{A \times B}((y_1, y_2) * (x_1, x_2)) \leq \xi_{A \times B}((z_1, z_2) * (y_1, y_2)) \wedge \xi_{A \times B}((z_1, z_2) * (x_1, x_2)) \leq t.$$

Then, $\zeta_{A \times B}((y_1, y_2) * (x_1, x_2)) \geq s$ and $\xi_{A \times B}((y_1, y_2) * (x_1, x_2)) \leq t$. Hence, $(y_1, y_2) * (x_1, x_2) \in (A \times B)_{s,t}$ and thus, $(A \times B)_{s,t}$ is a PMS-algebra of $X \times X$.

Conversely, assume that $(A \times B)_{s,t}$ is a PMS-algebra of $X \times X$. Let $(x, y) \in X \times X$ such that $\zeta_A(x, y) = s$ and $\xi_A(x, y) = t$. As, $(A \times B)_{s,t}$ is a PMS-Id of X , we have $(0, 0) \in (A \times B)_{s,t}$. This implies that $\zeta_A(0, 0) \geq s = \zeta_A(x, y)$ and $\xi_A(0, 0) \leq t = \xi_A(x, y)$. Therefore, $\zeta_A(0, 0) \geq \zeta_A(x, y)$ and $\xi_A(0, 0) \leq \xi_A(x, y)$, for all $(x, y) \in X \times X$. Also, let $(x_1, x_2), (y_1, y_2), (z_1, z_2) \in X \times X$ such that $s = \zeta_A((z_1, z_2) * (y_1, y_2)) \wedge \zeta_A((z_1, z_2) * (x_1, x_2))$ and $t = \xi_A((z_1, z_2) * (y_1, y_2)) \vee \xi_A((z_1, z_2) * (x_1, x_2))$. Then $\zeta_A((z_1, z_2) * (y_1, y_2)) \geq s$, $\zeta_A((z_1, z_2) * (x_1, x_2)) \geq s$ and $\xi_A((z_1, z_2) * (y_1, y_2)) \leq t$, $\xi_A((z_1, z_2) * (x_1, x_2)) \leq t$. Thus, $(z_1, z_2) * (y_1, y_2), (z_1, z_2) * (x_1, x_2) \in A_{s,t}$. Since $A_{s,t}$ is a PMS-Id of X , it implies that $(y_1, y_2) * (x_1, x_2) \in A_{s,t}$. Therefore, we have $\zeta_A((y_1, y_2) * (x_1, x_2)) \geq s = \zeta_A((z_1, z_2) * (y_1, y_2)) \wedge \zeta_A((z_1, z_2) * (x_1, x_2))$ and $\xi_A((y_1, y_2) * (x_1, x_2)) \leq t = \xi_A((z_1, z_2) * (y_1, y_2)) \vee \xi_A((z_1, z_2) * (x_1, x_2))$. Therefore, $A \times B$ is an ILF PMS-Id of $\in X \times X$. $\square$

Theorem 22. Let A_B be the strongest intuitionistic L -fuzzy relation on X , where $B = (\zeta_B, \xi_B)$ be an ILFS of a PMS-algebra X . If B is an ILF PMS-Id of X , then A_B is an ILF PMS-Id of $X \times X$.

Proof. Assume that B is an ILF PMS-Id of X . Let $x_1, x_2 \in X$. Then, $\zeta_{A_B}(0, 0) = \zeta_B(0) \wedge \zeta_B(0) \geq \zeta_B(x_1) \wedge \zeta_B(x_2) = \zeta_{A_B}(x_1, x_2)$ and $\xi_{A_B}(0, 0) = \xi_B(0) \vee \xi_B(0) \leq \xi_B(x_1) \vee \xi_B(x_2) = \xi_{A_B}(x_1, x_2)$ for $x_1, x_2 \in X$. Hence, $\zeta_{A_B}(0, 0) \geq \zeta_{A_B}(x_1, x_2)$ and $\xi_{A_B}(0, 0) \leq \xi_{A_B}(x_1, x_2)$. Also, let $(x_1, x_2), (y_1, y_2), z = (z_1, z_2) \in X \times X$. Then

$$\begin{aligned} \zeta_{A_B}((y_1, y_2) * (x_1, x_2)) &= \zeta_{A_B}(y_1 * x_1, y_2 * x_2) \\ &= \zeta_B(y_1 * x_1) \wedge \zeta_B(y_2 * x_2) \\ &\geq (\zeta_B(z_1 * y_1) \wedge \zeta_B(z_1 * x_1)) \wedge (\zeta_B(z_2 * y_2) \wedge \zeta_B(z_2 * x_2)) \\ &= (\zeta_B(z_1 * y_1) \wedge \zeta_B(z_2 * y_2)) \wedge (\zeta_B(z_1 * x_1) \wedge \zeta_B(z_2 * x_2)) \\ &= \zeta_{A_B}(z_1 * y_1, z_2 * y_2) \wedge \zeta_{A_B}(z_1 * x_1, z_2 * x_2) \\ &= \zeta_{A_B}((z_1, z_2) * (y_1, y_2)) \wedge \zeta_{A_B}((z_1, z_2) * (x_1, x_2)), \end{aligned}$$

and,

$$\begin{aligned} \xi_{A_B}((y_1, y_2) * (x_1, x_2)) &= \xi_{A_B}(y_1 * x_1, y_2 * x_2) \\ &= \xi_B(y_1 * x_1) \vee \xi_B(y_2 * x_2) \\ &\leq (\xi_B(z_1 * y_1) \vee \xi_B(z_1 * x_1)) \vee (\xi_B(z_2 * y_2) \vee \xi_B(z_2 * x_2)) \end{aligned}$$

$$\begin{aligned}
&= (\xi_B(z_1 * y_1) \vee \xi_B(z_2 * y_2)) \vee (\xi_B(z_1 * x_1) \vee \xi_B(z_2 * x_2)) \\
&= \xi_{A_B}(z_1 * y_1, z_2 * y_2) \vee \xi_{A_B}(z_1 * x_1, z_2 * x_2) \\
&= \xi_{A_B}((z_1, z_2) * (y_1, y_2)) \wedge \xi_{A_B}((z_1, z_2) * (x_1, x_2)).
\end{aligned}$$

Therefore, A_B is an ILF PMS-Id of $X \times X$. $\square$

The converse of the Theorem 22 is also valid as it has been proved in Theorem 23.

Theorem 23. *If A_B is an ILF PMS-Id of $X \times X$, then $B = (\zeta_B, \xi_B)$ is an ILF PMS-Id of X .*

Proof. Let A_B be an ILF PMS-Id of $X \times X$. Then for $(x_1, x_2), (y_1, y_2), (z_1, z_2) \in X \times X$, we have

(i).

$$\begin{aligned}
\zeta_B(0) \wedge \mu_A(0) &= \zeta_{A_B}(0, 0) \geq \zeta_{A_B}(x_1, x_2) = \zeta_B(x_1) \wedge \zeta_B(x_2) \\
&\Rightarrow \zeta_B(0) \wedge \zeta_B(0) \geq \zeta_B(x_1) \wedge \zeta_B(x_2) \\
&\Rightarrow \zeta_B(0) \geq \zeta_B(x_1) \quad \text{or} \quad \zeta_B(0) \geq \zeta_B(x_2),
\end{aligned}$$

and

$$\begin{aligned}
\xi_B(0) \vee \mu_A(0) &= \xi_{A_B}(0, 0) \leq \xi_{A_B}(x_1, x_2) = \xi_B(x_1) \vee \xi_B(x_2) \\
&\Rightarrow \xi_B(0) \vee \xi_B(0) \leq \xi_B(x_1) \vee \xi_B(x_2) \\
&\Rightarrow \xi_B(0) \geq \xi_B(x_1) \quad \text{or} \quad \xi_B(0) \geq \xi_B(x_2).
\end{aligned}$$

(ii).

$$\begin{aligned}
\zeta_B(y_1 * x_1) \wedge \zeta_B(y_2 * x_2) &= \zeta_{A_B}(y_1 * x_1, y_2 * x_2) \\
&= \zeta_{A_B}((y_1, y_2) * (x_1, x_2)) \\
&\geq \zeta_{A_B}((z_1, z_2) * (y_1, y_2)) \wedge \zeta_{A_B}((z_1, z_2) * (x_1, x_2)) \\
&= \zeta_{A_B}((z_1 * y_1), (z_2 * y_2)) \wedge \zeta_{A_B}((z_1 * x_1), (z_2 * x_2)) \\
&= (\zeta_B(z_1 * y_1) \wedge \zeta_B(z_2 * y_2)) \wedge (\zeta_B(z_1 * x_1) \wedge \zeta_B(z_2 * x_2)) \\
&= (\zeta_B(z_1 * y_1) \wedge \zeta_B(z_1 * x_1)) \wedge (\zeta_B(z_2 * y_2) \wedge \zeta_B(z_2 * x_2)).
\end{aligned}$$

If we put $x_2 = y_2 = z_2 = 0$, then $\zeta_B(y_1 * x_1) \geq \zeta_B(z_1 * y_1) \wedge \zeta_B(z_1 * x_1)$.

(iii).

$$\begin{aligned}
\xi_B(y_1 * x_1) \vee \xi_B(y_2 * x_2) &= \xi_{A_B}(y_1 * x_1, y_2 * x_2) \\
&= \xi_{A_B}((y_1, y_2) * (x_1, x_2)) \\
&\leq \xi_{A_B}((z_1, z_2) * (y_1, y_2)) \vee \xi_{A_B}((z_1, z_2) * (x_1, x_2)) \\
&= \xi_{A_B}((z_1 * y_1), (z_2 * y_2)) \vee \xi_{A_B}((z_1 * x_1), (z_2 * x_2)) \\
&= (\xi_B(z_1 * y_1) \vee \xi_B(z_2 * y_2)) \vee (\xi_B(z_1 * x_1) \vee \xi_B(z_2 * x_2)) \\
&= (\xi_B(z_1 * y_1) \vee \xi_B(z_1 * x_1)) \vee (\xi_B(z_2 * y_2) \vee \xi_B(z_2 * x_2)).
\end{aligned}$$

Similarly, if $x_2 = y_2 = z_2 = 0$, we get $\xi_B(y_1 * x_1) \leq \xi_B(z_1 * y_1) \vee \xi_B(z_1 * x_1)$. Therefore, B is an ILF PMS-Id of X . $\square$

6 Conclusion

In this paper, we used the concept of intuitionistic L-fuzzy set to PMS-algebras. We introduced the notion of the intuitionistic L-fuzzy PMS-ideal of a PMS-algebra and investigated several associated properties. We provide a condition for an intuitionistic L-fuzzy set in a PMS-algebra to be an intuitionistic L-fuzzy PMS-ideal. We characterized intuitionistic L-fuzzy PMS-ideals of a PMS-algebra in terms of their level subsets. We also discussed the algebraic nature of intuitionistic L-fuzzy PMS-ideals of a PMS-algebra under homomorphism and obtained incredible results. Moreover, we studied the Cartesian product of the intuitionistic L-fuzzy PMS-ideals of a PMS-algebra and investigated some interesting results. Finally, we explored the relationship between the strongest intuitionistic L-fuzzy relations in a PMS-algebra and intuitionistic L-fuzzy PMS-ideals of a PMS-algebra. We believe that the results of this study will add new dimensions to the structures of PMS-algebras based on intuitionistic L-fuzzy sets and serve as a foundation for future studies in the intuitionistic L-fuzzification of PMS-algebra. In our future work, we will extend this concept to interval valued intuitionistic L-fuzzy PMS-ideals of a PMS-algebra. Furthermore, we will develop the Neuro-fuzzy algebraic structures for PMS-subalgebras of PMS-algebras.

Acknowledgments

The author sincerely appreciates the editor and referee for careful reading, valuable comments and constructive suggestions, which have greatly contributed to improving this manuscript.

References

- [1] B. A. Alaba, Y. G. Wondifraw and B. L. Derseh, *On intuitionistic fuzzy PMS-ideals of a PMS-algebra under homomorphism and cartesian product*, International Journal of Computational Intelligence Systems, (68) **16**(2023),1-13.
- [2] N. Anitha and K. Arjunan, *Notes on intuitionistic fuzzy ideals of hemiring*, Applied Mathematical Science, (68) **5** (2011), 3393-3402.
- [3] K. T. Atanassov, *More on intuitionistic fuzzy sets*, Fuzzy Sets and Systems, (1) **33** (1989), 37-45.
- [4] K. T. Atanassov, *Intuitionistic fuzzy sets*, Fuzzy Sets and Systems, (1) **20** (1986), 87-96.
- [5] K. T. Atanassov and S. Stoeva, *Intuitionistic L-fuzzy sets*, Cybernetics and Systems Research, **2** (1984), 539-540.
- [6] M. Chandramouleeswaran and P. Muralikrishna, *On intuitionistic L-fuzzy subalgebras of BG-algebras*, International Mathematical Forum, (20) **5** (2010), 995-1000.
- [7] F. P. Choudhury, A. B. Chakraborty and S. S. Khare, *A note on fuzzy subgroup and fuzzy homomorphism*, J. Math. Anal. Appl., (2) **131** (1988), 537-553.

- [8] Y. Christopher Jefferson and M. Chandramouleeswaran, *L-fuzzy BP-algebras*, IRA-International Journal of Applied Sciences, (1) **4** (2016), 69-75.
- [9] Y. Christopher Jefferson and M. Chandramouleeswaran, *L-fuzzy BP-ideal*, International J. of Math. Sci. and Engg. Appls. (IJMSEA), (1) **10** (2016), 233-242.
- [10] B. L. Derseh, *A study of intuitionistic L-fuzzy PMS-subalgebras within PMS-algebras* (communicated)
- [11] B. L. Derseh, B. A. Alaba and Y. G. Wondifraw, *Intuitionistic fuzzy PMS-subalgebra of a PMS-algebra*, Korean J. Math., (3) **29** (2021), 563-576.
- [12] B. L. Derseh, B. A. Alaba and Y. G. Wondifraw, *On homomorphism and cartesian products of intuitionistic fuzzy PMS- subalgebra of a PMS-algebra*, Bulletin of the Section of Logic, (1) **52** (2023), 19-38.
- [13] J. A. Goguen, *L-fuzzy sets*, J. Math. Anal. Appl., **18** (1967), 145-174.
- [14] K. Iseki and S. Tanaka, *An introduction to the theory BCK-algebras*, Math Japon, **23** (1978), 1-26.
- [15] K. Iseki, *On BCI-algebra*, Seminar Notes, **8** (1980), 125-130.
- [16] P. Muralikrishna and M. Chandramouleeswaran, *Intuitionistic L-fuzzy ideals of BF-algebras*, International Journal of Engineering Science and Technology, (10) **2** (2010), 5413- 5418.
- [17] P. Muralikrishna and M. Chandramouleeswaran, *Generalisation of cartisean product of intuitionistic L-fuzzy BF-ideals*, International Journal of Contemporary Mathematical Sciences, (14) **6** (2011), 671-679.
- [18] S. Naganathan, K. Arjunan and N. Palaniappan, *A Study on intuitionistic L-fuzzy subgroups*, International journal of applied mathematical sciences, (53) **3** (2009), 2619-2624.
- [19] S. Naganathan, K. Arjunan and N. Palaniappan, *Level subsets of intuitionistic L-fuzzy subgroups of a group*, International journal of computational and applied mathematics, **4** (2009), 177-184.
- [20] M. Panigrahi and S. Nanda, *Intuitionistic fuzzy relations over intuitionistic fuzzy sets*, Journal of Fuzzy Mathematics, (3) **15** (2007), 675-688.
- [21] T. Priya and T. Ramachandran, *Homomorphism and cartesian product of fuzzy PS-algebra*, Appl. Math. Sci., (67) **8** (2014), 3321-3330.
- [22] A. Rosenfield, *Fuzzy subgroups*, J. Math. Anal. Appl., **35** (1971), 512-517.
- [23] P. K. Sharma, *Homomorphism of intuitionistic fuzzy groups*, International Mathematics Forum, (64) **6** (2011), 3169-3178.
- [24] P. M. Sithar Selvam and K. T. Nagalakshmi, *On PMS-algebras*, Transylvanian Review, **24** (2016), 1622-1628.

- [25] S. Sowmiya and P. Jeyalakshmi, *Intuitionistic L-fuzzy structures in Z-algebras*, International Journal of Engineering Research and Technology, (2) **10** (2021), 497-501.
- [26] Y. G. Wondifraw et al., *Intuitionistic fuzzy PMS-ideals of a PMS-algebra*, Thai J. Math., (4) **20** (2022), 1535-1548.
- [27] L. A. Zadeh, *Fuzzy sets*, Information and Control, **8** (1965), 338-353
- [28] A. Zarandi and A. B. Saeid, *Intuitionistic fuzzy ideals of BG-algebras*, World Academy of Science, Engineering and Technology, **5** (2005), 187-189.