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Dynamic modeling, system identification and robust control of unmanned helicopter using multilevel simulation environment

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Abstract. According to numerous capabilities and increasing applications (in both commercial and military fields) of radio-controlled helicopters, many investigations are being conducted on these unmanned aircraft vehicles. Due to complex dynamic behavior and manual control limitations, the automatic control ability of these UAVs has gained high importance. The multilevel dynamic modeling, system identification, and control of an unmanned helicopter have been investigated in this paper. The analysis has been performed in a multilevel simulation environment designed for these purposes. A detailed complex analytical dynamic model of unmanned helicopter is used as the target case and an approximate model is used for designing control system. The system identification method has been used to tune the approximate model parameters to make sure that this model resembles the detailed complex analytical model behaviors. After designing a hierarchical robust control system for the identified approximate model, its implementation on the detailed model is performed by designing an appropriate Kalman filter to estimate un-measured state variables. The results show that the control system designed based on the identified approximate model can properly control the detailed complex analytical model and can be investigated more in next steps by in-flight experiment.

Keywords: Unmanned rotorcraft, flapping dynamics, dynamic modeling, system identification, Kalman filter, robust control
AMS Subject Classification 2010: 93B30, 93C40, 93C85

1 Introduction

Despite relatively short time since popularity of UAVs, many applications have been intended for these devices including navigation, patrolling, search and rescue, and imaging. Advancements in various fields of technology have led to the manufacturing of smaller and lighter electronic and electromechanical devices [26]. As a result, research on UAV platforms has gained more attention in the past few years [4].

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Among different types of UAVs, helicopters have greater importance because of their numerous benefits and capabilities. Some of these capabilities can be pointed as ability of vertical take-off and landing, hover flight, high agility, performing complex maneuvers, flying in different directions and also ability to pass from obstacles and limited and narrow routes. These capabilities create numerous applications in military, civil and commercial fields for unmanned helicopters [31].

Despite numerous capabilities of unmanned helicopters, different disadvantages and problems can be seen in their utilization. Dynamic performance of helicopters is much more complicated relative to other fixed wing aerial vehicles. Many factors such as complex nonlinear dynamic model with numerous states and limited inputs, inherent instability, coupled dynamics, multiplicity of effecting parameters, effects of external disturbances, and existing unmodeled dynamics, will lead to difficulties in their control [27]. Also, high degree of skill and experience is required to fly an RC helicopter due to distance, continuous change of helicopter's position and direction relative to pilot, existing errors in estimating distances and velocities, and also local wind disturbances. In addition to these factors, the restricted field of view for operators and limited flight endurance during manual flight have underscored the significance of integrating automatic flight control systems for unmanned helicopters.

Implementing the control system on real hardware and performing flight tests is the best way to investigate its capabilities. But occurring an accident during a flight test of unmanned helicopters will be very dangerous and will have relatively high financial costs. In order to decrease the risks of faults and dangers in the first steps of control system design, some testbeds have been designed and being used by different people around the world (such as [9, 12, 19, 20]). Because of significant effects of testbeds on helicopter flight behavior, this way is not the completely proper method to evaluate control system during free flight.

The common way of evaluating control system in early steps of research is using software simulation. Some investigations have been accomplished using commercial simulator software such as X-plane [21, 30, 39], FlightLab [40] and other specific environments [6]. These simulators are not normally open-source and relations and dynamic systems behind their interface are not available for extensive research and analysis. Others normally used first principle approaches for dynamic modeling of unmanned helicopters (such as [25, 29, 33]) and then use the same model for both control system design and controller evaluation. Using the same model in both steps, in addition to elimination of necessity of system identification step, will ignore lots of affecting items and complicating factors.

In this paper, we propose to use two modeling methods with different complexity degrees for conducting the control system evaluation procedure. For control system design, a common first principle modeling approach has been used. Then a relatively complex and detailed analytical model of helicopter has been used to resemble real flight behavior of helicopter with higher accuracy. First, the simpler model is tuned to the analytical model using system identification methods, then a hierarchical control system is designed using robust control techniques and at the end, the designed controller is applied to the detailed analytical model to evaluate the ability of control system for trajectory tracking. The results show that the simple-model-based controller can control the detailed model properly and would be used in the next steps for flight experiments.

2 Dynamic modeling of unmanned helicopter

In order to achieve effective control over unmanned helicopters, it is imperative to possess an accurate dynamic model that accurately simulates the flight behavior of the helicopter under desired conditions. Numerous established sources have put forth various methodologies for dynamic modeling of helicopters, as

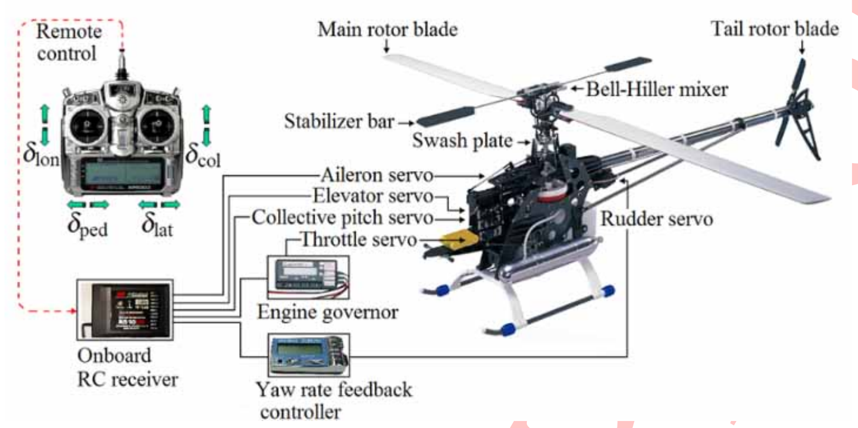

Figure 1: Control inputs (on remote control) and operating principles (servomotors and other control mechanisms) of an unmanned helicopter [4]

documented in [3, 7, 27]. In recent years, there has been a rise in the utilization of unmanned rotorcrafts, leading to the emergence of new literature pertaining to the modeling of these types of vehicles (such as [4, 26, 31]) and are being used extensively by research teams. In the field of unmanned helicopters, the process of dynamic modeling is typically carried out using analytical modeling, system identification, or a combination of both approaches. Analytical modeling, also referred to as first principle modeling, is further divided into various levels, each distinguished by the assumptions and methods employed in modeling the main rotor system and its flapping dynamics. Due to smaller scale and faster dynamics of unmanned helicopters, the most common method used for the main rotor flapping dynamic modeling is the first order approximate model [10, 23, 24].

In this paper both analytical and approximate modeling methods have been used to model a TRex-600e radio-controlled helicopter for design and evaluation of control system. A general approximate first principle model is used as the base structure for control system design because of its simplicity and capability of easier analysis. Then the designed control system is evaluated by the detailed analytical target model which is more complex and behaves closer to the actual helicopter. Of course, due to differences in states existing between the two models, an estimator is used to generate information needed for control input calculations. A hierarchical robust technique combined with a Kalman filter is used for control system design, state estimation and signal improvements. The modelling procedure can be accessed in [18].

The states utilized in the modeling process encompass the position of the helicopter's center of gravity, its translational and rotational velocities in body axes, the Euler angles representing the orientation of the helicopter, the states governing the dynamics of the main rotor flapping, and an additional state incorporated to account for the impact of gyro rate feedback from the internal PI controller. In RC helicopters, there are a total of four control inputs, with three of them affecting the main rotor and one affecting the tail rotor (Figure 1) and are represented by four normalized values ranging between -1 and 1, which exert influence on the servo motors. A comprehensive depiction of the states and control inputs can be found in Table 1. Furthermore, Figure 2 illustrates the overall diagram of the dynamic system employed for the unmanned helicopter.

The components responsible for generating force and moment in unmanned helicopters consist of the main rotor, tail rotor, fuselage, and horizontal and vertical fins. Among these, the modeling of the main rotor is of utmost significance and complexity. There are commonly three methods which are compared

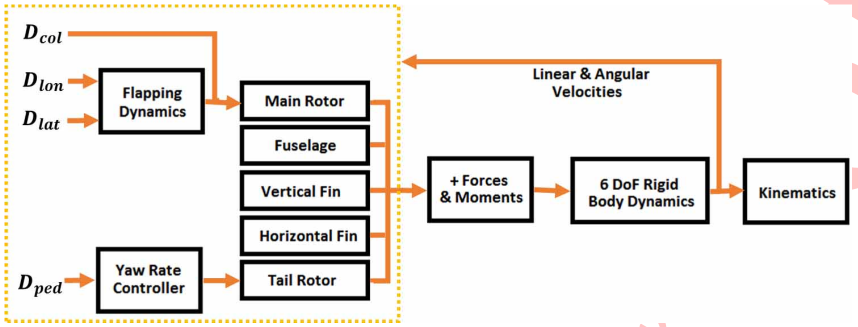

Figure 2: Schematic diagram of unmanned helicopter dynamic model [18]

Table 1: Space states and inputs of the unmanned helicopter dynamic model

Parameter	Unit	Description
$\mathbf{p}_n = [x_n\ y_n\ z_n]^T$	m	CG position (in earth frame)
$\mathbf{v}_b = [u\ v\ w]^T$	m/s	Translational velocity (in body frame)
$\omega^b_{b/n} = [p\ q\ r]^T$	rad/s	Rotational velocity (in body frame)
$\mathbf{\Theta} = [\phi\ \theta\ \psi]^T$	rad	Euler angles
$\mathbf{a} = [a_s\ b_s\ c_s\ d_s]^T$	-	Main rotor flapping angles
x_g	-	Yaw rate feedback PI controller state
δ_{lat}	-	Aileron servo input (normalized, -1 to 1)
δ_{lon}	-	Elevator servo input (normalized, -1 to 1)
δ_{col}	-	Collective servo input (normalized, -1 to 1)
δ_{ped}	-	Rudder servo input (normalized, -1 to 1)

briefly in Table 2. As mentioned before, control design strategy is implemented using the approximate model. After tuning the approximate model by system identification, the analytical model is used for designed control system evaluation. Detailed description of the used models is available in [4, 31].

Table 2: Comparison of different methods used for main rotor modeling

	Simple Model	Approximate Model	Analytical Model
Flapping Modeling	-	1st order system	Blade element theory
Flapping States	-	a_s, b_s	a_s, b_s, c_s, d_s
Stabilizer Bar Modeling	-	1st order system	1st order system
Stabilizer Bar States	-	a_s, b_s	a_s, b_s
Force and Moment Calculation	Perpendicular to TPP	Perpendicular to TPP	Analytical relations
Main Applications	• Basic analysis of navigation systems • Analysis of team operations	• Control system design • Fast and approximate simulation • Controller preliminary evaluation	• Real flight simulation • Control system evaluation • Ability of complex maneuvers check

There are different methods for determination of the required parameters. These methods contain direct measurement (for parameters such as mass, dimensions and motor rpm), performing specific tests (for CG location and moments of inertia), performing wind tunnel tests (for aerodynamic coefficients) and system identification techniques (for final tuning).

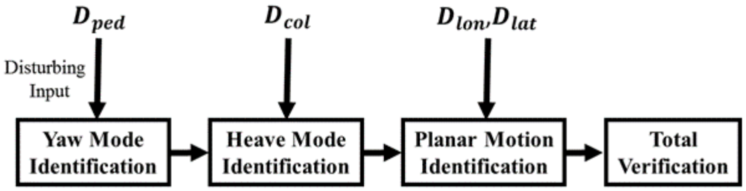

Figure 3: Diagram of dynamic system parameters' identification step-by-step procedure

3 System identification

System identification methods can generally be categorized as time domain and frequency domain. By studying the literature, it can be seen that system identification process is performed for different kinds of models such as linear model [2, 24, 37], nonlinear model [34], fuzzy or neural network type systems (such as [28, 35]) and transfer function representation of system [32]. The methods and applications of system identification for different types of UAVs have been investigated and categorized in [14]. Based on the approximate modeling method used here, a grey-box time domain system identification will be used in this research.

System identification process is performed using MATLAB System Identification Toolbox. The analytical target model which is implemented in Simulink environment, is used to generate input-output data. The control commands and states are recorded for different types of exciting signals during simulation. A grey-box model is created by using approximate model equations and parameters. Based on excited part of system dynamics, proper states have been chosen and related parameters have been tuned so that the output of approximate model closely follows the identification data generated by the analytical model. The optimization is done by system identification toolbox functions. Based on time cost and quality of identified system, Levenberg-Marquardt technique has been chosen.

Due to variety of parameters needed to be identified, system identification process is performed in 3 steps. Based on dynamic behavior of helicopter, the system is divided into yaw, heave and in-plane motions. For each of these modes, effective inputs, outputs and parameters are determined. In order to create input-output data, different excitation signals have been used for each mode. Since the dynamic system of unmanned helicopter is inherently unstable, it should be augmented with a control system in order to keep the helicopter stable and make it possible to generate data needed for identification. In this step, the control system is designed for analytical model using its own equations. A disturbing command signal is added to controller signals to excite desired modes. The excitation signals include step, pulse, rectangular and sinusoidal. A brief description of system identification procedure is presented in Figure 3.

To identify the yaw mode, r and ψ are considered as model outputs. Parameters that should be tuned are chosen in relations of tail rotor thrust calculation and internal PI controller. The yaw mode is excited by several types and values of tail rotor collective input (δ_{ped}). The results of initial and identified system responses are shown in Figure 4 and Figure 5 for pulse and rectangular disturbing input respectively. The identified parameters values are slightly different for various inputs; the mean values are finally considered as result.

In heave mode system identification, w and z_n are chosen as system outputs. It can be divided into longitudinal and lateral sections by itself, but due to similarities exist between these two motions

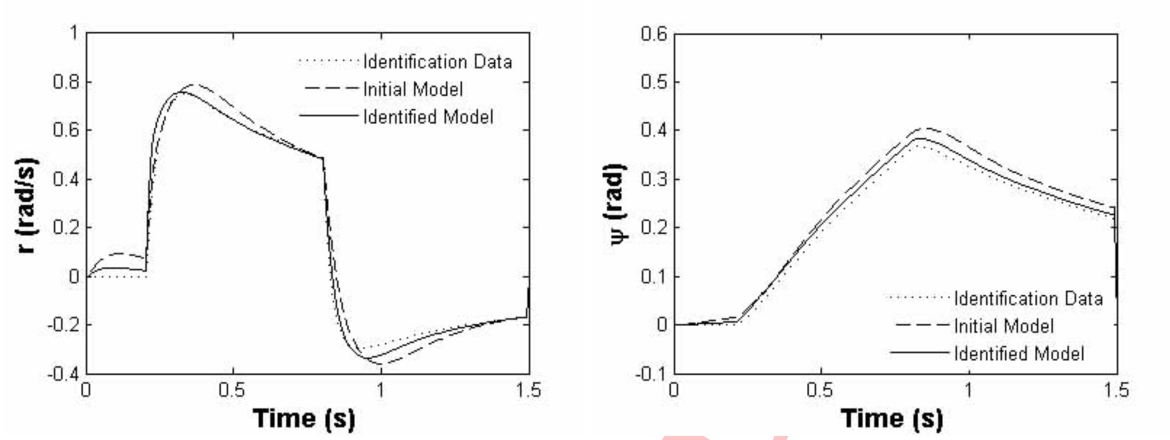

Figure 4: Yaw response of initial and identified models for pulse δ_{ped} input

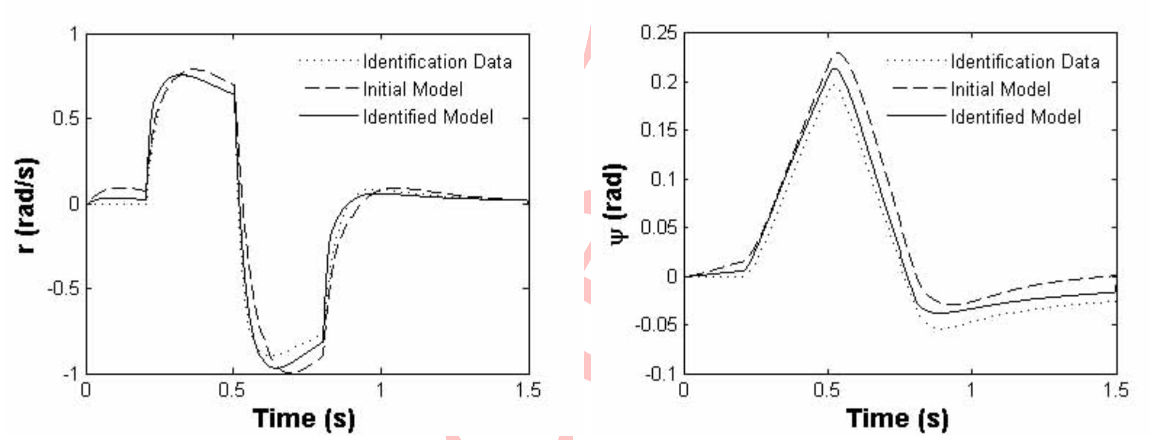

Figure 5: Yaw response of initial and identified models for rectangular δ_{ped} input

and couplings in the flapping dynamics, related parameters are identified simultaneously. The chosen parameters are mainly having great role in thrust force generation. The excitation is performed on the main rotor collective input (δ_{col}). The results for initial and identified system for some inputs are shown in Figure 6 and Figure 7 for rectangular and sinusoidal excitation inputs. As can be seen, the identified model exactly follows the identification data in this mode.

System identification of planar motion is the most sophisticated part. The output states are chosen as u, v, ϕ and θ. The parameters to be identified include coefficients of flapping dynamics equations and main rotor's longitudinal and lateral forces and moments. The excitation signal is considered in cyclic inputs (longitudinal cyclic, lateral cyclic and both at the same time) with different types and values. Again, mean values of resulted identified parameters are considered as final values. Some sample responses of the system are shown in Figure 8 and Figure 9.

As can be seen in the above results, the identified model follows the data acceptably. To ensure performance on general situation, the identified model is verified by using some general motion data. For this purpose, some data sets are generated by simultaneously exciting all input signals and are compared with the response of identified system to the same inputs. Figure 10 shows an example of these verifica-

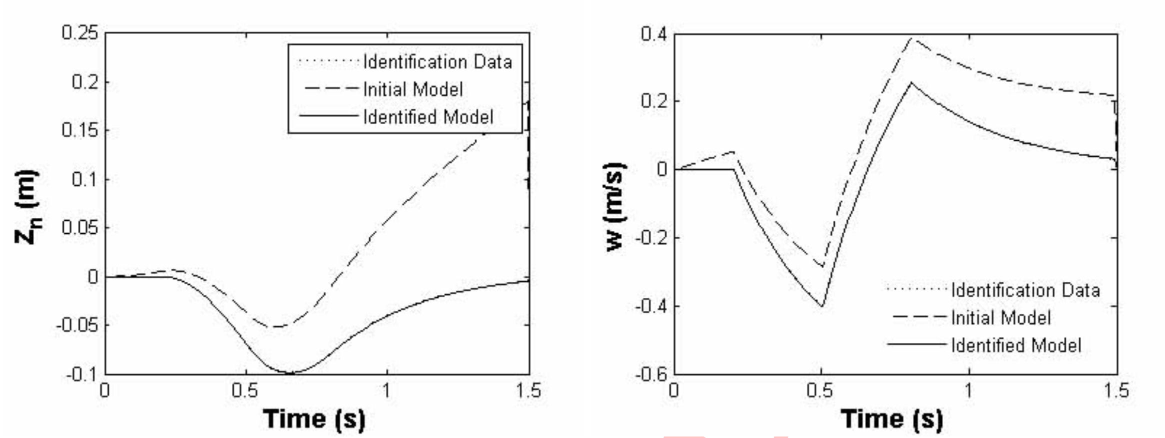

Figure 6: Heave response of initial and identified models for rectangular δ_{col} input

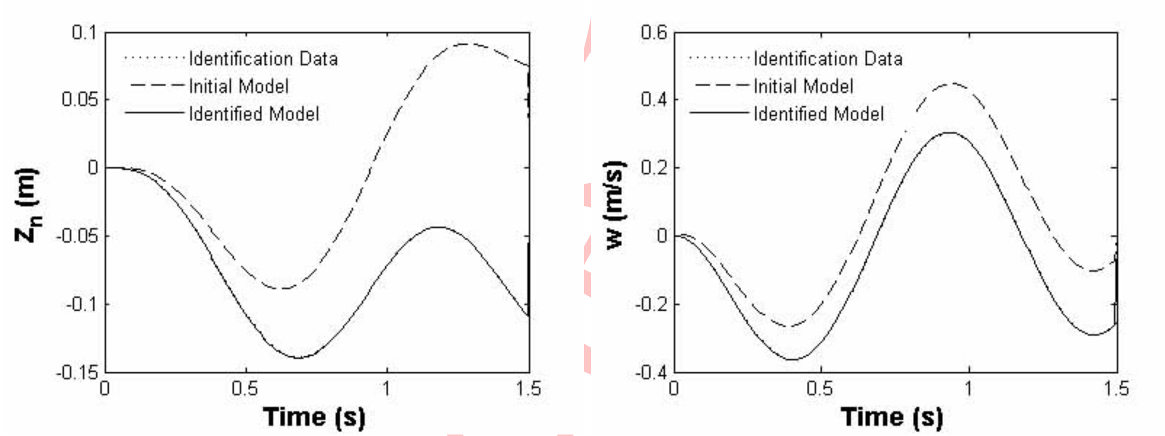

Figure 7: Heave response of initial and identified models for sinusoidal δ_{col} input

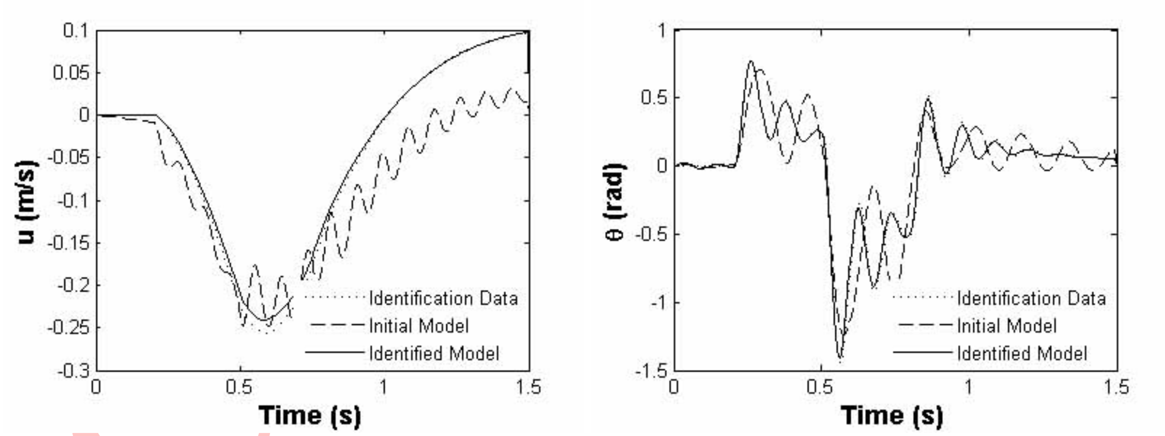

Figure 8: Longitudinal response of initial and identified models for rectangular δ_{lon} input

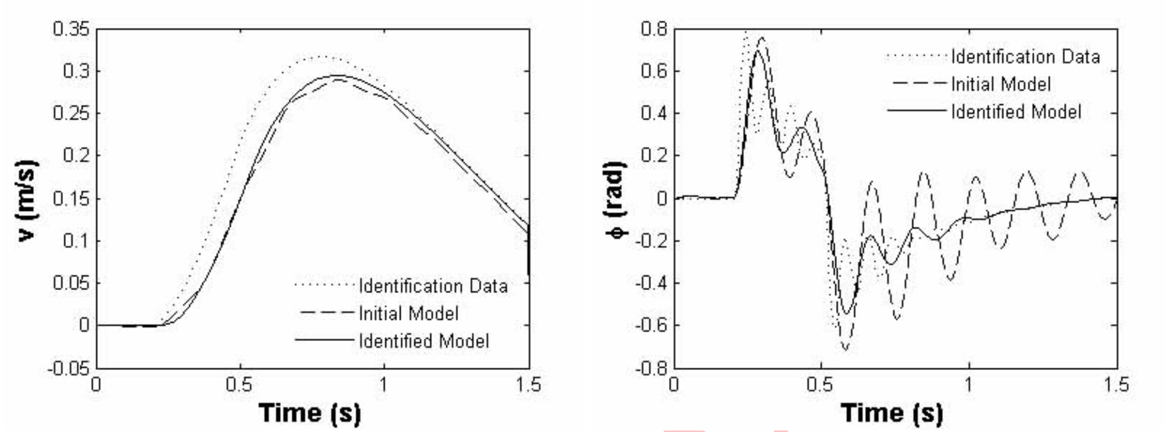

Figure 9: Lateral response of initial and identified models for sinusoidal δ_{lat} input

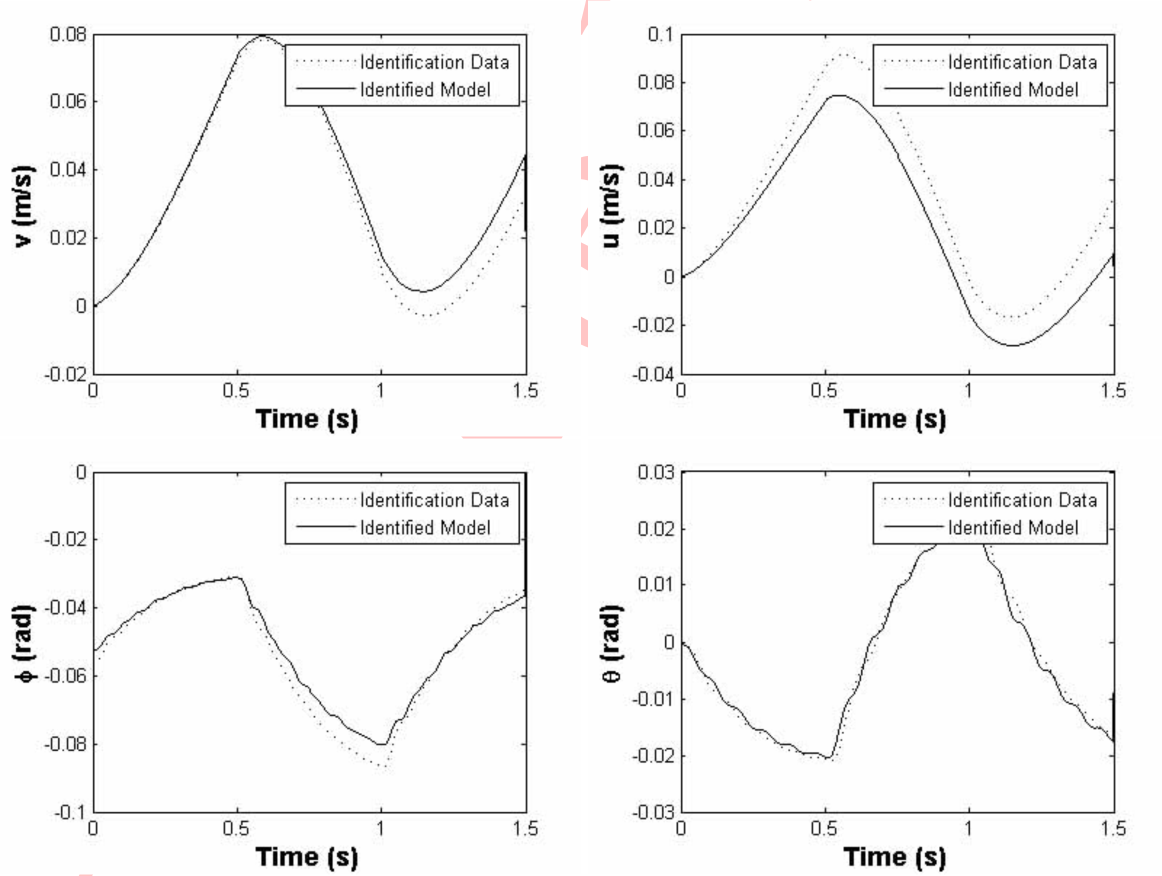

Figure 10: Custom response of identified model for a general motion of helicopter

tion data sets. As the results show, the identified approximate model successfully follows the behavior of target data and can be used in the next steps.

4 Control system design

Searching various references shows that applied control systems for unmanned helicopters cover a wide range of techniques and methods. Comprehensive survey studies can be found in [1, 17]. Although the application of linear, hierarchical, and robust ([13])control methods is more common due to the inherent dynamics and existing limitations of unmanned helicopters, as reported in [4, 5, 8, 18, 22], other control approaches such as optimal control [38], nonlinear control [25], adaptive control [11], and fuzzy control [15] have also recently attracted considerable attention. The performance of the designed controller is usually improved by employing robust techniques, since simplifications, uncertainties, and unmodeled dynamics affect the system behavior. In order to choose a proper control method, many factors should be considered, including the required precision and model complexity, computational cost, limitations on parameters and inputs, the considered flight envelope and desired maneuvers, implementation feasibility, assurance of satisfactory performance, and minimization of operational risks. In [16], the implementation of both hardware and software aspects of a control system for an unmanned aerial vehicle testbed is investigated. This study examines various practical considerations associated with implementing a real UAV control system.

Because of the large number of states and the complexity of the helicopter dynamic model, the control system is partitioned into hierarchical layers. One layer addresses the fast rotational dynamics of the helicopter, while another layer is responsible for the slower translational dynamics, as illustrated in Figure 11. Controllers corresponding to each layer are designed separately. . Outer layer includes position and linear movements of the helicopter and generate the desired values for helicopter Euler angles for tracking the predefined trajectory. In inner layer, angular dynamics is controlled and track the commanded values.

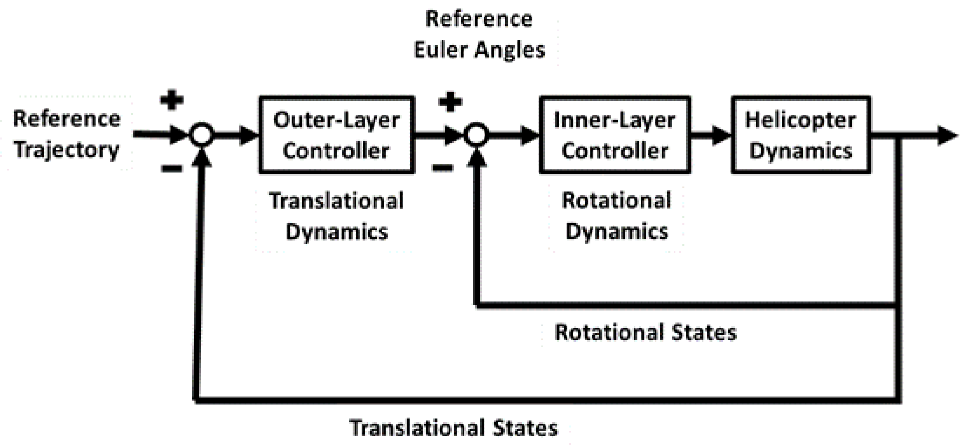

Figure 11: System's dynamic separation to fast (inner loop, rotational dynamics) and slow (outer loop, translational dynamics) parts and the proposed control structure [18]

The controllers for the inner and outer layers of a helicopter can be designed using different methods. The inner layer, which is crucial for stability and control, takes into account external disturbances and their impact on angular dynamics. For this reason, robust control methods like H_∞ have been employed. The outer layer, responsible for helicopter movements and maneuvers, can be selected based on the desired maneuvers and required performance quality. In this study, the PRT (Robust and Perfect Tracking) method was utilized for the outer layer controller.

The design process of the control system begins by determining the hover trim operating point, specifying the system states and inputs, and linearizing the system around this hover point. Once the state and input matrices (A and B) of the inner layer system are calculated, the feedback law for the inner layer control is obtained by defining the necessary parameters for the robust H_∞ controller design and solving its algebraic Riccati equation. Additionally, a Kalman filter is developed for the purpose of estimating the main rotor flapping angles and finalizing the feedback states. Afterwards, the closed loop system of the inner layer is treated as a new system and the transfer function connecting the helicopter orientation angles with the corresponding linear accelerations is determined using perturbation techniques. This matrix is then utilized to convert the desired linear accelerations into appropriate Euler angles that will induce those accelerations. Finally, the control system for the outer layer is designed to finalize the overall control system.

4.1 Inner layer control system design

The inner layer dynamic system has been linearized around the hover trim operating point using the single perturbation theory and its accompanying numerical methods. The states and inputs considered for this layer are given in (1) and (2), which are defined as the difference between the actual and trim values:

$$x^T = [\phi, \theta, p, q, a_s, b_s, c_s, d_s, r, \delta_{ped,int}, \psi] \tag{1}$$

and

$$u^T = [\delta_{lat}, \delta_{lon}, \delta_{ped}]. \tag{2}$$

Once linearized, the system's state and input matrices ($A_{11\times 11}$ and $B_{11\times 3}$) can be derived. Consequently, the inner layer's linear dynamic system can be regarded as a relationship between these matrices as relation (3):

$$\dot{x} = Ax + Bu + B_{col}(\delta_{col} - \delta_{col,0}). \tag{3}$$

In the proposed hierarchical structure, δ_{col} is assumed to be determined by outer layer controller, thus it will not be included in inner layer control system inputs. Actually, the designed inner layer controller should be able to counter the effects of δ_{col} changes as external disturbance source. System structure needed for designing robust control system is as (4):

$$\Sigma : \begin{cases} \dot{x} = Ax + Bu, \\ y = C_1 x, \\ h_{out} = C_{out} x, \\ h_{in} = C_2 x + D_2 u. \end{cases} \tag{4}$$

In (4) y, h_{out} and h_{in} are measurable system output and combination of states and inputs considered for controller design respectively. The output y can be defined as (5) matrix format:

$$y = [\phi, \theta, p, q, r, \psi]_{actual} - [\phi, \theta, p, q, r, \psi]_{trim} = C_1 x. \tag{5}$$

Outputs considered as targets which should be stabilized and controlled, will be defined in h_{out} term. Based on system configuration, these states include Euler angles of the helicopter. According to (6), h_{out} is difference of these states with their trim values:

$$h_{out} = [\phi, \theta, \psi]_{actual} - [\phi, \theta, \psi]_{trim} = C_{out} x. \tag{6}$$

Actually, the primary objective of inner loop control system is to reach h_{out} to zero. To address constraints during control system design, term h_{in} is defined as (7), in which C_2 and D_2 matrices include weighting parameters to tune importance and effects of different states and inputs:

$$h_{in} = C_2 x + D_2 u. \tag{7}$$

The elements of the weight matrices are determined through a process involving multiple trials. The P matrix is determined from solving algebraic Riccati equation of H_∞ controller (relation (8)). By calculation of control input coefficients (as relations (9) and (10)), control feedback law can be obtained from equation (11):

$$A^T P + PA + C_2^T C_2 - (PB + C_2^T D_2)(D_2^T D_2)^{-1}(D_2^T C_2 + B^T P) = 0, \tag{8}$$

$$F = -(D_2^T D_2)^{-1}(D_2^T C_2 + B^T P), \tag{9}$$

$$G = -[C_{out}(A + BF)^{-1} B]^{-1}, \tag{10}$$

$$u = Fx + G(r - h_{out}). \tag{11}$$

The reference signal vector for helicopter Euler angles, denoted as r, is generated by the outer layer control system as (12):

$$r = [\phi_r, \theta_r, \psi_r]. \tag{12}$$

As can be seen clearly, calculation of control input from equation (11) needs the values of all system states. So as mentioned above, unknown system states should be estimated by designing proper Kalman filter. Having the state and input matrices of the system, Kalman filter can be designed using relations (13)-(15):

$$\dot{P}_{KF} = P_{KF} A^T + A P_{KF} + R_1 - P_{KF} C^T R_2^{-1} C P_{KF}, \tag{13}$$

$$K = P_{KF} C^T R_2^{-1}, \tag{14}$$

$$\dot{\hat{X}} = A\hat{X} + K(Y - C\hat{X}) + BU. \tag{15}$$

In these relations, matrices R_1 and R_2 and initial value of P_{KF} matrix is determined after several trial and errors. After conducting several simulations, it becomes evident that the Kalman filter coefficient converges to a constant matrix within a short period of time. This constant matrix can then be utilized to estimate the states by disregarding the dynamics of the Kalman filter system. The results demonstrate that the designed Kalman filter performs optimally when employing both the dynamic and static coefficient matrix.

4.2 Outer layer control system design

By appropriately defining the inputs and outputs and linearizing the inner closed loop system, the required information for designing the outer controller can be obtained following the design of the inner layer controller. Doing this will lead to (16) matrix equation which relates the inner layer reference commands to desired body linear accelerations:

$$\begin{bmatrix} \delta_r \\ \phi_r \\ \theta_r \end{bmatrix} = (G_{in,cl,0})^{-1} \begin{bmatrix} a_{xb,r} \\ a_{yb,r} \\ a_{zb,r} \end{bmatrix}. \tag{16}$$

The primary function of the outer layer controller is to determine the required linear accelerations based on predefined position, velocity, and acceleration signals. It is also responsible for calculating the reference commands that will be used by the inner layer controller. By studying the linear dynamics and frequency response of a helicopter, it is evident that for near hover flight, the longitudinal, lateral, and vertical flight modes can be separated. This finding is supported by [4]. For ease of implementation, the control system is developed independently for each of these modes. One such mode is the dynamic system governing the longitudinal movement of the helicopter such as (17):

$$\begin{bmatrix}\dot{x}_n\\ \dot{u}_n\end{bmatrix}=\begin{bmatrix}0&1\\0&0\end{bmatrix}\begin{bmatrix}x_n\\ u_n\end{bmatrix}+\begin{bmatrix}0\\1\end{bmatrix}a_{x_n}. \tag{17}$$

In (17) x_n, u_n and a_{x_n} are longitudinal position, velocity and acceleration respectively. By adding the reference trajectory information of longitudinal movement ($x_{n,r}$, $u_{n,r}$ and $a_{x,n,r}$) and its dynamics, the augmented system can be defined as (18)-(20):

$$\dot{x}_x=\begin{bmatrix}\dot{x}_{n,r}\\ \dot{u}_{n,r}\\ \dot{a}_{x,n,r}\\ \dot{x}_n\\ \dot{u}_n\end{bmatrix}=\begin{bmatrix}0&1&0&0&0\\0&0&1&0&0\\0&0&0&0&0\\0&0&0&0&1\\0&0&0&0&0\end{bmatrix}\begin{bmatrix}x_{n,r}\\ u_{n,r}\\ a_{x,n,r}\\ x_n\\ u_n\end{bmatrix}+\begin{bmatrix}0\\0\\0\\0\\1\end{bmatrix}a_{x_n}+\begin{bmatrix}0\\0\\1\\0\\0\end{bmatrix}\dot{a}_{x_n}, \tag{18}$$

$$y_x=x_x, \tag{19}$$

$$e_x=\begin{bmatrix}-1&0&0&1&0\end{bmatrix}x_x. \tag{20}$$

In (20) e_x is the error of helicopter longitudinal position and control system is aimed to reach that to zero. Using RPT control method [4,5], equation (21) can be derived for required longitudinal acceleration of system:

$$a_{x,n}=\begin{bmatrix}-\frac{\omega_{n,x}^2}{\varepsilon_x^2}&-\frac{2\zeta_x\omega_{n,x}}{\varepsilon_x}&\frac{\omega_{n,x}^2}{\varepsilon_x^2}&\frac{2\zeta_x\omega_{n,x}}{\varepsilon_x}&1\end{bmatrix}x_x. \tag{21}$$

In (21) ε_x is the tuning parameter and $\omega_{n,x}$ and ζ_x are the natural frequency and damping ratio of the final closed loop dynamic system respectively. In a similar manner, the method and relationship can be applied to the lateral and vertical modes. Once the required accelerations have been determined and transferred into the body axes, the reference command signal of the inner layer can be calculated using equation (16).

5 Control implementation results and discussion

In order to evaluate the control system designed based on identified approximate model, it is applied on the detailed analytical model. Since the performance of controller highly depends on the ability of the Kalman filter to correctly estimate the states, designed estimator has been checked in different situations. The comparison between estimated and actual data for a specific flight condition is shown in Figure 12. Obtained results show that the performance of design Kalman filter is completely acceptable.

The hovering ability and stability performance of the controller were evaluated by examining the system response under various untrimmed initial conditions. The performance of identified model controller is compared with the base analytical model controller which is designed with the same procedure.

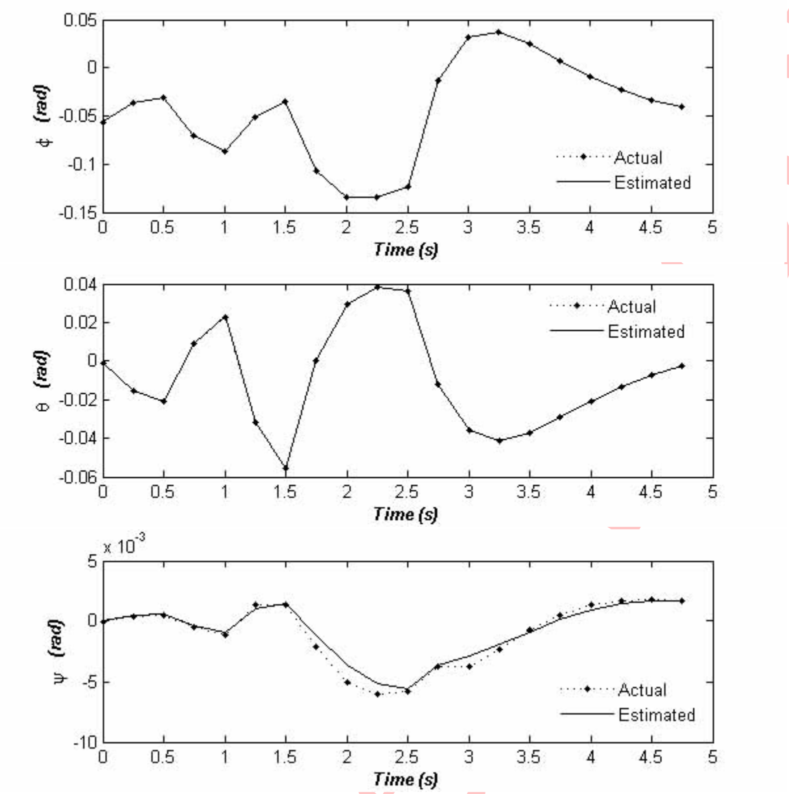

Figure 12: The comparison of actual and Kalman filter estimated Euler angles for a general movement

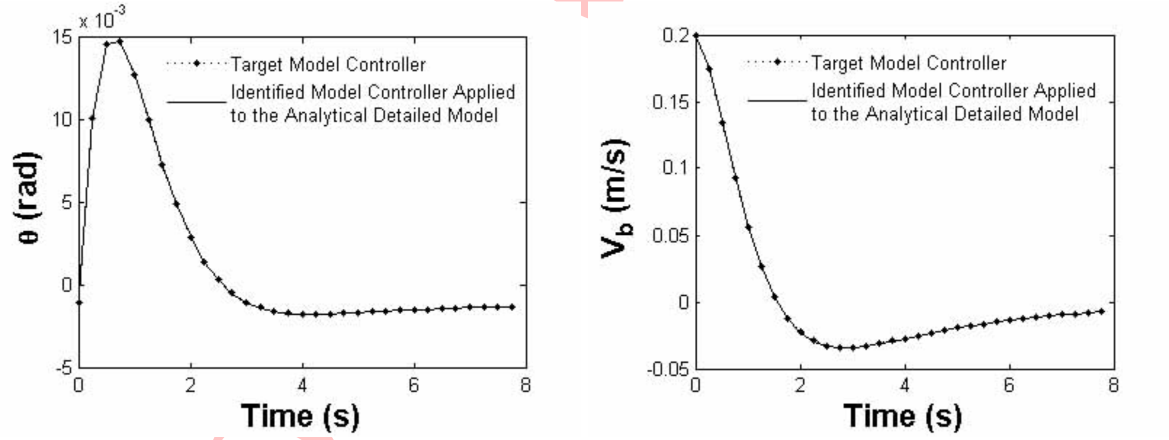

Figure 13: The pitch and longitudinal velocity response of the controlled system to 0.2 m/s initial longitudinal velocity

Figure 13 and Figure 15 show the related system state changes after longitudinal velocity and excess roll angle initial conditions respectively. Also, the control effort of longitudinal input related to system response to proposed longitudinal velocity initial condition is shown in Figure 14 which demonstrate the action imposed on the system to bring it back to the zero-velocity hover flight.

Both controllers' performance is completely matched when the disturbance is considered for linear

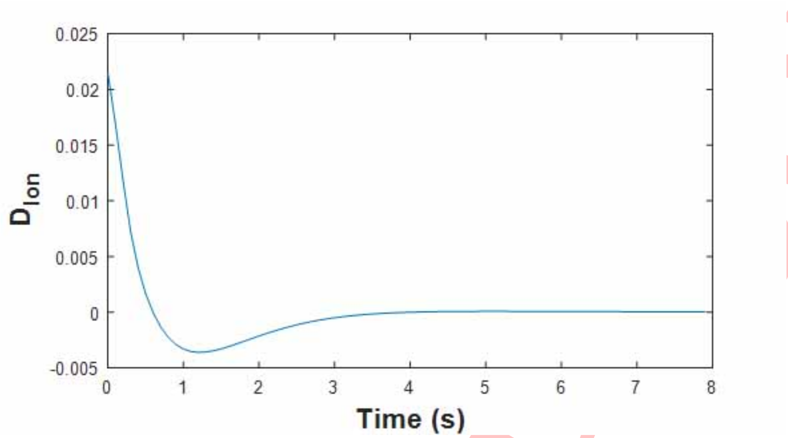

Figure 14: The control effort of longitudinal input for the response of controlled system to 0.2 m/s initial longitudinal velocity

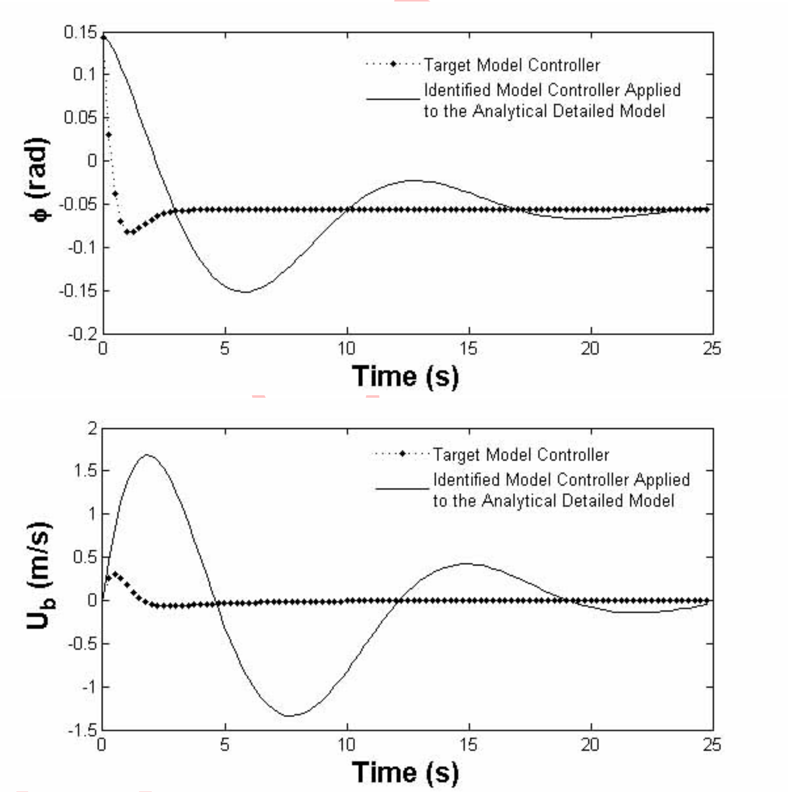

Figure 15: The roll and lateral velocity response of the controlled system to 0.2 rad excess roll angle in initial condition

velocities. But identified controller's ability is decreased when disturbances are inserted on rotational dynamics of helicopter. It is mainly because of the much simpler model used for the main rotor flapping dynamics which increases time response of the system against changes in these modes. The results show that control systems are able to stabilize the system (with different time constants) and hovering ability

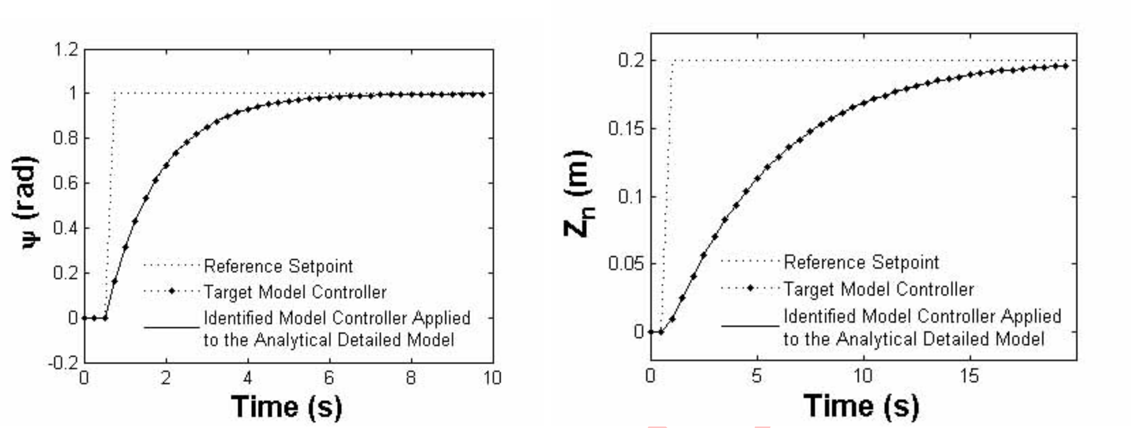

Figure 16: The response of the controlled system exposed to set point change in yaw angle (left) and height (right) separately

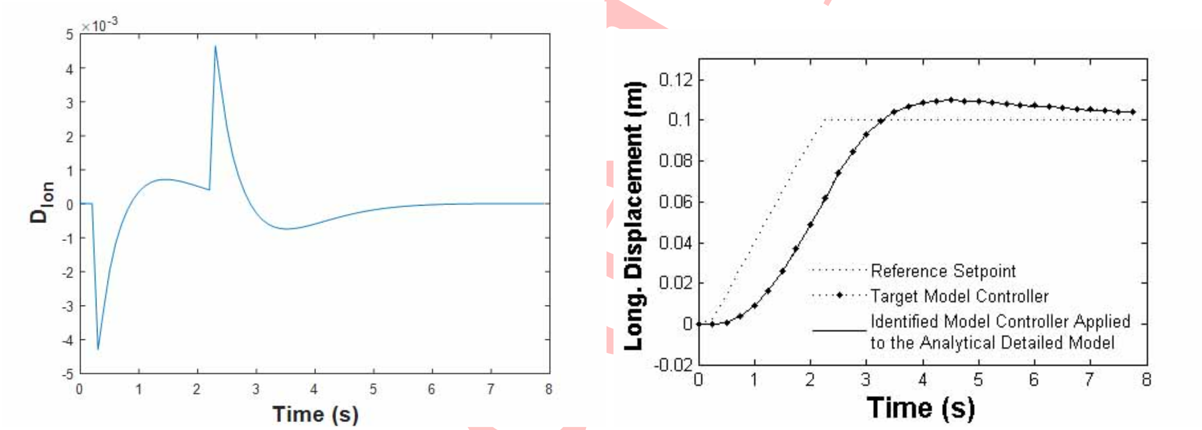

Figure 17: The control effort and longitudinal displacement response of the controlled system exposed to predefined longitudinal trajectory

of both controllers is totally acceptable.

Various reference trajectories are employed to assess the performance of the control system. These trajectories consist of following set-points, predetermined positions and velocities, as well as simple closed loop paths. Several simulated outcomes are provided below, arranged in order of increasing complexity. To commence the evaluation, the yaw and heave motions of the helicopter are analyzed. Figure 16 illustrates the system's response to adjustments in the set-point for the helicopter's yaw angle and height. Generally, the controllers have the same performance while just these two modes are interfered which is acceptable.

Next, we examine the control system's capability to track a predetermined position and velocity in a single direction. This is demonstrated in Figure 17, where both controllers exhibit identical responses for specific longitudinal movements. By simulating additional trajectories of this nature, it becomes apparent that the control system performs adequately when the timescale of these maneuvers closely aligns with the system's time response.

Figure 18 depicts the results of the system's response when attempting to track a predetermined

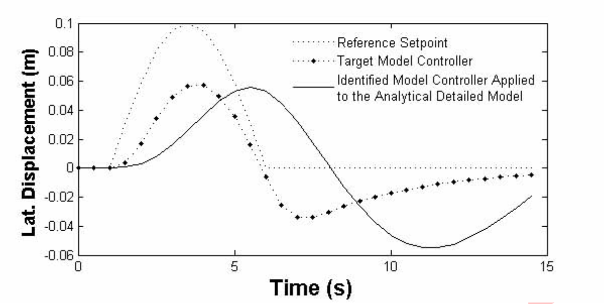

Figure 18: The lateral displacement response of the controlled system exposed to predefined sinusoidal lateral trajectory

sinusoidal trajectory in the lateral direction. It is evident from the graph that, due to the slower time response of the system, it is unable to reach the maximum point of the trajectory and begins to decrease prematurely as the trajectory enters the falling section. This issue is exacerbated in the identified model controller, as it has an even slower response. Based on these findings, we can conclude that the designed control systems are unable to accurately follow trajectories that require a relatively faster response compared to their time constant.

Lastly, an investigation is conducted to assess the capability of control systems to accurately track more complex trajectories. The simulation results for rectangular and nose-in circular trajectories are shown in Figure 19 and Figure 20. The findings indicate that both control systems perform adequately in accurately tracking linear trajectories with waypoints, as long as the system time response is comparatively slower. However, when there is a continuous change in the desired trajectory, issues arise once again. These problems stem primarily from the control systems' low bandwidth, resulting in sluggish responses.

Based on the comparison and considering the characteristics of unmanned helicopters, it can be concluded that the control system created using the target analytical model performs better overall, although the identified model controller also performs adequately. However, both control systems encounter difficulties when the desired trajectory changes rapidly, exceeding their time response capabilities influenced by the bandwidth of the closed loop system. Nevertheless, this investigation successfully achieves hovering ability. Improving the tracking results would require greater focus and effort in defining compatible trajectories in terms of position, velocity, and acceleration.

6 Conclusion

Due to the importance of reaching the control ability of unmanned helicopters, the problem of system identification and control of these UAVs are investigated in this paper. Because of helicopter dynamic system complexity and simplification used during modeling stage, using the same model for control system design and evaluation seems to be not proper. So analysis has been performed in a multilevel simulation environment designed for these purposes. A complex analytical dynamic model of unmanned helicopter is used as the target case and an approximate model is used for designing the control system.

The main difference between the target and the approximate model is in the main rotor and its flapping dynamics modeling. In the target model, an analytical method (based on blade element theory) is used to model flapping dynamics which lead to a complex 2nd order dynamic system with 6 states. Force and moments components are calculated by nonlinear complex equations directly derived from the blade

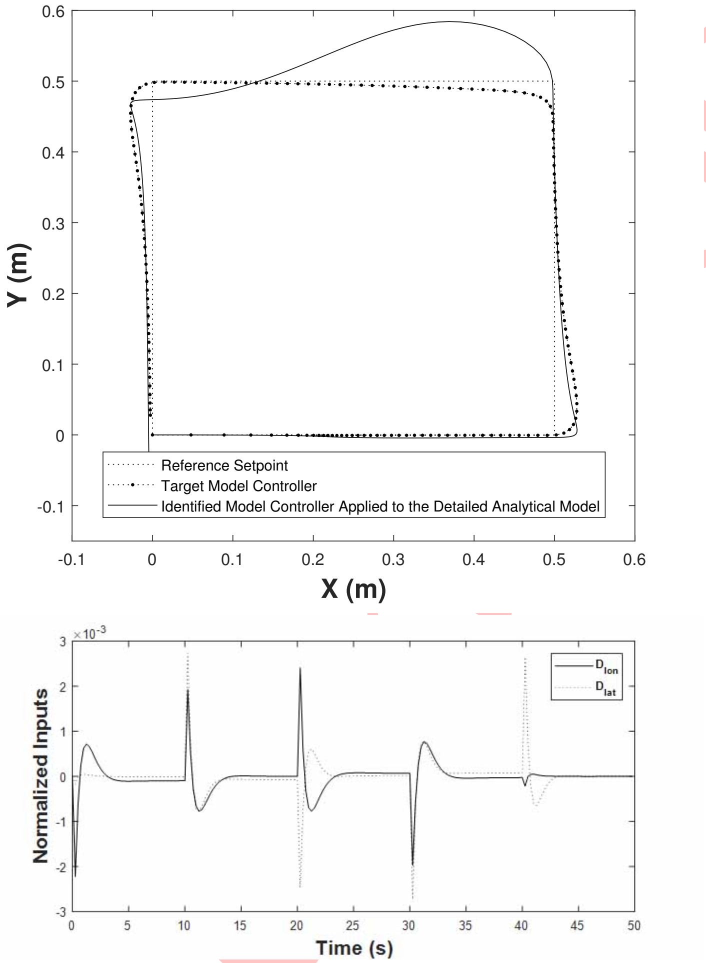

Figure 19: The X-Y position response and longitudinal and lateral control efforts of the controlled system to track rectangular CCW trajectory (from origin)

element theory. In the approximate model, flapping dynamics treats as a coupled 1st order system with 2 states (as flapping angles) and the main rotor thrust force is assumed to be perpendicular to TPP. Other elements such as stabilizer bar, swashplate and Bell-Hiller mechanism, tail rotor, fuselage and vertical and horizontal tails are considered in both models.

System identification method has been used to tune the approximate model parameters to make sure that it resembles the complex analytical model behaviors. System identification procedure is performed by MatLab System Identification Toolbox. Identification is divided to yaw mode, heave mode and planar motion and verification is done by custom flight data. It can be seen that the approximate model can properly predict analytical model behavior after system identification and can be used for control purposes.

The control system is designed in hierarchical configuration due to numerous states of the dynamic system. A robust H_∞ control system is designed for the inner layer which controls the rotational dynamics of helicopter and tracks the reference values of Euler angles and has an important role on the stability of system. The outer layer controller is designed by RPT control method and generate the proper reference

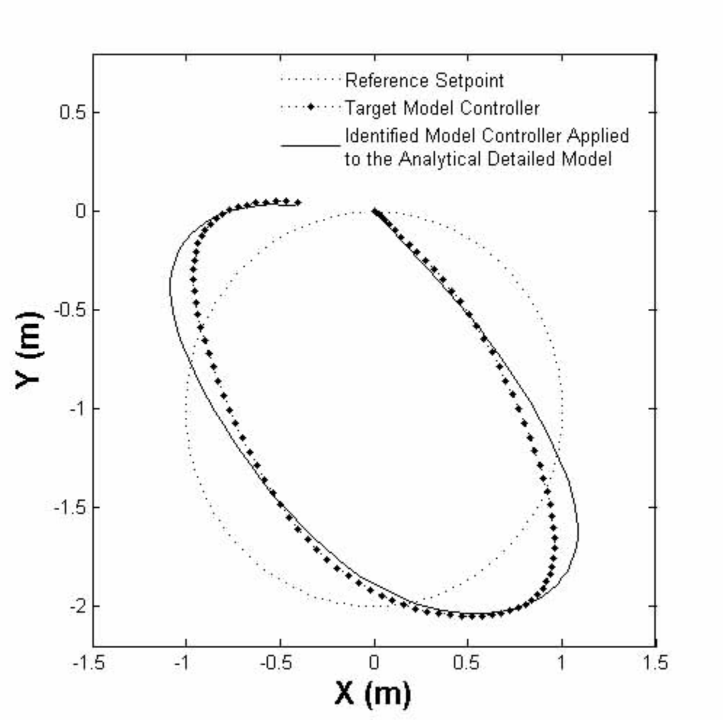

Figure 20: The X-Y position response of the controlled system to track circular CW trajectory (nose-in from origin)

values for the inner layer to track the predefined trajectories. Also, a Kalman filter is designed to estimate unavailable states and enhance signal properties.

After designing a hierarchical robust control system for identified approximate model, it is implemented on the analytical detailed complex target model and its performance is evaluated. The results show that the control system designed based on identified approximate model, can properly control the complex analytical model. Its stability and waypoint tracking is acceptable, but more complex trajectory tracking which needs high bandwidth for the closed loop system should be improved. According to simulation results, it can be concluded that the introduced approximate model and designed control system, and also the multi-layer simulator based on the analytical model can be used as a primary test bed which can serve as pre-flight experiments with relatively lower risk of fault or dangers.

The next step of this investigation would be implementing the same procedure for actual unmanned helicopter. This needs the design and implementation of sensors and avionic board for the proposed TRex600 helicopter and performing actual test on the test bed or free flight experiments. Additionally control methods used for the inner and outer control layer should be analyzed more so that performance of control system improves for faster responses and more complex maneuvers.

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Conflict of Interest

The authors declare no competing financial interests or personal relationships that could have influenced this work.

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