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The influence of S-semiembedded subgroups

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Abstract. A subgroup H of a finite group G is said to be S-embedded in G if G has a normal subgroup T such that HT is S-permutable in G and $H \cap T \leq H_{sG}$. H is said to be S-semiembedded in G if there is an S-permutable subgroup T of G such that HT is S-permutable in G and $H \cap T \leq H_{\bar{s}G}$; where $H_{\bar{s}G}$ is an S-semipermutable subgroups of G contained in H. In this paper, we present some new characterization of supersolubility and nilpotency of finite groups under S-semiembedding property.

Keywords: S-semiembedded subgroup, nilpotent group, supersolvable group.
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1 Introduction

Throughout this paper, all groups are finite. We use conventional notions and notation as in Doerk and Hawkes [3]. Let $|G|$ denote the order of G, $\pi(G)$ denote the set of all primes dividing $|G|$ and $Z(G)$, $\Phi(G)$, $F(G)$ and $F^*(G)$ is the center, the Frattini subgroup, the Fitting and the generalized Fitting subgroup of G respectively. Let $\mathcal{F}$, $\mathcal{U}$, $\mathcal{N}$ and $\mathcal{N}_p$ will denote a formation, the class of all supersoluble, nilpotent and p-nilpotent groups respectively. We use $G^{\mathcal{N}}$ as the $\mathcal{N}$-residual of G and $G^{\mathcal{N}_p}$ as the $\mathcal{N}_p$-residual of G. The symbol $\mathcal{A}(p-1)$ denotes the formation of all abelian groups of exponent dividing $p-1$.

Two subgroups H and K of a group G are said to be permutable, if $HK = KH$. If H permutes with all Sylow subgroups of G, then we say H is S-permutable subgroup of G. This concept was introduced by Kegel [10] and has been studied extensively by many authors. A well known result of Kegel [3] shows that S-permutable subgroups are subnormal. On the other hand, Srinivasan [16] proved that a finite group G is supersolvable if all maximal subgroups of every Sylow subgroups of G are S-permutable in G. Various generalizations of permutability and S-permutabliliy have been defined that in particular, mentioned by "semipermutability" and "S-semipermutability". A subgroup H is semipermutable in G if H permutes with all p-subgroups

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of G for primes p not dividing $|H|$, and less restrictively, H is S-semipermutable in G if H permutes with every Sylow p-subgroup of G for primes p not dividing $|H|$. In [9], Isaacs proved that H^G is solvable if H is an S-semipermutable nilpotent Hall subgroup of G.

Wang in [19] introduced the following concept: A subgroup H of G is said to be c-normal in G, if G has a normal subgroup T such that $G = HT$ and $H \cap T \leq H_G$; where H_G is the normal core of H in G.

As a generalization of c-normal subgroup, the concept of S-embedded subgroup was extended by Guo et al in [5]: A subgroup H of G is said to be S-embedded in G, if G has a normal subgroup T such that HT is S-permutable in G and $H \cap T \leq H_{sG}$, where H_{sG} denotes the largest S-permutable subgroup of G contained in H. Using this idea, many authors gave new definitions and proved many results about soluble groups and nilpotent groups [6, 13, 14]. Mao et al in [12] introduced the following concept:

Definition 1. *Let H be a subgroup of G. H is called S-semiembedded in G if G has an S-permutable subgroup T such that HT is S-permutable in G and $H \cap T \leq H_{\overline{s}G}$; where $H_{\overline{s}G}$ is an S-semipermutable subgroups of G contained in H.*

It is easy to observe that every S-embedded or S-semipermutable subgroup of G is S-semiembedded in G. In general, a S-semiembedded subgroup of G does not need to be S-embedded or S-semipermutable. For instance:

Example 1. Let $G = A_5$, the alternative group of degree 5. A_4 is S-semiembedded subgroup of G, but it is not S-embedded in G.

Example 2. Let $G = S_4$, then $H = \langle (1\ 2) \rangle$ is S-semiembedded subgroup of G but it is not S-semipermutable in G.

This paper is going to investigate the influence of the S-semiembedded subgroups on the structure of finite groups.

2 Preliminaries

Lemma 1. *Let G be a group. H is an S-permutable subgroup of G and N is a normal subgroup of G.*

1. *[10] H is subnormal in G.*
2. *[10] If $H \leq K \leq G$, then H is S-permutable in K.*
3. *[2] If $M \leq G$, then $H \cap M$ is S-permutable in M.*
4. *[21] If H is a p-group, then $H \leq O_p(G)$.*
5. *[15] If H is a p-group, then $O^p(G) \leq N_G(H)$.*
6. *[10] HN/N is S-permutable in G/N*
7. *[15] If K is S-permutable in G, then $H \cap K$ is S-permutable in G.*

8. *[11] If H is Hall subgroup of G, then $H \trianglelefteq G$.*

Lemma 2. *Let G be a group and H is an S-semipermutable subgroup of G and N is a normal subgroup of G.*

1. *[22] If $H \leq K \leq G$, then H is S-semipermutable in K.*
2. *[22] If H is a p-group for some prime $p \in \pi(G)$, then HN/N is S-semipermutable in G/N*
3. *[22] If $H \leq O_p(G)$, then H is S-permutable in G.*
4. *[11] If H is a p-group for some prime $p \in \pi(G)$, then $H \cap N$ is S-semipermutable in G.*

Lemma 3 ([12], Lemma 2.3)**.** *Let H be a S-semiembedded subgroup of G and N be a normal subgroup of G.*

1. *If $H \leq K \leq G$, then H is S-semiembedded in K.*
2. *If H is a p-group for some prime $p \in \pi(G)$ and $N \leq H$, then H/N is S-semiembedded in G/N.*
3. *If H is a p-group for some prime $p \in \pi(G)$ and N is a normal p'-group, then HN/N is S-semiembedded in G/N.*

Lemma 4 ([7], III, 5.2 and IV, 5.4)**.** *Suppose that G is a minimal non-nilpotent group.*

1. *G has a normal Sylow p-subgroup P for some prime p and $G = PQ$, where Q is a non-normal cyclic q-subgroup for some prime $q \neq p$.*
2. *$P/\Phi(P)$ is a minimal normal subgroup of $G/\Phi(P)$.*
3. *The exponent of P is p or 4.*

Lemma 5. *Let H be an S-semiembedded subgroup of G and N a normal subgroup of G such that $H \leq N \trianglelefteq G$ then G has a S-permutable subgroup L contained in N such that HL is S-permutable and $H \cap L \leq H_{\bar{s}G}$.*

Proof. It can be deduced directly by Definition 1 and Lemma 3. □

Lemma 6 ([8], Chapter X)**.** 1. *If N is normal in G, then $F^*(N) = F^*(G) \cap N$.*

2. *If N is normal in G and $N \leq F^*(G)$, then $F^*(G)/N \leq F^*(G/N)$.*
3. *$F(G) \leq F^*(G) = F^*(F^*(G))$. If $F^*(G)$ is soluble, then $F^*(G) = F(G)$.*
4. *$F^*(G) = F(G)E(G)$ and $F(G) \cap E(G) = Z(E(G))$ where $E(G)$ is the layer of G.(see [8], p. 128).*
5. *$C_G(F^*(G)) \leq F(G)$.*

Lemma 7 ([18], Lemma 2.2)**.** *Let E be a normal p-subgroup of a group G. If $E \leq Z_{\mathcal{U}}(G)$, then $(G/C_G(E))^{\mathcal{A}(p-1)} \leq O_p(G/C_G(E))$.*

3 Main Results

Theorem 1. *Let N be a normal subgroup of G such that G/N is nilpotent. If every minimal subgroup of N is contained in $Z_\infty(G)$ and every cyclic subgroup of N with order 4 is S-semiembedded in G or lies in $Z_\infty(G)$, then G is nilpotent.*

Proof. Suppose that G is a counterexample of minimal order. We will derive a contradiction in several steps.
Step (1). G is a minimal non-nilpotent group.
Let H be a proper subgroup of G. By nilpotency of G/N, we have $H/(H \cap N) \cong HN/N$ is nilpotent. If H_1 be a minimal subgroup of $H \cap N$, then $H_1 \leq H \cap Z_\infty(G) \leq Z_\infty(G)$. By Lemma 3, every cyclic subgroup of $H \cap N$ is S-semiembedded in H or lies in $Z_\infty(H)$. Hence every proper subgroup H of G satisfies the hypothesis of the theorem, thereby H is nilpotent.
Step (2). $G = PQ$, where P is a normal Sylow p-subgroup of G and Q is a non-normal cyclic q-subgroup of G, $P/\Phi(P)$ is a chief factor of $G/\Phi(P)$ and the exponent of P is p or 4.
It is result of Step (1) and Lemma 5.
Step (3). $p = 2$ and every cyclic subgroup of $P \leq N$ of order 4 is S-semiembedded in G.
We have $G/(P \cap N)$ is nilpotent. Thus $Q(P \cap N) = Q \times (P \cap N)$ and $Q \; ch \; Q(P \cap N)$. By $G/(P \cap N) = P/(P \cap N) \times Q(P \cap N)/(P \cap N)$, $Q(P \cap N)/(P \cap N) \trianglelefteq G/(P \cap N)$ and $Q(P \cap N) \trianglelefteq G$. Thus $Q \trianglelefteq G$, that is a contradiction. Therefore $P \leq N$. If $P > 2$ or every element with order 4 of G lies in $Z_\infty(G)$, then $P \leq Z_\infty(G)$. By Lemma 7, $G = P \times Q$ is nilpotent. That is a contradiction.
Step (4). For every $x \in P \setminus \Phi(P)$, $o(x) = 4$.
Suppose that $x \in P \setminus \Phi(P)$ such that $o(x) = 2$. $\langle x \rangle^G \leq Z_\infty(G)$ and $\langle x \rangle^G \Phi(P)/\Phi(P) \trianglelefteq G/\Phi(P)$. By Step (2), $P = \langle x \rangle^G \Phi(P) = \langle x \rangle^G \leq Z_\infty(G)$. By Lemma 7, $G = P \times Q$ is nilpotent, a contradiction.
Step (5). Some minimal subgroup of $P/\Phi(P)$ is not S-permutable in $G/\Phi(P)$.
Suppose that every minimal subgroup of $P/\Phi(P)$ is S-permutable in $G/\Phi(P)$. By [23, Lemma 2.5], $P/\Phi(P)$ has a maximal subgroup which is normal in $G/\Phi(P)$. By Step (2), $\mid P/\Phi(P) \mid = p$. By Step (3), P is a cyclic subgroup of order 4. By [20, Lemma 2.2], G is nilpotent. That is a contradiction.
Step (6). The final contradiction.
By previous step, there exists a minimal subgroup $X/\Phi(P)$ of $P/\Phi(P)$ that is not S-permutable in $G/\Phi(P)$. Let $x \in X/\Phi(P)$. By Step (3) and Step (4), x is of order 4 and $\langle x \rangle$ is S-semiembedded in G. Hence there exists an S-permutable subgroup T of G contained in P such that $\langle x \rangle T$ is S-permutable in G and $\langle x \rangle \cap T \leq \langle x \rangle_{\bar{s}G}$. By Lemma 2 and $\langle x \rangle_{\bar{s}G} \leq P$, $\langle x \rangle \cap T$ is S-permutable in G. By Step (2), $T \leq \Phi(P)$ or $T = P$. If $T \leq \Phi(P)$, then $X/\Phi(P) = \langle x \rangle T \Phi(P)/\Phi(P)$ is S-permutable in $G/\Phi(P)$. That is a contradiction. If $T = P$, then $\langle x \rangle$ is S-permutable in G and hence $X/\Phi(P) = \langle x \rangle \Phi(P)/\Phi(P)$ is S-permutable in $G/\Phi(P)$, a contradiction. The last contradiction completes the proof. □

Theorem 2. *If every minimal subgroup of $F^*(G^{\mathcal{N}})$ lies in $Z_\infty(G)$ and every cyclic subgroup of $F^*(G^{\mathcal{N}})$ with order 4 is S-semiembedded in G, then G is nilpotent.*

Proof. Assume the assertion is not true and let G be a counterexample of minimal order. We will derive a contradiction in several steps.

Step (1). If $E \trianglelefteq G$, then E is nilpotent.
By Lemma 6, $E/(E \cap G^{\mathcal{N}}) \cong EG^{\mathcal{N}}/G^{\mathcal{N}}$ and $E^{\mathcal{N}} \trianglelefteq E \cap G^{\mathcal{N}} \trianglelefteq G^{\mathcal{N}}$. We have $F^*(E^{\mathcal{N}}) \trianglelefteq F^*(E \cap G^{\mathcal{N}}) \trianglelefteq F^*(G^{\mathcal{N}})$. We have also $E \cap Z_\infty(G) \le Z_\infty(E)$, therefore E satisfies the hypothesis of the theorem. Thus E is nilpotent.
Step (2). $F(G)$ is the unique maximal normal subgroup of G.
Since the class of nilpotent groups is a Fitting class, it follows that G has a unique maximal normal subgroup M. By Step (1), M = F(G).
Step (3). $G = G^{\mathcal{N}} = G'$ and $F^*(G) = F(G) < G$.
If $G^{\mathcal{N}} < G$, then $G^{\mathcal{N}}$ is nilpotent by Step (1). By Lemma 6, $F^*(G^{\mathcal{N}}) = G^{\mathcal{N}}$. By Theorem 1, G is nilpotent, a contradiction. Hence, $G = G^{\mathcal{N}}$. By Theorem 1, if $F^*(G) = G$, then G is nilpotent, that is a contradiction. Thus $F^*(G) < G$. By Lemma 6, $F^*(G) = F(G)$.By Step (2), $G/F(G)$ is simple. If $G/F(G)$ is cyclic of prime order, then G is nilpotent. By Theorem 1, $G/F(G)$ is a non-abelian simple group. $G' \nleq F(G)$, therefore $G' = G$.
Step (4). The final contradiction.
Let p be a minimal prime divisor of $\mid F(G) \mid$ such that $O_p(G) \neq 1$. Let $\overline{G} = O_p(G)Q$, where $Q \in Syl_q(G), (q \neq p)$. We have $\overline{G}^{\mathcal{N}} \le O_p(G)$ and $\overline{G} \cap Z_\infty(G) \le Z_\infty(\overline{G})$. By Theorem 1, $\overline{G}$ is nilpotent. Thus $\overline{G} = O_p(G) \times Q$ and $Q \le C_G(O_p(G))$. Thereby $G/C_G(O_p(G))$ is a p-group and $G = C_G(O_p(G))$ i.e $O_p(G) \le Z(G)$.
By Lemma 6, $F^*(G/O_p(G)) = F^*(G)/O_p(G)$. Therefore every element $\overline{\alpha}$ of prime order r in $F^*(G/O_p(G))$ can be viewed as the image of an element α of prime order r in $F^*(G)$, for every $r\rangle p$. Thus α lies in $Z_\infty(G)$ by hypotheses. By $O_p(G) \le Z(G)$, $Z_\infty(G/O_p(G)) = Z_\infty(G)/O_p(G)$. Therefore $\overline{\alpha}$ lies in $Z_\infty(G/O_p(G))$. It follows that $G/O_p(G)$ satisfies the hypothesis of theorem and hence $G/O_p(G)$ is nilpotent. Thus G is nilpotent. That is a contradiction. □

Theorem 3. *Suppose that* p *is a prime divisor of* $\mid G \mid$ *with* $(\mid G \mid, p-1) = 1$ *and* $P \in Syl_p(G)$. *If every cyclic subgroup of* $P \cap G^{\mathcal{N}_p}$ *with prime order or order* 4 *(if* $p = 2$ *and* P *is non-abelian) not having* p*-nilpotent supplement in* G *is* S*-semiembedded in* G*, then* G *is* p*-nilpotent.*

Proof. Assume the assertion is not true and let G be a counterexample of minimal order. We will derive a contradiction in several steps.
Step (1). $P = G^{\mathcal{N}_p}$ is not cyclic and G is a minimal non-nilpotent group.
By [20, Lemma 2.2], P is not cyclic. By $E/(E \cap G^{\mathcal{N}_p}) \cong EG^{\mathcal{N}_p}/G^{\mathcal{N}_p}$, we have $E^{\mathcal{N}_p} \le E \cap G^{\mathcal{N}_p}$. By Lemma 3, E satisfies the hypothesis of the theorem. Thus, E is p-nilpotent. Therefore G is a minimal non-nilpotent group. By Lemma 4, $P = G^{\mathcal{N}_p}$ and $G = PQ$, where P is a normal Sylow p-subgroup of G and Q is a non-normal cyclic q-subgroup of G, $P/\Phi(P)$ is a chief factor of $G/\Phi(P)$ and the exponent of P is p or 4.
Step (2). Some minimal subgroup $K/\Phi(P)$ of $P/\Phi(P)$ is not S-permutable in $G/\Phi(P)$.
By Lemma 4, $P/\Phi(P)$ is a minimal normal subgroup of $G/\Phi(P)$. If every subgroup $K/\Phi(P)$ of $P/\Phi(P)$ be S-permutable in $G/\Phi(P)$, then $P/\Phi(P)$ has a maximal subgroup that is normal in $G/\Phi(P)$ [17, Lemma 2.11]. That is a contradiction.
Step (3). The final contradiction.
Let $k \in K/\Phi(P)$. Then $\langle k \rangle$ is a cyclic group of prime order or order 4. Let T be any supplement of $\langle k \rangle$ in G. We have $G = \langle k \rangle T$ and $P = P \cap G = P \cap (\langle k \rangle T) = \langle k \rangle (P \cap T)$. $(P \cap T)\Phi(P)/\Phi(P) \trianglelefteq G/\Phi(P)$, thus $(P \cap T)\Phi(P) \trianglelefteq G$. Therefore $P \cap T \le \Phi(P)$ or $P \cap T = P$. If $P \cap T \le \Phi(P)$, then

$P = \langle k\rangle$. That is a contradiction. If $P \cap T = P$ for any supplement T, then $T = G$. Therefore $\langle k\rangle$ don't have p-nilpotent supplement in G. By Lemma 5, there exists an S-permutable subgroup $L \leq P$ of G such that $\langle k\rangle L$ is S-permutable and $\langle k\rangle \cap L \leq \langle k\rangle_{\overline{s}G}$. Since $P/\Phi(P)$ is a chief factor of G, $L = P$ or $L \leq \Phi(P)$. If $L \leq \Phi(P)$, then $K/\Phi(P) = \langle k\rangle L\Phi(P)/\Phi(P)$ is S-permutable in $G/\Phi(P)$, a contradiction. If $L = P$, then $K/\Phi(P) = \langle k\rangle\Phi(P)/\Phi(P)$ is S-permutable in $G/\Phi(P)$. That is a contradiction. □

Theorem 4. *Let G be a finite group and N be a normal subgroup of G. Suppose that there exists a normal subgroup L of G such that L is supersolvable, $F^*(N) \leq L \leq N$ and for any non-cyclic Sylow subgroup L' of L, every cyclic subgroup of L' with order p or 4(if L' is a non-abelian 2-group) is S-semiembedded in G, then $N \leq Z_{\mathcal{U}}(G)$.*

Proof. By [18, Theorem C], if $F^*(N) \leq L \leq Z_{\mathcal{U}}(G)$, then $N \leq Z_{\mathcal{U}}(G)$. Thus let (G, L) be a counterexample of minimal order. Let p be a minimum prime divisor of $\mid L \mid$ and $L' \in Syl_p(L)$. We will derive a contradiction in the following steps.

Step (1). $L = L'$ is a non-cyclic p-nilpotent subgroup.

By the hypothesis and Theorem 3, L is p-nilpotent. Let M be the normal p-complement of L' in L. By $L \trianglelefteq G$ and $K\ ch\ L$, we have $K \trianglelefteq G$. By Lemma 3, if $K \neq 1$, then (G, K) and $(G/K, L/K)$ satisfies the hypothesis of the theorem. Thus $K \leq Z_{\mathcal{U}}(G)$ and $L/K \leq Z_{\mathcal{U}}(G/K)$, therefore $L \leq Z_{\mathcal{U}}(G)$. That is a contradiction. Hence $K = 1$ and $L = L\prime$. If $L = L'$ be a cyclic subgroup then L is hypercyclically embedded in G that is contract with the hypothesis.

Step (2). L is not a normal minimum subgroup of G and every cyclic subgroup of L' from order p or order 4 (if L' is a non-abelian 2-group) is S-embedded in G.

Let M be a maximal subgroup of L. Thus ML is S-permutable in G and $M \cap L = M \leq L_{sG} = L$. Thus M is S-embedded in G. By [5, Lemma 2.2], L is not normal minimal subgroup of G. By Lemma 2, every cyclic subgroup of L' from order p or 4 (if L' is a non-abelian 2-group) is S-embedded in G.

Step (3). There exists a normal subgroup $Q \leq L$, $Q \neq 1$ such that L/Q is a non-cyclic chief factor of G and $Q \leq Z_{\mathcal{U}}(G)$. If $R \trianglelefteq G$ and $R \subsetneq L$, then $R \leq Q$.

By Step (2), L is not normal minimal subgroup of G. Let $Q \neq 1$ normal subgroup of G such that L/Q is a chief factor of G. (G, Q) satisfies the hypothesis of the theorem, hence $Q \leq Z_{\mathcal{U}}(G)$. Therefore L/Q is not cyclic. If $R \lhd G$ and $R < L$, then $F^*(R) < L$. By minimal choice of (G, L), $F^*(R) \leq Z_{\mathcal{U}}(G)$. By [18, Theorem C], $R \leq Z_{\mathcal{U}}(G)$. If R is not a subgroup of Q, then by $L/Q = RQ/Q \cong R/R \cap Q$, we have $L \leq Z_{\mathcal{U}}(G)$, a contradiction, thus $R \leq Q$.

If L is a non-abelian 2-group, We use Ω to denote the subgroup $\Omega_1(L)$, otherwise, $\Omega = \Omega_2(L)$.

Step (4). $C_G(\Omega)/C_G(L)$ is a p-group.

This follows from [4, Theorem 2.4].

Step (5). $\Omega \nleq Z_{\mathcal{U}}(G)$.

Let $\Omega \leq Z_{\mathcal{U}}(G)$. By Lemma 7, $(G/C_G(\Omega))^{\mathcal{A}(p-1)}$ is a p-group. By Step (4), $(G/C_G(L/Q)) \in \mathcal{A}(p-1)$. Thus $\mid L/Q \mid = p$. That is a contradiction.

Step (6). The final contradiction.

By Steps (3) and (5), $\Omega = L$. Let $R_1, \ldots, R_m$ be the set of all cyclic subgroups of L of order p or 4 which is not contained in Q. By Step (2), every R_i is S-embedded in G. Hence, there exists an S-permutable subgroup O_i of G contained in L such that R_iO_i is S-permutable in

G and $R_i \cap O_i \leq (R_i)_{sG}$. By Lemma 1, if any R_i be S-permutable in G, then R_iQ/Q is S-permutable in G/Q. If R_i is not S-permutable in G, then $1 < O_i < L$. By Lemma 1, $O_i \leq Q$ and $R_iQ/Q = R_iO_iQ/Q$ is S-permutable in G/Q. Since $L/Q = (R_1Q/Q)(R_2Q/Q)\dots(R_mQ/Q)$ is an elementary abelian group and for every $1 \leq i \leq m$, R_iQ/Q is cyclic, then every minimal subgroup of L/Q is S-permutable in G/Q. By [23, Lemma 2.5], L/Q is a cyclic group of order p, that is a contradiction. □

Theorem 5. *Let $\mathcal{F}$ be a saturated formation containing $\mathcal{U}$. If G has a normal subgroup N such that $G/N \in \mathcal{F}$ and there exists a normal subgroup L of G such that L is supersolvable, $F^*(N) \leq L \leq N$ and for any non-cyclic Sylow subgroup L' of L, every cyclic subgroup of L' with order p or 4(if L' is a non-abelian 2-group) is S-semiembedded in G, then $G \in \mathcal{F}$.*

Proof. By Theorem 4, $N \leq Z_{\mathcal{U}}(G)$. By [1, Lemma 2.6], $G \in \mathcal{F}$. □

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