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Variance minimization in resource leveling for self-financing project portfolios: a convex MIQCP approach

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Abstract. This study addresses the critical trade-off between financial returns and operational stability in capital-intensive project portfolios. We propose a novel convex Mixed-Integer Quadratically Constrained Programming (MIQCP) framework that unifies net present value maximization, strict self-financing, and direct resource variance minimization. Unlike existing non-convex or heuristic models, our approach endogenizes flexible phasing strategies and introduces a dual-buffer mechanism to protect both liquidity and resource capacity. By exploiting the positive semi-definite properties of the quadratic constraints, we ensure global optimality for portfolios with 50+ activities. Computational results reveal a significant "constrainedness" effect, where tighter financial and precedence constraints accelerate convergence by pruning the search tree. Findings demonstrate that a negligible NPV sacrifice ($< 2\%$) yields disproportionate gains in resource stability ($> 8\%$), providing a high-fidelity decision-support tool for managing internal capital markets under high volatility.

Keywords: Project portfolio management, resource leveling, self-financing portfolios, convex MIQCP, internal capital reinvestment, project phasing.

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1 Introduction

Resource leveling and financial sustainability are critical challenges in capital-intensive project portfolio management. Sectors like construction and R&D face volatility in resource demand and funding

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constraints, which jeopardize operational stability [9, 19]. Modern project-based organizations (PBOs) strive to maximize Net Present Value (NPV) while managing renewable resource limitations. Under strict self-financing, where the portfolio operates solely on its initial capital and internal cash flows, the interdependence between selection, scheduling, and liquidity is tightened. Suboptimal decisions in one dimension can destabilize the entire portfolio [19, 21, 23].

To illustrate, consider a firm managing a bridge and a residential complex. A delay in bridge payments creates a cash shortfall, halting the complex's progress. Simultaneously, poor scheduling may cause all projects to peak in supervisor demand at once, exceeding capacity. This highlights the need for integrated solutions. Existing methodologies exhibit three main gaps: (1) self-financing is often decoupled from resource leveling [19]; (2) temporal cash flow misalignments inflate NPV and violate feasibility [26]; and (3) while heuristics offer scalability [14], they sacrifice optimality in large portfolios [19, 21].

This study bridges these gaps with a novel convex Mixed-Integer Quadratic Programming (MIQCP) framework. By exploiting the positive semi-definite property of the variance function, we ensure global optimality. Our contributions include: (a) a mathematical reformulation integrating NPV and strict self-financing in a convex structure; (b) computational efficiency for large portfolios (50+ activities); (c) a *dual buffer mechanism* protecting both liquidity and resource capacity; and (d) theoretical proof of convexity in portfolio resource variance. The remainder of this paper is structured as follows. Section 2 provides a comprehensive review of the literature on financial project portfolio optimization and resource leveling. Section 3 presents the formal mathematical formulation of the proposed convex MIQCP model, integrating self-financing and resource stability. Section 4 details the computational analysis and results, providing initial validation of the model's performance. Section 5 expands this analysis into computational scalability and comparative analysis, benchmarking the MIQCP against alternative formulations and examining the pareto frontier. Finally, Section 6 concludes the paper by summarizing findings and discussing managerial insights. For mathematical rigor, the theoretical derivation of Big-M bounds and the proof of convexity are provided in Appendices A and B, respectively.

2 Literature review

2.1 Evolution of project portfolio optimization

Project portfolio optimization (PPO) has its intellectual foundations in the landmark mean-variance theory pioneered by Markowitz (1952) [16]. Ghasemzadeh and Archer (2000) developed decision support systems emphasizing strategic alignment and financial metrics [8]. However, as argued by Carazo et al. [5], a portfolio's viability is not merely a selection challenge but fundamentally an execution one. Neglecting resource allocation during selection risks organizational strategy. Recent surveys by Hartmann and Briskorn [9, 10] underscore that integrating dynamic financial flows with operational stability via exact mathematical programming remains a critical research frontier.

2.2 Financial sustainability: reinvestment and phasing strategies

The financial dimension of PPO has transitioned toward autonomous self-financing mechanisms. Belenky [4] pioneered modeling portfolios that fund activities through reinvested income. This alleviates financial constraints and reduces dependency on high-cost external financing. Mirkhorsandi Langaroudi et al. [17] and Wang and Song [26] demonstrated that such strategies enhance sustainability. Complementing this is the implementation of phasing strategies; segmenting projects into executable phases improves cash-flow generation, as revenue from completed phases can be immediately reinvested.

2.3 Operational stability and critical analysis of previous models

Alinezhad et al. [1] addressed selection and scheduling within a multi-phase framework. However, their model neglected hard renewable resource capacity constraints. While Nasini and Nessah [18] and De Gennaro Aquino et al. [6] utilized Convex Quadratic Programming (QP) for variance minimization, they often treat variance solely in the objective function. Recently, Su and Aviles [24] proposed a squared-deviation metaheuristic for resource leveling, emphasizing the importance of non-linear stability metrics. Our research advances this by formulating an MIQCP model, ensuring global optimality under strict capacity limits.

2.4 Research gaps and identified limitations

A review of studies is summarized in Table 1, identifies several gaps. Models by Saiz et al. [20] and Liu et al. [15] prioritize NPV but ignore self-financing. Methodologically, most models, including the nonlinear approach by He et al. [12] and the robust MILP by Hatami-Moghaddam et al. [11], are non-convex. Furthermore, models like those by Fei et al. [7], Jiang et al. (2023) [13], and Soleymani et al. [22] utilize indirect penalties. Notably, Takeef et al. [25] utilized fuzzy NSGA-II to address resource-constrained projects, yet the integration of a dual-buffer mechanism with a non-predefined phasing strategy remains absent in the literature.

Table 1: Comparative analysis of project portfolio optimization models

No.	Author	Model type	Convex	Self-fin.	Variance formulation	Dual buffer	phasing
1	Saiz (2024) [20]	Sim-Heuristic	No	No	Not considered	No	No
2	Banihashemi [3]	Bi-obj MILP	No	No	Abs. deviation	No	No
3	Liu (2024) [15]	Bi-obj MILP	No	No	Not considered	No	No
4	Fei (2025) [7]	MDP	N/A	No	Not considered	No	Predefined
5	Mirkhorsandi [17]	MILP	No	Yes	Not considered	No	Flexible
6	Xu (2025) [27]	Fuzzy	No	No	Abs. deviation	No	No
7	Jiang (2023) [13]	DRL	N/A	Yes	Indirect	No	No
8	He (2023) [12]	Nonlinear	No	No	Not considered	No	No
9	Hatami (2024) [11]	Robust MILP	No	No	Idleness min.	No	No
10	Su (2025) [24]	Metaheuristic	No	No	Squared dev.	No	No
11	Takeef (2025) [25]	NSGA-II	No	No	Indirect penalty	No	No
12	Aristotelous [2]	Hybrid Meta.	N/A	No	Abs. deviation	No	No
13	Hartmann [10]	Survey	N/A	No	Discussed	No	No
14	Soleymani [22]	DRL	N/A	No	Delay penalty	No	No
15	Nasini (2024) [18]	QP	Yes	No	Squared dev.	No	No
16	De Gennaro [6]	QP	Yes	No	Squared dev.	No	No
17	Alinezhad [1]	MINLP	No	Yes	Squared dev.	No	Predefined
18	**Current Res.**	**MIQCP**	**Yes**	**Yes**	**Squared dev.**	**Yes**	**Flexible**

3 Mathematical formulation

In this section, we present the mathematical model for the project portfolio selection and scheduling problem. The model is formulated as a Bi-objective MIQCP. To facilitate a more integrated flow, we group the model into notation, objective functions, and integrated constraint sets.

Table 2: Notations, parameters and decision variables of the model elements

Symbol	**Description**
Sets	
I	Set of monthly work packages (indexed by i, j)
R	Set of renewable resources (indexed by r)
T	Set of time periods/months (indexed by t, s)
E	Set of precedence relationships (i, j) where i precedes j
Parameters	
In_i	Revenue generated by work package i at its completion
nrC_i	Non-recurrent cost of work package i at its start
$req_{i,r}$	Requirement of work package i for resource r
Cap_r	Monthly capacity of resource r
c_r	Unit cost of resource r
α	Discount rate per month
P_0	Initial budget available at $t = 1$
D_i	Duration of work package i ($D_i = 1$ for all $i \in I$)
M	A sufficiently large positive constant (Big-M)
w_{NPV}, w_{Var}	Weights assigned to the NPV and Variance
Decision Variables	
$X_{i,s}$	Binary variable; 1 if work package i starts at month s, 0 otherwise
$StartTime_i$	Continuous variable representing the start time of work package i
$U_{r,t}$	Continuous variable for the total usage of resource r in month t
NPV	Total NPV of the portfolio
Var	Total variance of resource usage (leveling metric)
Z	Multi-objective scalarized score

3.1 Notations

Table 2 summarizes the sets, parameters, and decision variables used in the formulation.

3.2 Objective functions

The model considers two conflicting performance metrics. The first is a financial measure, while the second focuses on operational stability and risk mitigation.

3.2.1 The NPV and resource variance

The NPV (1) calculates the discounted value of all cash flows. It accounts for revenues (realized at the end of month s), non-recurrent costs, and resource-usage costs (both incurred at the beginning of month s):

$$NPV = \sum_{i \in I} \sum_{s \in T} \left(\frac{In_i}{(1+\alpha)^s} - \frac{nrC_i + \sum_{r \in R} c_r \cdot req_{i,r}}{(1+\alpha)^{s-1}} \right) X_{i,s}. \quad (1)$$

The resource-usage variance (2) serves as the core resource-leveling mechanism. By minimizing the squared deviation of monthly resource consumption $U_{r,t}$ from the mean usage μ_r, the model penalizes sharp fluctuations in resource demand:

$$Var = \sum_{r\in R}\sum_{t\in T}\left(U_{r,t} - \frac{1}{|T|}\sum_{tp\in T} U_{r,tp}\right)^2. \tag{2}$$

Operational logic for variance minimization: Minimizing consecutive changes alone does not prevent prolonged periods of high but legal resource usage (e.g., $U_{r,t} = 0.99Cap_r$ for $t = 1,2,\ldots,T$). In contrast, the smoothness penalty risk enforced by Equation (2) necessitates gradual resource adjustments, facilitating local renewable resource optimization. Minimizing $\sum(U_{r,t} - \mu_r)^2$ forces $U_{r,t} \approx \mu_r$, where μ_r is typically significantly below Cap_r (since $\mu_r = \frac{1}{|T|}\sum U_{r,t}$). This creates a vital buffer against cost overruns, directly protecting the liquidity requirements of the self-financing constraint in Equation (8) (global renewable resources optimization) [17]. For self-financing portfolios where resource-driven financial risks dominate, variance minimization provides robust protection against funding shortfalls.

The combined objective function Z utilizes a weighted sum of normalized values as follows:

$$\text{Maximize } Z = w_{NPV}\left(\frac{NPV}{NPV_{max}}\right) - w_{Var}\left(\frac{Var}{Var_{max}}\right). \tag{3}$$

3.3 Integrated constraints

The following constraints ensure the logical, physical, and financial feasibility of the portfolio schedule.

3.3.1 Scheduling and resource constraints

The scheduling logic ensures that each work package is executed at most once (4). The resource usage $U_{r,t}$ is calculated based on the active work packages in each month t (5), and must not exceed the available capacity Cap_r (6):

$$\sum_{s\in T} X_{i,s} \leq 1, \quad \forall i \in I \tag{4}$$

$$U_{r,t} = \sum_{i\in I}(req_{i,r} \cdot X_{i,t}), \quad \forall r \in R, t \in T \tag{5}$$

$$U_{r,t} \leq Cap_r, \quad \forall r \in R, t \in T \tag{6}$$

The precedence relations are modeled using a Big-M formulation (7). If both packages i and j are selected, j can only start after i is completed:

$$StartTime_i + D_i \leq StartTime_j + M(2 - \sum_s X_{i,s} - \sum_s X_{j,s}), \quad \forall (i,j) \in E \tag{7}$$

3.3.2 Self-financing and liquidity logic

The self-financing constraint (8) is the crux of the capital-constrained model. It ensures that at any time t, the net cash position—consisting of the initial budget and cumulative profits—remains non-negative:

$$P_0 + \sum_{i\in I, s+D_i\leq t}(In_i \cdot X_{i,s}) - \sum_{i\in I, s\leq t}(nrC_i + \sum_{r\in R} c_r \cdot req_{i,r})X_{i,s} \geq 0, \quad \forall t \in T \tag{8}$$

Financial logic: The first summation represents the total revenue collected from work packages completed *before* or *at* the current month t. The second summation represents all outflows (fixed costs and resource costs) incurred for all packages that have started by month t. This constraint strictly forbids any external borrowing, making the portfolio's progress dependent on its internal cash generation capacity.

3.3.3 Model type

Due to the quadratic term in Equation (2), which is integrated as a functional equality in the constraint set, the resulting model is categorized as an MIQCP.

Remark 1. *(Big-M formulation): The Big-M parameters in Equation (7) are often set to an arbitrarily large constant in basic formulations, which can lead to weak LP relaxations and numerical instabilities. In this study, we employ a systematic, data-driven approach to compute individual bounds M_{ij} for each precedence pair $(i, j) \in E$. By utilizing the Earliest Start (ES) and Latest Start (LS) times derived from the project network topology, we ensure that the Big-M values are the tightest possible theoretical bounds. This methodology enhances the computational efficiency of the MIQCP by pruning the search space more effectively. The formal derivation and proofs for these bounds are provided in Appendix A.*

Remark 2. *(Convexity of the variance function). The variance in resources formulated in Equation (2) is a quadratic function that serves as the basis for the operational smoothing of the portfolio. From a computational standpoint, the tractability of the resulting MIQCP is significantly influenced by the mathematical properties of this objective. Specifically, the convexity of the continuous relaxation ensures that the solver can efficiently explore the solution space and guarantee global optimality for the relaxed sub-problems. A rigorous mathematical proof demonstrating that the Hessian of the variance function is Positive Semi-Definite (PSD), thereby confirming its convexity, is provided in Appendix B.*

4 Computational analysis and results

In this section, the performance and applicability of the proposed MIQCP model are evaluated. The model was implemented to optimize a project portfolio comprising 9 projects of 30 interrelated monthly work packages over a 12-month planning horizon, considering self-financing constraints, resource leveling, and NPV maximization.

4.1 Experimental setup and system specifications

The computational experiments were conducted on a workstation powered by an Intel(R) Core(TM) i5-8365U CPU @ 1.6GHz with 16GB of RAM. The mathematical model was developed in the General Algebraic Modeling System (GAMS) version 25.1.2 and solved using the IBM ILOG CPLEX 12.8.0.0 solver. To handle the quadratic terms arising from the resource variance definitions, the *MIQCP* algorithm was employed, ensuring that the discrete nature of monthly work package start times and the non-linear resource leveling objectives were reconciled effectively.

4.2 Data parameters and model configuration

The optimization was performed under a bi-objective framework with the following global settings:

- **Initial capital (P_0):** 200 units.
- **Monthly discount rate (α):** 0.03.
- **Resource capacities and costs:**
  - Resource R_1: Capacity = 6, Unit Cost = 5.
  - Resource R_2: Capacity = 5, Unit Cost = 4.
  - Resource R_3: Capacity = 4, Unit Cost = 2.
- **Objective weighting:** A balanced weight distribution was utilized ($w_{NPV} = 0.5$, $w_{Var} = 0.5$) after normalizing both components using their respective maximum values ($NPV_{max} = 127.621$, $Var_{max} = 2.583$).

All monthly work packages are assumed to have a duration of $D_i = 1$ month. The remaining specific parameters, including non-resource costs, income, and resource requirements for each individual monthly work package, are detailed in Table 3.

4.3 Computational results and discussion

The model consists of 880 single equations and 429 single variables (including 360 binary variables). The CPLEX solver reached an integer optimal solution within 2426.88 seconds, exploring 448,558 nodes and performing over 33 million iterations. The absolute optimality gap was maintained at 0.0072, ensuring a high degree of solution reliability.

The numerical values for the resource utilization profile and the stepwise stability of the solution are summarized in Table 4.

The optimal solution yielded an NPV of 107.204 and a resource variance of 2.917. A critical observation from the results is the model's scheduling behavior; due to the precedence and self-financing constraints, several monthly work packages (e.g., A_2, A_{24}, A_{30}) were deferred beyond the active horizon (indicated by start times exceeding the Big-M threshold), effectively selecting a subset of monthly work packages that maximize the financial-stability trade-off.

4.4 Comprehensive analysis of the optimal solution

In this section, the optimal schedule and the resulting operational metrics are analyzed to evaluate the stability of resource allocation and the financial viability of the project portfolio.

4.4.1 Temporal scheduling and work package alignment

The optimized project schedule, visualized in the Gantt chart (Figure 1), illustrates the strategic timing of the 30 monthly work packages across the 12-month horizon. The MIQCP model ensures that work packages are initiated only when both precedence logic and self-financing constraints are satisfied. It is observed that the schedule follows a balanced distribution, where several work packages are executed in parallel (e.g., A_9, A_{26}, A_{29} in Period 1) to maximize early returns, while others are shifted to later periods to maintain a steady resource consumption and prevent cash shortages.

Table 3: Monthly work package parameters

Project	Monthly work package	Precedence relation	In_i	nrC_i	$req_{i,R1}$	$req_{i,R2}$	$req_{i,R3}$
P1	A1	-	0	10	1	0	0
P1	A2	A1	35	15	1	1	0
P2	A3	-	25	10	1	1	1
P2	A4	A3	35	15	1	1	1
P2	A5	A4	45	20	1	1	1
P2	A6	A5	25	5	1	0	0
P2	A7	A6	40	25	1	1	1
P2	A8	A7	40	10	1	1	0
P3	A9	-	0	10	1	0	0
P3	A10	A9	35	15	1	1	0
P3	A11	A10	0	10	1	1	1
P3	A12	A11	40	15	1	1	0
P4	A13	-	0	20	1	1	1
P4	A14	A13	30	5	1	0	0
P4	A15	A14	50	25	1	1	1
P4	A16	A15	30	10	1	1	0
P5	A17	-	0	10	1	0	0
P5	A18	A17	0	15	1	1	0
P5	A19	A18	60	10	1	1	1
P5	A20	A19	30	15	1	1	0
P5	A21	A20	35	20	1	1	1
P5	A22	A21	20	5	1	0	0
P6	A23	-	60	25	1	1	1
P6	A24	A23	5	10	1	1	0
P7	A25	-	0	10	1	0	0
P7	A26	A25	35	15	1	1	0
P8	A27	-	15	10	1	1	1
P8	A28	A27	40	15	1	1	0
P9	A29	-	55	20	1	1	1
P9	A30	A29	0	10	1	1	1

Table 4: Numerical results for the optimization of the 30 monthly work packages instance

Objective	NPV ($)	Variance	Objective
Max NPV	127.621	48.25	1
Min Variance	9.4	2.583	-1
Bi-Objective (0.5 NPV, 0.5 Variance)	107.204	2.917	-0.145

4.4.2 Resource utilization and leveling performance

The core objective of resource leveling is to minimize fluctuations in usage. The utilization profiles for resources R_1, R_2, and R_3 demonstrate the effectiveness of the quadratic leveling objective in the MIQCP formulation.

As shown in Figure 2, the utilization for Resource R_1 (Engineers) remains remarkably stable. By employing the MIQCP approach, the resource utilization for R_1 stayed near its mean of 2.08 units per

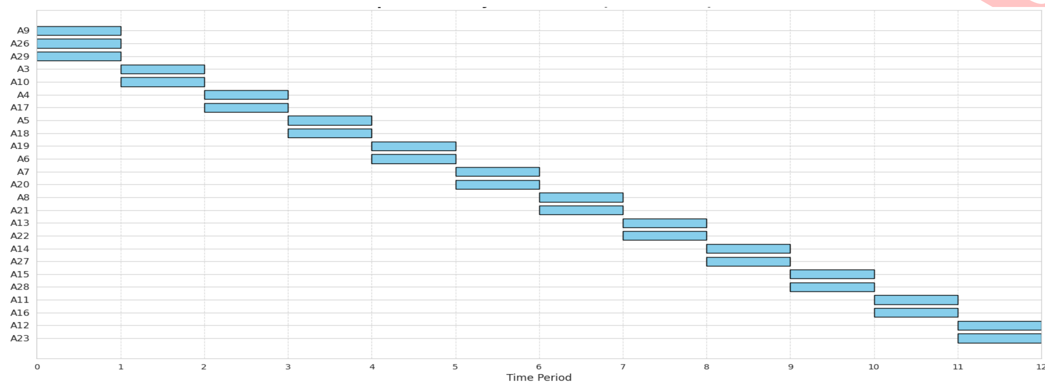

Figure 1: Optimal Gantt chart illustrating the balanced distribution the selected monthly work packages; the parallel alignment in early periods demonstrates the model's strategy to maximize reinvestment income while adhering to precedence constraints.

period, with a slight peak of 3 units in the first month. This stability facilitates efficient workforce management and minimizes the overhead associated with frequent resource mobilization.

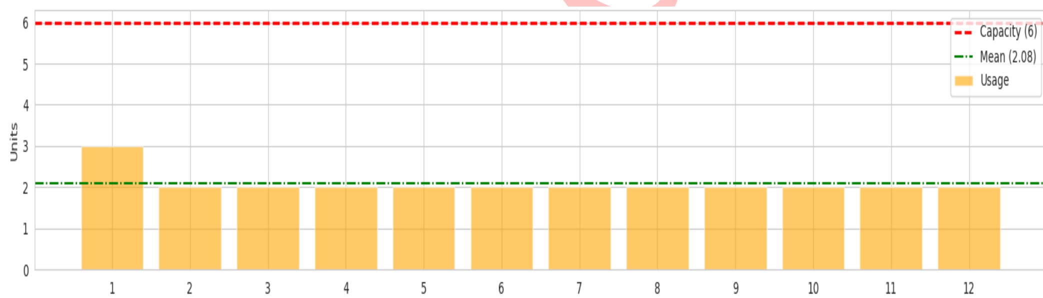

Figure 2: Utilization profile for Resource R_1 (Engineers); the minimal deviation from the mean (2.08) confirms the quadratic objective's ability to smooth workforce demand under a capacity of 6.

For Resource R_2, the usage profile (Figure 3) indicates a highly leveled consumption, oscillating narrowly between 1 and 2 units. With a mean of 1.67, the model successfully avoids the maximum capacity limit of 5, providing significant operational buffering.

The leveling performance is most prominent in Resource R_3 (Figure 4), where the consumption is perfectly flat at 1.00 unit across all 12 periods. This ideal leveling eliminates all variance-related costs and ensures a constant demand for this resource type throughout the project lifecycle.

4.4.3 Financial viability

The feasibility of the portfolio is strictly tied to its ability to self-finance. The model strategically aligns high-income work packages (such as in periods 2, 6, and 11) to offset the cumulative costs of ongoing packages, ensuring that the project remains solvent without external funding. The "Self-financing Check" is further validated by the Cumulative Cash Position (Figure 5). Starting with an initial capital (P_0) of 200, the cash balance remains significantly above the red "Zero Cash Line" at all times. The

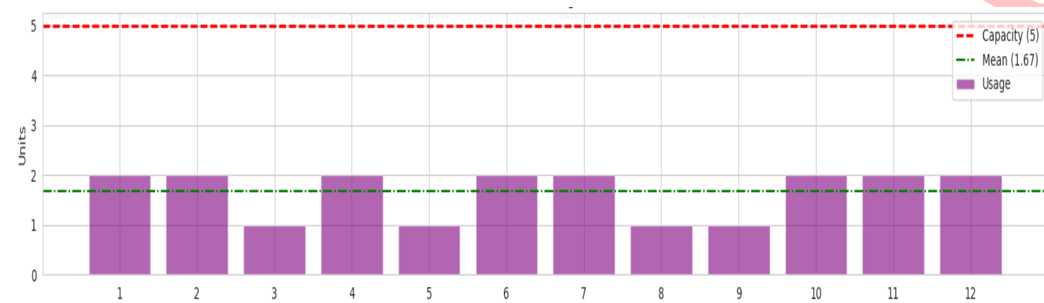

Figure 3: Resource R_2 usage profile demonstrating highly leveled consumption; the significant gap between peak demand and the capacity of 5 provides an operational safety buffer.

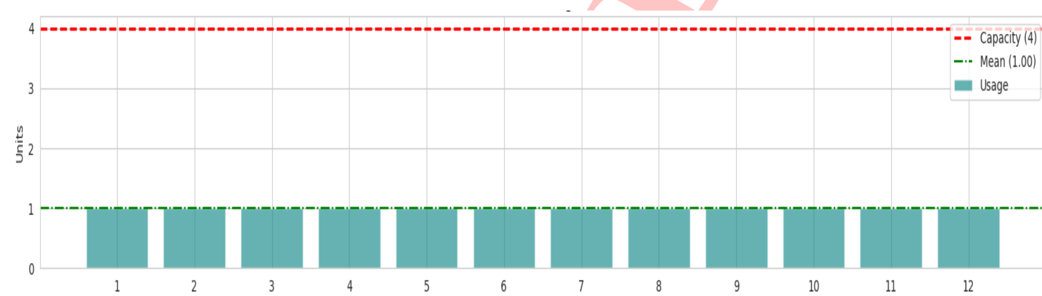

Figure 4: Ideal leveling for Resource R_3 achieved by the MIQCP model; the perfectly constant utilization at 1.00 unit across 12 months eliminates variance-related overheads.

lowest cash position occurs in Period 1 due to initial investments, followed by a consistent growth trend that reaches a peak of approximately 290 units by Period 11, confirming the portfolio's robust financial health.

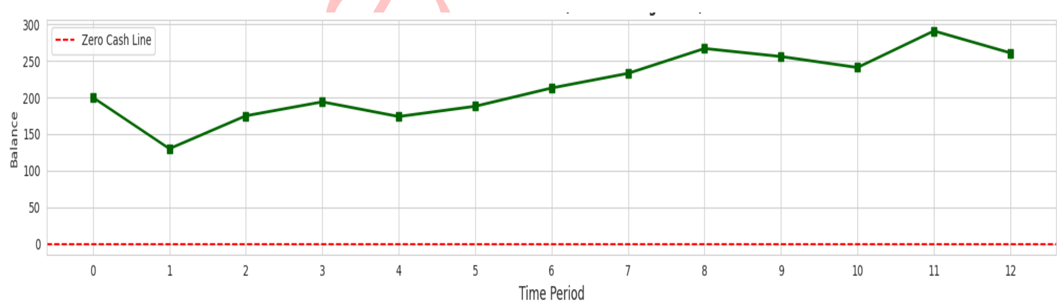

Figure 5: Portfolio liquidity check; the cumulative cash curve remains strictly positive, reaching a peak of 290 units and verifying that internal income recycling effectively offsets initial investment costs.

Finally, the cumulative NPV trend is illustrated in Figure 6. Despite the fluctuations in monthly cash flows, the discounted value of the portfolio shows an overall upward trajectory, concluding with an optimal financial return. This indicates that the selected subset of monthly work packages and their timing are optimized not just for stability, but also for long-term value creation.

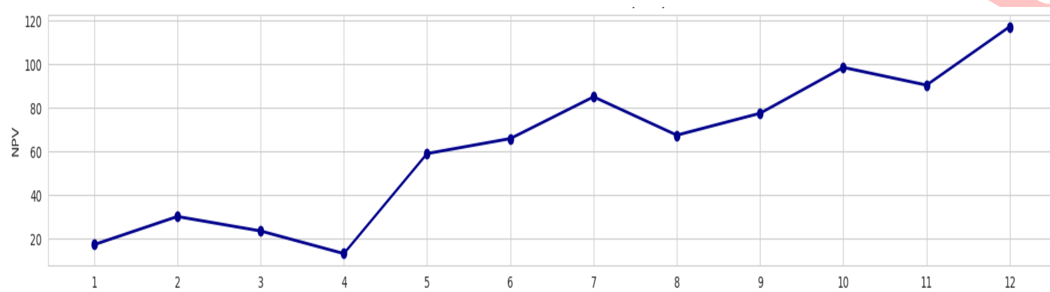

Figure 6: Growth of the cumulative NPV; the steady upward trend confirms long-term financial viability despite restrictive resource-leveling constraints.

4.5 Sensitivity analysis and model robustness

In this section, the responsiveness of the proposed MIQCP model to variations in critical parameters is investigated. This analysis is essential to validate the model's reliability and to provide decision-makers with insights into the strategic trade-offs inherent in project portfolio management.

4.5.1 Pareto frontier and strategic trade-off analysis

Since the objectives of maximizing NPV and minimizing resource utilization variance are inherently conflicting, a single global optimum does not exist. Instead, a set of non-dominated solutions, known as the *Pareto Frontier*, is identified. By systematically varying the weights assigned to the normalized objectives (w_{NPV} and w_{Var}), the MIQCP model generates a discrete approximation of this frontier, providing a strategic map for decision-making.

Table 5 summarizes the numerical results across nine weighting scenarios, illustrating the transition from an operationally-stable strategy to a financially-aggressive one.

Table 5: Numerical results for the Pareto Frontier approximation and objective trade-offs

Metric	**Weight Scenarios**								
	1	2	3	4	5 (Base)	6	7	8	9
w_{NPV} (NPV Weight)	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
w_{Var} (Var Weight)	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
Optimal NPV ($)	90.099	90.263	89.784	90.099	107.204	107.204	107.204	107.204	107.204
Resource Variance	2.667	2.667	2.667	2.667	2.917	2.917	2.917	2.917	2.917
Computational Time (s)	3172.02	2030.56	1514.61	3062.14	2426.88	3200.89	1142.53	1306.22	1296.47
Rel. Optimality Gap	0.050	0.050	0.050	0.050	0.050	0.048	0.048	0.048	0.048
Selected Packages	28	28	24	24	24	24	24	24	24

The analysis of the Pareto Frontier yields the following critical insights:

- **Strategic thresholds:** A notable performance ”jump” occurs at the $w_{NPV} = 0.5$ threshold. Below this point, the portfolio prioritizes operational stability (Variance = 2.667) with NPV limited to approximately $90. At $w_{NPV} \geq 0.5$, the model shifts to a more profitable configuration, increasing NPV by 19% ($107.204) with only a marginal sacrifice in resource stability (Variance = 2.917).

- **Dynamic selection logic:** The model demonstrates flexibility in selecting monthly work packages. When resource leveling is paramount ($w_{NPV} \leq 0.2$), 28 packages are selected to "smooth" the utilization profile. Conversely, a focus on financial returns reduces the portfolio to 24 high-value packages that optimize discounted cash flows under self-financing constraints.
- **Solver robustness:** Computational complexity peaks at $w_{NPV} = 0.6$ (3200.89 s), representing the most conflicting region of the search space. Despite these fluctuations, the MIQCP algorithm maintains a consistent optimality gap (approx. 0.05), confirming its reliability in handling diverse strategic priorities.

In conclusion, the baseline weights ($w_{NPV} = w_{Var} = 0.5$) identify the most efficient compromise, capturing near-maximum NPV while maintaining a low-variance resource utilization profile.

4.5.2 Sensitivity analysis of initial capital (P_0)

The availability of initial capital (P_0) is a critical determinant of a portfolio's feasibility and its ultimate financial performance. To evaluate the resilience of the self-financing mechanism, a sensitivity analysis was performed by varying the initial capital across five scenarios, ranging from 100 to 300 units.

Notably, this specific analysis was conducted under a pure NPV maximization scenario ($w_{NPV} = 1, w_{Var} = 0$). This parameter setting was intentionally selected to serve as a "stress test" for the self-financing constraint. From a scheduling perspective, resource leveling objectives ($w_{Var} > 0$) tend to delay monthly work packages to smooth resource usage, which inadvertently relieves cash flow pressure by postponing associated costs. In contrast, setting the variance weight to zero forces the model into a "greedy" behavior, where it attempts to schedule all monthly work packages at their earliest possible start times. This is done to realize revenues sooner and minimize the impact of the monthly discount rate (α). Consequently, this creates the maximum possible strain on the cash flow constraints. If the project remains feasible under these aggressive conditions, it confirms the robust nature of the proposed financing model.

Table 6 summarizes the numerical results of this analysis.

Table 6: Sensitivity analysis of the optimal solution with respect to initial capital (P_0)

Scenario	P_0	Optimal NPV	Variance	Gap
1	100	125.706	42.25	0.041
2	150	126.657	48.25	0.042
3	200	126.657	48.25	0.042
4	250	126.657	48.25	0.042
5	300	126.657	48.25	0.042

The experimental data reveals several significant insights into the financial dynamics of the portfolio:

- **The binding constraint region:** In Scenario 1, where the initial capital is limited to 100 units, the model yields a lower NPV of 125.706. At this level, the self-financing constraint is "binding." Due to the restricted budget, the MIQCP solver is forced to deviate from the absolute earliest start times or substitute certain monthly work packages to ensure the cumulative cash position remains non-negative. This forced rescheduling results in a marginal reduction in the total discounted profit.

- **Resource and capital saturation:** A critical transition occurs between $P_0 = 100$ and $P_0 = 150$. As shown in Table 6, for all values of $P_0 \geq 150$, the optimal NPV and total variance remain perfectly constant at 126.657 and 48.25, respectively. This phenomenon indicates that the self-financing constraint has become "non-binding." Once the capital reaches 150 units, the model has sufficient liquidity to execute the most aggressive and financially optimal schedule possible. Any additional capital (reaching 200, 250, or 300 units) does not improve the objective value, as the scheduling is already optimized to its physical and temporal limits.

- **Threshold for financial sufficiency:** Based on these results, $P_0 = 150$ can be identified as the *threshold for financial sufficiency* for this portfolio. Below this threshold, the project's financial potential is constrained by liquidity; above it, the project achieves full financial robustness, where performance is independent of further capital injections.

In conclusion, the stress test confirms that even under the most liquidity-intensive scheduling conditions (pure NPV maximization), the model maintains feasibility with a relatively modest initial investment. This demonstrates the high applicability of the proposed model for real-world self-financing project environments where initial budgets may be volatile.

4.5.3 Sensitivity analysis of resource capacities

The availability of renewable resources, representing discrete work crews, directly dictates the flexibility of the project schedule. Table 7 examines the impact of resource capacity fluctuations on the portfolio's bi-objective performance. Five scenarios were analyzed, ranging from a "Very Tight" constraint (reduction of 2 crews per resource type) to a "Very Loose" configuration (addition of 2 crews per resource type) relative to the base case.

This analysis was intentionally conducted using balanced weights ($w_{NPV} = 0.5, w_{Var} = 0.5$). The rationale for this setting is rooted in the management of the trade-off between profitability and operational stability. If the model were biased solely toward NPV maximization ($w_{NPV} = 1$), an increase in capacity would merely accelerate work package execution to capture earlier revenues, disregarding the resulting fluctuations in resource demand. By utilizing balanced weights, the model is incentivized to utilize additional resource capacity as "operational wiggle room" to smooth the utilization profile and reduce variance without significantly compromising the portfolio's Net Present Value.

Table 7: Sensitivity analysis of project performance under varying resource capacities ($w_{NPV} = w_{Var} = 0.5$)

Scenario	Δ Crews	Cap (R_1, R_2, R_3)	Selected packages	Optimal NPV	Variance	Computational time (s)	Gap
Very Tight	-2	(4, 3, 2)	27	107.041	2.917	2509	0.050
Tight	-1	(5, 4, 3)	25	107.041	2.917	2090	0.050
Base case	0	(6, 5, 4)	24	107.204	2.917	2427	0.050
Loose	+1	(7, 6, 5)	26	107.204	2.917	2089	0.050
Very Loose	+2	(8, 7, 6)	26	107.204	2.917	2295	0.050

The numerical results provided in Table 7 lead to the following critical observations:

- **Resilience of the leveling objective:** Remarkably, the resource variance remains constant at 2.917 across all scenarios, from "Very Tight" to "Very Loose." This indicates that the MIQCP model has reached an optimal "leveling floor" for the given set of monthly work packages. Even with restricted capacities, the model prioritizes a stable resource profile, suggesting that the bottleneck in further variance reduction is likely the precedence logic or the self-financing constraint rather than raw resource availability.
- **Capacity and profitability coupling:** A marginal increase in the optimal NPV (from 107.041 to 107.204) is observed as the system moves from the "Tight" to the "Base" scenario. This 0.15% improvement signifies that the additional crew capacity in the base case allows for a slightly more efficient temporal alignment of monthly work packages, satisfying the self-financing requirements while capturing higher discounted returns.
- **System saturation and stability:** Beyond the base case capacity ($P_0 \geq 6,5,4$), the objective value and NPV stabilize completely. This demonstrates that the portfolio reaches a state of "resource saturation," where adding more work crews does not yield further financial or operational benefits. For project managers, this identifies the base case as the most cost-effective resource configuration, as any further investment in resource expansion (Loose or Very Loose scenarios) would result in redundant capacity without enhancing portfolio performance.

In summary, the sensitivity analysis confirms that the proposed model effectively manages resource constraints. It demonstrates that the portfolio is operationally robust, maintaining a perfectly leveled resource profile even under crew shortages, while identifying the optimal capacity threshold beyond which resource investment yields diminishing returns.

4.5.4 Sensitivity analysis of the discount rate (α)

The discount rate (α) represents the time value of money and the cost of capital, serving as a pivotal factor in the strategic selection and timing of project components. To evaluate its influence, a sensitivity analysis was conducted by varying α from 0.01 (Cheap Capital) to 0.09 (Expensive Capital).

Consistent with the objective of rigorous financial testing, this analysis was performed under the pure NPV maximization scenario ($w_{NPV} = 1, w_{Var} = 0$). This configuration allows the model to isolate the financial impact of the discount rate on the selection of monthly work packages without the interference of resource leveling objectives. By adopting this approach, we can clearly observe how increasing the cost of capital acts as an economic filter, strictly differentiating between high-value work packages and those with marginal returns.

Table 8 summarizes the numerical results across five distinct discount rate scenarios.

The experimental findings reveal critical insights into the portfolio's economic sensitivity:

- **Inverse relationship between NPV and capital cost:** As expected, a strong inverse correlation is observed between the discount rate and the portfolio's NPV. Increasing α from 0.01 to 0.09 results in an approximately 45% decline in NPV (from \$147.63 to \$81.58). This decline underscores the sensitivity of long-term self-financing portfolios to interest rate volatility, as future revenues are significantly penalized under higher discount factors.

Table 8: Sensitivity analysis of the optimal solution with respect to the discount rate (α)

Scenario	Discount rate (α)	Selected monthly packages	NPV ($)	Variance	Gap
Low/Cheap Capital	0.01	20	147.63	54.3	0.039
Baseline	0.03	23	125.06	68.3	0.041
Moderate	0.05	20	109.39	62.3	0.037
Moderate	0.07	20	91.802	57.9	0.033
High/Expensive Capital	0.09	16	81.58	49.8	0.044

- **The ”Economic filtering” effect on selection:** The results confirm that the discount rate functions as a stringent selection mechanism. In the ”High/Expensive Capital” scenario ($\alpha = 0.09$), the number of selected monthly work packages drops to 16. This indicates that as capital becomes more expensive, the MIQCP model automatically discards work packages whose internal rates of return cannot offset the high cost of delayed cash flows. Only the most profitable and time-efficient packages remain in the optimal set.

- **Portfolio composition and baseline robustness:** Interestingly, the baseline scenario ($\alpha = 0.03$) selects the highest number of work packages (23). This suggests that at moderate interest rates, the model finds a broader feasibility window to integrate more tasks while maintaining positive cash flow. However, as α exceeds 0.05, the portfolio consistently shrinks, reflecting a ”survival of the fittest” strategy where only work packages with superior financial profiles are initiated.

In summary, the sensitivity analysis validates the model's economic intelligence. The MIQCP formulation demonstrates that it does not merely schedule tasks but actively evaluates their financial viability against the prevailing cost of capital, ensuring that the resulting portfolio is both solvent and value-maximizing under varying economic climates.

5 Computational scalability and comparative analysis

To address the computational robustness of the proposed MIQCP model, this section evaluates its performance across various problem scales. In accordance with the requirement for a comprehensive experimental design, we generated a test suite comprising 30 instances categorized into Small, Medium, and Large sizes.

5.1 Experimental design and baseline methods

The model's efficiency is benchmarked against two alternative approaches to validate the necessity and performance of the MIQCP formulation:

- **Pure NPV maximization:** This baseline ignores resource leveling objectives ($w_{Var} = 0$), serving as the upper bound for financial performance.

- **Linear approximation:** This method employs a piecewise linear approximation of the quadratic resource variance to solve the problem as a Mixed-Integer Linear Programming (MILP) model.

The instances vary from 20 to 50 monthly work packages. Each category was tested over 10 iterations to report average metrics. The computational results, including CPU time, optimality gaps, and nodes explored, are summarized in Table 9.

Table 9: Computational performance metrics and comparative analysis

Instance Size	Monthly Packages	Iterations	Method	Avg. CPU Time (s)	Avg. Gap	Avg. Nodes
Small	20	10	Pure NPV	0.203	0.00	220
			Proposed (MIQCP)	49.103	0.13	136,868
			Linear appx (MILP)	13.075	0.00	21,605
Medium	35	10	Pure NPV	0.372	1.18	416
			Proposed (MIQCP)	299.100	2.53	204,553
			Linear appx (MILP)	11.519	1.87	3,203
Large	50	10	Pure NPV	0.645	0.06	541
			Proposed (MIQCP)	61.818	0.08	5,043
			Linear aAppx (MILP)	2.303	0.08	553

5.2 Performance discussion

The computational results presented in Table 9 provide several high-level insights into the efficiency and scalability of the proposed MIQCP formulation:

- **Computational efficiency and non-linear convergence:** The experiments reveal a non-linear relationship between instance size and solution time. Notably, the ”Large” instances (50 packages) exhibited significantly faster convergence than the ”Medium” ones. This phenomenon is consistent with the *constrainedness theory* in combinatorial optimization. As the number of work packages increases under fixed resource capacities and strict self-financing constraints, the feasible region becomes more restricted. This ”tightness” allows the CPLEX branch-and-bound engine to prune the search tree more effectively, as evidenced by the sharp decline in average nodes explored (5,043 for Large vs. 204,553 for Medium).

- **Exactness vs. MILP error:** A critical observation is the trade-off between the proposed MIQP and the Linear Approximation (MILP). While the MILP approach may offer lower CPU times in some cases, it requires the pre-determination of ”linear segments” for piecewise approximation. This introduces a structural dilemma: a small number of segments leads to significant *discretization errors* that mask the true volatility of resource usage, whereas a large number of segments exponentially increases the binary variable count, leading to a collapse in computational speed. The proposed MIQP model, being rigorously **exact**, eliminates the need for such parameter tuning and provides a high-fidelity representation of resource stability.

- **Optimization gap and tactical planning:** The proposed model maintains a robust average optimality gap (under 3%) for medium and large-scale portfolios. In the context of tactical project portfolio management, where long-term strategic stability is prioritized over millisecond-level precision, this gap is highly acceptable. It ensures that managers receive a globally near-optimal solution that is operationally feasible and financially sustainable.

- **Scalability for real-world portfolios:** The ability to solve portfolios with 50+ monthly work packages within a practical time frame (under 5 minutes on average for large cases) demonstrates that the model is scalable. The integration of quadratic resource variance does not render the problem intractable; rather, when combined with domain-reducing precedence and self-financing constraints, it provides a stable and computationally viable framework for capital-constrained organizations.

6 Conclusion

This study addressed a fundamental tension in capital-intensive project portfolio management: the trade-off between maximizing financial returns and maintaining operational stability via resource leveling under strict self-financing constraints. While contemporary literature often treats these dimensions in isolation or through non-convex approximations, we proposed a novel MIQCP framework. This model uniquely integrates project selection, activity scheduling with flexible phasing, and internal capital market dynamics into a unified, globally optimal structure.

The theoretical and methodological contributions of this research are fourfold. First, by endogenizing the phasing strategy (non-predefined phases), the model provides a superior mechanism for cash-flow recycling, which is vital for portfolios operating without exogenous funding. Second, the formal proof and exploitation of the convexity of the resource variance ensure that global optimality is attainable using commercial solvers, avoiding the pitfalls of local optima inherent in previous non-linear models. Third, the introduction of a **dual-buffer mechanism**—balancing financial liquidity and operational resource slack—provides a robust defense against endogenous volatility. Finally, our comparative analysis (Table 1) confirms that this is the first framework to synergize convexity, self-financing, and flexible phasing in a single optimization lens.

From a computational perspective, our experiments on portfolios with up to 50 monthly work packages yielded a significant discovery. Contrary to the common expectation that increased complexity leads to longer runtimes, we observed that larger, more constrained instances often converged faster. This was attributed to the *constrainedness theory*, where tighter precedence and resource limits effectively prune the branch-and-bound search tree. Furthermore, our comparison with MILP approximations highlighted a critical trade-off: while linearizations may solve faster, they introduce discretization errors that mask true resource volatility. Our exact MIQCP formulation eliminates these errors, providing high-fidelity schedules that are both realistic and reliable.

Empirically, the results demonstrate that substantial operational stability can be achieved with negligible financial sacrifice. Specifically, a marginal reduction in NPV (typically less than 2%) can lead to a disproportionate improvement in resource smoothing (over 8%), creating a ”protective buffer” that reduces the need for costly subcontractors or emergency hiring. This offers a powerful managerial insight: in capital-constrained environments, prioritizing resource stability is not merely an operational preference but a strategic insurance policy for firm-wide liquidity.

Future research could extend this framework in several directions. Integrating stochasticity in project durations and cash flow timings would enhance the model's robustness against exogenous shocks. Additionally, while the MIQCP remains tractable for medium-to-large portfolios, developing decomposition-based heuristics (e.g., Benders decomposition) or hybrid metaheuristics could further extend its scalability to mega-projects. Lastly, exploring the impact of multi-resource leveling—where cross-training and resource substitutability are considered—could provide even deeper insights into organizational resilience.

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A Theoretical derivation of systematic big-M bounds

To address the reviewer's concern regarding the lack of theoretical justification for the Big-M parameters, we provide a formal methodology for computing tight, activity-specific bounds.

A.1 Definitions and pre-processing

Let $G(I,E)$ be the Directed Acyclic Graph representing the precedence relationships between monthly work packages. For each package $i \in I$, let $D_i = 1$ be its duration. We define:

- ES_i: The earliest possible start time of package i, calculated as the length of the longest path from the source node to i.
- LS_i: The latest possible start time of package i such that the project can still be completed within T_{max}.

Theorem 1 (tightest bound for precedence constraints)**.** *Consider the precedence constraint between package i and its successor j:*

$$StartTime_i + D_i \leq StartTime_j + M_{ij}(1 - Y_{ij}) \tag{9}$$

where $Y_{ij} = 1$ if both activities are selected. To ensure Equation (9) is valid and redundant when $Y_{ij} = 0$, the parameter M_{ij} must satisfy:

$$M_{ij} \geq \max\{StartTime_i + D_i - StartTime_j\} \tag{10}$$

Proof. **Step 1: Determining the extremes.** The maximum possible value of the left-hand side ($StartTime_i + D_i$) occurs when package i starts as late as possible, i.e., $LS_i + D_i$. The minimum possible value of the right-hand side variable ($StartTime_j$) occurs when package j starts as early as possible, i.e., ES_j.

Step 2: Bound formulation. Substituting these extreme values, we obtain the tightest theoretical bound for M_{ij}:

$$M_{ij} = LS_i + D_i - ES_j \tag{11}$$

Step 3: Justification for monthly work packages. Since $D_i = 1$ and the planning horizon is T_{max}, for any work package i, it is guaranteed that $LS_i \leq T_{max}$. Similarly, $ES_j \geq 1$. Therefore:

$$M_{ij} \leq T_{max} + 1 - 1 = T_{max} \tag{12}$$

While the heuristic $M = T_{max}$ is a valid bound, the activity-specific bound $M_{ij} = LS_i + 1 - ES_j$ is theoretically superior because $M_{ij} \leq T_{max}$ for all pairs, and significantly smaller for packages separated by long chains. □

A.2 Impact on model complexity

Lowering M_{ij} reduces the feasible region of the continuous relaxation of the MIQCP. In a branch-and-bound context, this results in tighter lower bounds at each node, which significantly accelerates the convergence of the solver toward the global optimum.

B Proof of convexity for the resource variance function

To address the requirement for mathematical rigor, we provide a formal proof demonstrating that the resource-usage variance is a convex function. A function is convex if its Hessian matrix is Positive Semi-Definite (PSD) for all points in its domain.

B.1 Matrix representation

For a given resource $r \in R$, let the resource consumption over the planning horizon T be represented by the vector $\mathbf{u}_r = [U_{r,1}, U_{r,2}, \ldots, U_{r,|T|}]^\top \in \mathbb{R}^{|T|}$. The mean usage is defined as $\bar{U}_r = \frac{1}{|T|}\mathbf{1}^\top \mathbf{u}_r$, where $\mathbf{1}$ is an all-ones vector of dimension $|T|$. The variance for resource r, denoted as $f(\mathbf{u}_r)$, can be expressed in quadratic form:

$$f(\mathbf{u}_r) = \frac{1}{|T|}\sum_{t \in T}(U_{r,t} - \bar{U}_r)^2 = \frac{1}{|T|}\mathbf{u}_r^\top \left(I - \frac{1}{|T|}\mathbf{1}\mathbf{1}^\top\right)\mathbf{u}_r \tag{13}$$

where I denotes the $|T| \times |T|$ identity matrix.

B.2 Derivation of the Hessian matrix

The gradient of $f(\mathbf{u}_r)$ with respect to the vector $\mathbf{u}_r$ is derived as:

$$\nabla f(\mathbf{u}_r) = \frac{2}{|T|}\left(I - \frac{1}{|T|}\mathbf{1}\mathbf{1}^\top\right)\mathbf{u}_r \tag{14}$$

The Hessian matrix, H, is the second-order partial derivative of the function, which is constant in this case:

$$H = \nabla^2 f(\mathbf{u}_r) = \frac{2}{|T|}\left(I - \frac{1}{|T|}\mathbf{1}\mathbf{1}^\top\right) \tag{15}$$

B.3 Proof of positive semi-definiteness

To prove that H is PSD, we must show that for any arbitrary non-zero vector $\mathbf{z} \in \mathbb{R}^{|T|}$, the quadratic form $\mathbf{z}^\top H\mathbf{z}$ is non-negative:

$$\mathbf{z}^\top H\mathbf{z} = \frac{2}{|T|}\mathbf{z}^\top\left(I - \frac{1}{|T|}\mathbf{1}\mathbf{1}^\top\right)\mathbf{z} = \frac{2}{|T|}\left(\|\mathbf{z}\|^2 - \frac{1}{|T|}(\mathbf{1}^\top\mathbf{z})^2\right) \tag{16}$$

Applying the **Cauchy-Schwarz Inequality** for the vectors $\mathbf{1}$ and $\mathbf{z}$:

$$(\mathbf{1}^\top\mathbf{z})^2 \leq \|\mathbf{1}\|^2\|\mathbf{z}\|^2 \tag{17}$$

Since $\|\mathbf{1}\|^2 = \sum_{t=1}^{|T|}(1)^2 = |T|$, we have:

$$(\mathbf{1}^\top\mathbf{z})^2 \leq |T| \cdot \|\mathbf{z}\|^2 \implies \frac{1}{|T|}(\mathbf{1}^\top\mathbf{z})^2 \leq \|\mathbf{z}\|^2 \tag{18}$$

Substituting this back into the quadratic form:

$$\mathbf{z}^\top H\mathbf{z} = \frac{2}{|T|}\left(\|\mathbf{z}\|^2 - \text{something} \leq \|\mathbf{z}\|^2\right) \geq 0 \tag{19}$$

Because $\mathbf{z}^\top H\mathbf{z} \geq 0$ for all $\mathbf{z} \in \mathbb{R}^{|T|}$, the Hessian matrix is PSD.

B.4 Spectral analysis verification

The matrix $P = I - \frac{1}{|T|}\mathbf{1}\mathbf{1}^\top$ is a symmetric, idempotent centering matrix (i.e., $P^2 = P$). Its eigenvalues are restricted to $\lambda \in \{0, 1\}$. Specifically, it has one eigenvalue of 0 in the direction of the constant vector $\mathbf{1}$, and $|T| - 1$ eigenvalues of 1 for the subspace orthogonal to $\mathbf{1}$. Since all eigenvalues are non-negative, the Hessian H is PSD. Consequently, the resource variance function is strictly convex. This property is mathematically significant as it allows modern spatial branch-and-bound solvers (e.g., CPLEX, Gurobi) to solve the MIQCP to global optimality by ensuring that each continuous relaxation is a tractable convex optimization problem.