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Optimal location of healthcare and treatment centers with complex structures based on performance

Azam Azodi[†], **Jafar Fathali**[†], **Mojtaba Ghiyasi**[‡*], **Tahereh Sayar**[†]

[†] *Faculty of Mathematical Sciences, Shahrood University of Technology, Shahrood, Iran*
[‡] *Faculty of Industrial Engineering and Management, Shahrood University of Technology, Shahrood, Iran*

Email(s): a.azodi.62@gmail.com, fathali@shahroodut.ac.ir, mog@shahroodut.ac.ir, t.sayar.t@gmail.com

Abstract. Optimal use of existing facilities and resources to improve the efficiency of healthcare and treatment centers, achieving social welfare, and responding to the needs of customers, is an important issue. Paying more attention to healthcare and treatment centers, allocating sufficient resources, and using them correctly will improve the health of the workforce and increase production and productivity in society. One of the important mechanisms for evaluating the performance and efficiency of healthcare and treatment centers is the use of data envelopment analysis. In this article, we propose a new mechanism for the proper distribution of facilities and healthcare and treatment centers in cities to reduce costs and also maximize the efficiency of healthcare and treatment centers with the aim of better quality of services. This is done by integrating the problem of p-median location and network data envelopment analysis. Proposed methods are applied for performance measurement, location-allocation, and distribution of 11 healthcare and treatment centers in Shahrood city in Iran. The primary results show potential of cost reduction that could be done when allocating clients, considering the performance of healthcare and treatment centers. Another important finding is to have centralized healthcare and treatment centers rather than diffused center to reach the optimal condition which is a vital information for health care policy makers.

Keywords: Efficiency, healthcare and treatment centers, p-median problem, data envelopment analysis, network data envelopment analysis.
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1 Introduction

In recent decades, due to the high costs of health and medical services and the government's problems in financing these costs, organizations believe that health and treatment should be considered not only

*Corresponding author
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from a social point of view but also from an economic point of view [26, 28]. Therefore, today, proper distribution of health facilities in cities and efficient use of these facilities are considered vital. Moreover, evaluating the performance of health service provider units as well as locating these units to minimize costs is a very important issue. This issue is very important for the managers of the health department to know which units are more efficient than other units and also what method can be used to improve the efficiency of the units. Healthcare and treatment, as organizational units that provide healthcare services to the general public, play a central role in providing quality services. To improve the quality of these services and increase the efficiency of units, a method is needed to allocate resources and facilities optimally. For this purpose, data envelopment analysis (DEA) approach is used to calculate the efficiency of healthcare and treatment centers.

The DEA non-parametric method for evaluating the efficiency or calculating the efficiency of a finite number of decision-making units (DMUs) in multi-input and multi-output modes. Analyzing the data by measuring the relative efficiency of the organizations ranks them, identifies the strengths and weaknesses of each organization, and provides suggestions for improving the efficiency of each organization. In 1957, Farrell was the first to present a non-parametric method for determining the efficiency in the case of two inputs and one output and presented the linear segment-segment convex hull method for boundary approximation. Farrell suggested that to determine efficiency, one should first specify an efficiency frontier and then interpret the distance from the efficient frontier as a measure of inefficiency. In 1978, Charnes, Cooper, and Rhodes [5] extended Farrell's non-parametric method to a system with multiple inputs and outputs using mathematical programming and called "DEA". Since then, many studies have been conducted in this field and many new models and articles have been presented. So far, more than a thousand references, including articles, specialized reports, etc., have been published in this field, and the importance and capabilities of data coverage analysis are becoming more apparent every day. This means that the method of DEA has been approved in terms of applicability, and its use in some fields has made it a suitable method for modeling practical processes.

Conventional DEA models consider systems as a closed set and ignore the process within the system, performance, and relationships between them. For this reason, researchers have objected to these models. This view, which is known as the "black box", loses a lot of valuable information about the DMUs and limits the analysis of the performance of DMUs to the initial inputs and final outputs [3,8]. Therefore, the performance measurement results with conventional DEA methods prevent the acquisition of valuable management information. To solve this problem, in an article, Fáre and Grosskopf [8], while pointing out the weakness of the conventional DEA model, introduced "envelopment analysis of network data" and described its importance in a more accurate analysis of the efficiency of DMUs. In this model, a DMU is considered a network structure with all the internal parts and the connections between them. Since in network models the internal processes of each DMU can be checked, these models provide a more accurate picture of the efficiency of DMUs. In these models, the limitation of internal processes is added to the limitation of the total process, as a result, the total efficiency score will be smaller than the efficiency score of internal processes. Network production systems have different structures, and the discussion of performance measurement is usually based on the system structure. For example, Kao [16] has divided network models into three categories: series, parallel, and dynamic. In a multi-departmental DMU, when the activities of the departments are placed along each other, the system has a series structure. The serial network structure consists of two or more processes that are serially connected through intermediate sections. In this case, the input of the whole system enters the first part and the final output of the system leaves the last part. Another structure, as opposed to the series, is the parallel structure, where several parts operate independently in the system, each consuming several externally supplied inputs and producing several final outputs. A parallel structure is efficient when all its processes are efficient. In the field of series network coverage analysis, many studies have been done so far, and most authors have used the two-stage structure of series, but parallel systems, unlike series, are not widely discussed in the literature. Kao [17] studied a problem of efficiency measurement for the forest districts in Taiwan, where each district uses the inputs of land, labor, forest stocks, and operating expenses to generate timber, recreational visitors, and a soil conservation effect. Each district is divided into several working circles performing the same functions. In this example, the district is the DMU, and the working circles are divisions. We will use the idea from Kao [17] to discuss the efficiency measurement of multi-component systems.

Among the works that have been done in the field of efficiency of health and treatment centers with DEA methods, we can refer to Sherman's article [35], which was the first to use DEA in the field of healthcare and evaluate the effective use of resources with relative efficiency. After that, researchers have widely evaluated the DEA model in the medical and health system in different ways. In [38], the relative efficiency of Greek healthcare and treatment centers was measured using the DEA model, the result of which was improvement of policies for inefficient centers according to performance indicators. Keshtkar et al. [19] conducted a study titled determining the efficiency of health units in the cities of Golestan Province. Retzlaff-Roberts et al. [30], used health data from the Organization for Economic Co-operation and Development (OECD) and data coverage analysis. Also, Marschall and Flessa [22] evaluated the efficiency of rural healthcare and treatment centers in Burkina Faso, identified the efficiency and inefficiency of these centers. Ghiyasi et al. [9] used resource allocation models to find the optimal allocation of resource for the wards of Imam Reza hospital. Farahabadi et al. [7] also analyzed the efficiency of selected urban healthcare and treatment centers affiliated with Isfahan University of Medical Sciences. In addition to calculating the efficiency of each center, they provided suitable solutions to improve the efficiency of these centers. The cost efficiency of parallel wards of hospitals in Mashahd is investigated in [10]. This was done by developing the cost efficiency models for dealing with the network models with the parallel structure. Jiang et al. [15] used the DEA model to calculate the efficiency of 1,105 sample hospitals in 31 Chinese provinces and confirmed that healthcare efficiency in China is relatively low, with the highest efficiency in eastern China and the lowest in western China. In [11] the authors proposed an inverse DEA model for dealing with ratio data called inverse DEA-R for hospital efficiency analysis. In a study, Chai et al. [4] used DEA model, social network analysis (SNA), cluster analysis, and regression analysis to analyze the structural characteristics of the economic network structure and efficiency of healthcare in China. In [34] an inverse DEA structure is proposed for dealing with the budgeting and planning models in hospitals. Nayar et al. in [24] evaluated the efficiency differences of different types of hospitals. The authors in [12] proposed a semi-additive technology for dealing with the integer data and then used the proposed models for analyzing the performance of hospitals in Mashhad. Zare Ahmadabadi et al. [37] used a quasi-experimental method based on mathematical modeling to identify and investigate the behavior of variables that explain the efficiency of healthcare and treatment centers. Recently, Afonso et al. [2] presented a new approach based on the network DEA in Portuguese hospitals to evaluate quality and access in healthcare. In this method, a network DEA model is combined with the weak disposability of outputs to handle undesirable outputs related to the poor quality of care or the lack of access to appropriate and safe care. Also, Yousefi et al. [36], relying on artificial neural networks (ANN), have used DEA approach based on the theory of economy of scale principles to increase the future efficiency of Covid-19 treatment centers. In [20] the authors proposed a new intuitionistic fuzzy scheme of DEA for evaluating rural comprehensive health service centers. Mahmoodirad et al. in [21] proposed a DEA based performance evaluation approach for hospitals, by implementing a novel picture fuzzy BCC model.

In healthcare, incorrect locations decision of healthcare and treatment centers, have serious impact on society beyond the simple criteria of cost and service. For example, healthcare facilities that are difficult to access are likely to be associated with increased morbidity and mortality. From this point of view, facility location modeling for healthcare is much more important than similar modeling for other areas. It can be said that the first location allocation study in the field of healthcare facilities was presented by Gould and Leinbach, which considered the problem of locating Guatemalan hospitals and determining their capacities as a p-median problem [27]. There have been many studies in this field, but none of them have considered the network efficiency of healthcare and treatment centers. Considering that no study has been done in the field of locating health centers with the network efficiency approach, in this article, we are looking for the location of some centers with the p-median problem so that the weighted sum of the distances between these p centers and customers is minimized. For this purpose, we consider the network efficiency of the centers as the weights of the distances between the centers in the p-median problem. The p-median problem is one of the most applied types of location problems, which was the first time introduced with the Fermat-Weber problem in the space of R^2. Several versions of the problem have been defined in the literature, and it has been used in different applications varying from the location of industrial plants and warehouses or public facilities like schools [31]. Mathematical program formulation of locating the p-median was proposed by Hakimi [14] and ReVelle and Swaim [32]. The p-median problem is classified as NP-hard [18] so to solve it we find in the literature a huge number of exact methods and metaheuristic approaches that look for a good solution (sometimes the optimal solution) when the problem is characterized by a big number of demand points and facilities. Reese [29] summarized the exact solution methods for the p-median problem in past studies. In the field of heuristics, Rolland et al. [33] proposed a tabu search algorithm for the p-median problem. Plastino et al. [25] presented an evaluation of heuristics for the p-median problem with investigating the effect of spatial distribution of destination locations, and the number of sources and destinations on the performance of algorithms for varying problem sizes using synthetic and real datasets. Also, in the field of metaheuristics, Chiyoshi and Galvão [6] presented a statistical analysis of simulated annealing for the p-median problem. There are many other metaheuristic algorithms proposed to solve the p-median problem, see e.g. [13, 23]. Afsharian [1] integrated DEA into the analysis of the p-median problem and used real-world data from German hospitals to illustrate the proposed approach.

The main aim of this paper is developing a mathematical modeling framework for considering the efficiency of parallel production systems in the p-median location problem which is the theoretical contribution of the current paper. Considering the practical view, we apply the proposed approaches in the real world application on evaluation and reconsideration of healthcare and treatment centers in Shahrood city of Iran.

We develop a hybrid approach considering the healthcare and treatment centers as production units, which are parallel networks with three sub-units (departments) that operate independently of each other. This issue has not been discussed in the existing literature, thus this is the novel development to consider the efficiency of parallel systems in the p-median problem in the literature.

The theory of network DEA and the proposed mathematical model to calculate the efficiency of the network is explained in Section 2. In Section 3, we describe the p-median location problem. In Section 4, we discuss the location of healthcare and treatment centers in Shahrood City using existing models

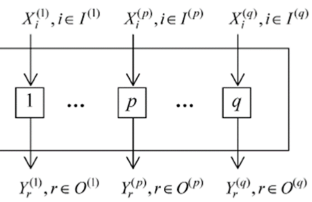

Figure 1: Structure of a parallel DMU

and present the results of solving this model, and in Section 5, we express the conclusion.

2 Performance assessment

To assess the performance of the production or survival unit, we should consider the whole structure of the production. Thus, to calculate the efficiency of each DMU, taking their internal processes, we face a network structure. Several models have been developed to measure the performance of a network system. Some models measure DMU and process efficiency simultaneously to derive mathematical relationships between them and identify the most effective way to improve the efficiency of a DMU. In this research, we use Kao's parallel network model [16] and decompose the efficiency of each DMU into the efficiency of the internal processes of that DMU. Figure 1 shows the parallel structure of a DMU. As shown in Figure 1, DMU_j has sections or q processes and each section applies the same inputs $X_i, i = 1, \ldots, m$ to produce identical outputs $Y_r, r = 1, \ldots, s$. The total inputs used by all q processes of the DMU_j are equal to $\sum_{k=1}^{q} X_{ij}^{(k)} = X_{ij}$. Also, the sum of all outputs produced by all q processes is equal to the system outputs, i.e. $\sum_{k=1}^{q} Y_{rj}^{(k)} = Y_{rj}$. In the parallel system, there are no intermediate products to connect the sections and all the sections operate independently. To measure the relative efficiency of a DMU in a network environment, Kao [16] proposed a relational model that requires the same factor to have the same coefficient to maintain the relationship of the sections in the system. For example, the inputs of X_{ij} have the same coefficient of v_i and the outputs of Y_{rj} have the same coefficient of u_r. Based on this idea while considering the operation of all sections, the efficiency measurement model of this system under constant return to scale is as follows:

$$
\begin{aligned}
P(1): \quad & E_k = \max \sum_{r=1}^{s} u_r Y_{rk} & \\
\text{s.t.} \quad & \sum_{i=1}^{m} v_i X_{ik} = 1, & \\
& \sum_{r=1}^{s} u_r Y_{rj} - \sum_{i=1}^{m} v_i X_{ij} \leq 0, & j = 1, \ldots, n, \\
& \sum_{r \in O^{(p)}} u_r Y_{rj}^{(p)} - \sum_{i \in I^{(p)}} v_i X_{ij}^{(p)} \leq 0, & p = 1, \ldots, q,\ j = 1, \ldots, n, \\
& u_r, v_i \geq \varepsilon, & r = 1, \ldots, s,\ \ i = 1, \ldots, m.
\end{aligned}
$$

The limitations of this model show that the production frontier is constructed by the sections of all DMUs. The system efficiency for a parallel system is equal to the weighted average of the efficiency of the sections, where the weight of the p process is equal to the total input consumed by the p process divided by the input consumed by all the q processes.

For systems composed of several divisions performing different functions, there are usually joint inputs being shared by all, or several, divisions, in addition to their specific inputs. For example, the area in a healthcare and treatment centers in the terminology of DEA, is jointly used by the sections of each center, which, together with other section-specific inputs, produce different outputs. Most parallel systems in the real world have shared input, and most studies in parallel systems also concentrate on these.

The system efficiency can be measured based on the total input it consumes and the total output it produces while considering the operations of all divisions. Following the structure of the parallel system in Figure 2, the ratio-form model under constant returns to scale is:

$$
\begin{aligned}
P(2): \quad & E_0 = \max \sum_{r=1}^{s} u_r Y_{r0} \\
\text{s.t.} \quad & \sum_{i=1}^{m} v_i X_{i0} + \sum_{l=1}^{q} t_l X_{l0}^{s} = 1, \\
& \sum_{r=s^{(k-1)}+1}^{s^{(k)}} u_r Y_{rj}^{(k)} - \left(\sum_{i=m^{(k-1)}+1}^{m^{(k)}} v_i X_{ij}^{(k)} + \sum_{l=1}^{q} t_l \alpha_l^{(k)} X_{lj}^{s} \right) \le 0, \qquad k = 1, \ldots, p, \;\; j = 1, \ldots, n, \\
& t_l, u_r, v_i \ge \varepsilon, \qquad l = 1, \ldots, q, \;\; r = 1, \ldots, s, \;\; i = 1, \ldots, m.
\end{aligned}
$$

In many cases, it is not clear how input is shared by all segments. For example, in healthcare and treatment centers, the space that a midwife uses to carry out her responsibility is not exactly clear, but it is clear that it is in a reasonable range such as the interval $[L_l, U_l]$. In this case, we can consider the $\alpha_l^{(k)}$ $(l = 1, \ldots, q,\ k = 1, \ldots, p)$ ratio as a variable and search for the most desirable value that brings the most efficiency to the system. Based on this method, the following restrictions are added to the $P(2)$ model.

$$
\begin{aligned}
& L_l^{(k)} \le \alpha_l^{(k)} \le U_l^{(k)}, \qquad l = 1, \ldots, q, \;\; k = 1, \ldots, p, \\
& \sum_{k=1}^{p} \alpha_l^{(k)} = 1, \qquad l = 1, \ldots, q.
\end{aligned}
$$

3 Median problem

The p-median problem seeks to find the location of p facilities among the candidate points so that the weighted sum of the distances of customers to the nearest facility is minimized.
To formulate the p-median problem, consider the following notations:

I: set of demand points,

J: set of possible locations for the facilities,

d_{ij}: distance between customer i and potential facility j,

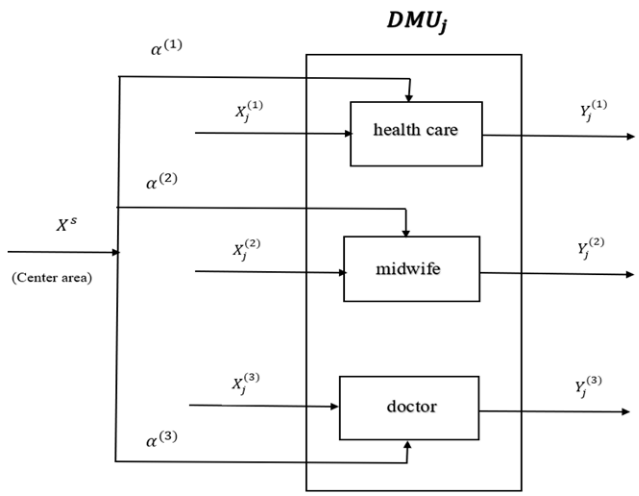

Figure 2: Structure of a DMU with parallel departments

p: total number of facilities to be located,

w_i: weight associated with each demand point (demand or number of customers).

The allocation decisions, namely which facility j satisfies the demand expressed by the customer i, are defined by the following variables:

$$x_{ij} = \begin{cases} 1, & \text{if demand point } i \text{ is allocated to facility } j, \\ 0, & \text{otherwise,} \end{cases} \qquad \forall i, j \in J,$$

and location decisions are represented with:

$$y_j = \begin{cases} 1, & \text{if a point facility is located at point } j, \\ 0, & \text{otherwise,} \end{cases} \qquad \forall j \in J.$$

The p-median problem can be formulated as follows (see e.g., Hakimi [14], ReVelle and Swain [32]):

$$P(3): \quad \min \sum_{i \in I, j \in J} w_i d_{ij} x_{ij}$$

$$\text{s.t.} \quad \sum_{j \in J} x_{ij} = 1, \qquad \forall i \in I, \tag{3.1}$$

$$x_{ij} \leq y_j, \qquad \forall i \in I, \ \forall j \in J, \ i \neq j, \tag{3.2}$$

$$\sum_{j \in J} y_j = p, \tag{3.3}$$

$$x_{ij}, y_j \in \{0, 1\}, \qquad \forall i \in I, \ \forall j \in J. \tag{3.4}$$

Constraints (3.1) ensure that all the demand points are allocated. Constraints (3.2) guarantee that a point receives allocation only if it is a plant. Constraint (3.3) fixes the number of plants to p. Constraints (3.4) state that all variables are binary.

4 Locating and analyzing the efficiency of healthcare and treatment centers in Shahrood city

Shahrood city is located in Semnan province of Iran and has 11 healthcare and treatment centers that cover a population of 165,789 people. We consider each of these centers as a decision-making unit (DMU), which includes three healthcare departments, midwives, and doctors who provide services to clients independently. In each DMU, there is a common input for each department, which produces different outputs along with specific inputs.

Analyzing the efficiency of a center makes it possible to evaluate the strengths and weaknesses or the efficiency and inefficiency of that center, and by eliminating the shortcomings and strengthening the strengths, an important step is taken toward the optimal use of the existing facilities. The most appropriate tool for analyzing the efficiency of healthcare and treatment centers is the use of data envelopment analysis, especially network data.

In this article, we use the p-median model for locating the healthcare and treatment centers to select five centers as health service centers. We aim to minimize the weighted sum of distances between the selected centers and demands, and also maximize the total efficiency of the centers. Considering the efficiency in locating the p-median problem of healthcare and treatment centers helps the decision-maker to reach more effective information and better analysis of the problem, which is realized by DEA methods. Since the conventional DEA methods ignore the efficiency of internal processes in calculating the total efficiency, therefore, in this article, we have used the network DEA method. In this method, the efficiency of the internal parts of each base affects the efficiency of the whole center, and by knowing the strengths and weaknesses of each part, the efficiency of each center can be increased.

As shown in Figure 2, the common input of all departments in each DMU is the area of the healthcare and treatment centers. In each DMU, it is not clear how the area is shared by all segments, but it is clear that it is in a reasonable range such as the interval $[L, U]$.

We use the p-median problem to determine the optimal locations for five healthcare and treatment centers to serve a set of customers, to minimize the total weighted distance or cost of serving customers.

According to Figure 3, we are going to choose 5 centers as the main centers among the 11 healthcare service centers in Shahrood city, so that taking into account the efficiency of these centers, the total costs and also the access time of customers will decrease. For this purpose, we first calculate the efficiency of each of the departments and DMUs with the help of model $P(2)$, then we use the complementary efficiency measure $1-E_i$ of the DMUs as the weight assigned to each center. In model $P(3)$, we set the values of w_i equal to $1-E_i$ and solve the p-median problem (Figure 4). In this formulation, centers with higher efficiency levels are assigned smaller weights, which increases their likelihood of being selected as optimal centers in the minimization process. As a result, the efficiency of centers plays an important role in choosing them as optimal facilities, and the health organization can decide to locate healthcare and treatment services in more efficient centers, thereby reducing the overall cost of people's access to these facilities.

The presented models are solved by Lingo software. Table 1 shows the inputs and outputs of each of the DMUs and subunits. In the first stage of the analysis, we measure the internal and external efficiency of all healthcare and treatment centers, reported in Table 2. We find five efficient centres out of eleven centres that support our efficiency estimation and its discrimination power. Furthermore, even for those efficient centres we still need some improvements in internal sections. Overall, we have a strong and

Figure 3: Healthcare and treatment centers of Shahrood city

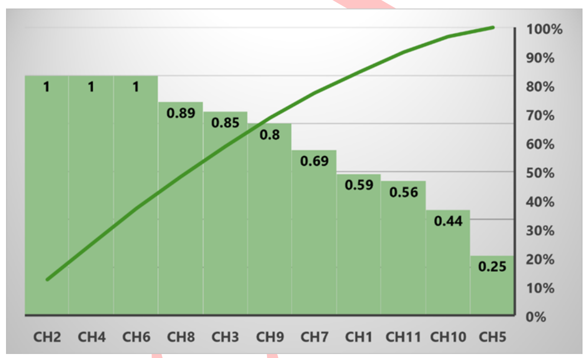

Figure 4: Network efficiency of healthcare centers

reasonable efficiency estimation at this stage. These efficiency measures reflect the internal and external performance of all centres. Thus, more productive centres should have more potential for providing services. This important key issue is considered in the proposed models in the current article that we used for our case study. However, there is another important issue on the location side which is the transportation cost.

There might be a healthcare and treatment center that performs efficiently and its distance to certain clients may be far. On the other side, considering the total expenditure of transportation to transfer patients may sound economical. Our case study is even more sensitive since we are deciding about the patient's transportation and timing may be vital in some cases. Therefore, we should consider the transportation cost of the patients in the transferring process. This is done using the distance data provided in Table 3.

Thus, we simultaneously consider the efficiency of healthcare and treatment centres through the complementary measure $1 - E_i$ as a capacity-related weight, along with transportation costs represented by distances, when transferring and allocating patients to the healthcare and treatment centers.

Table 1: The input and output values of each healthcare center

Decision Making Unit (DMU)	Subunits (Departments)	Number of Staff	Area (m^2)	Number of services to clients
CH1	HealthCare	1	270	3250
	Midwife	5		2895
	Doctor	1		1935
CH2	HealthCare	2	325	5863
	Midwife	4		4530
	Doctor	2		9417
CH3	HealthCare	3	270	8925
	Midwife	3		6870
	Doctor	1		4015
CH4	HealthCare	2	160	11000
	Midwife	2		6058
	Doctor	1		726
CH5	HealthCare	3	250	3270
	Midwife	1		1778
	Doctor	1		933
CH6	HealthCare	7	800	11940
	Midwife	1		6990
	Doctor	2		5350
CH7	HealthCare	2	160	5873
	Midwife	3		4520
	Doctor	1		2163
CH8	HealthCare	1	300	4925
	Midwife	4		4809
	Doctor	1		1300
CH9	HealthCare	2	325	5390
	Midwife	3		5079
	Doctor	1		3805
CH10	HealthCare	1	325	2346
	Midwife	3		2885
	Doctor	1		2100
CH11	HealthCare	3	150	2963
	Midwife	3		3550
	Doctor	1		2680

Table 3 also shows the distance of each of the centers that we use to solve model $P(3)$ and we consider in the location problem. After solving the p-median problem by considering the efficiency of the centers, the results are shown in Table 4. This table presents the optimal solutions of the p-median problem along with the set of centers covered by each selected median. The selection of CH4 and CH6 is fully consistent with the efficiency evaluation results, as both centers are identified as efficient units with an efficiency score equal to one. Given the structure of the objective function in the p-median model, these centers contribute less to the total objective value while serving demand points, which makes them attractive candidates for selection when distance considerations do not dominate. In particular, CH6 is selected as a median and covers several neighboring centers (CH1, CH5, CH9, and CH11), indicating that its efficiency advantage is reinforced by its spatial proximity to these centers. CH8 is selected as a median to serve CH2 and CH3, reflecting the role of distance minimization in the allocation process. Although these centers do not necessarily exhibit the highest efficiency levels, their geographical closeness leads to

Table 2: Internal and external efficiency of health centers

Efficiency of DMUs	Efficiency of subunits	Subunits	DMUs
0.59	0.59	HealthCare	CH1
	0.28	Midwife	
	0.41	Doctor	
1	0.53	HealthCare	CH2
	0.37	Midwife	
	1	Doctor	
0.85	0.54	HealthCare	CH3
	0.74	Midwife	
	0.85	Doctor	
1	1	HealthCare	CH4
	1	Midwife	
	0.15	Doctor	
0.25	0.19	HealthCare	CH5
	0.44	Midwife	
	0.19	Doctor	
1	0.31	HealthCare	CH6
	1	Midwife	
	0.56	Doctor	
0.69	0.46	HealthCare	CH7
	0.74	Midwife	
	0.53	Doctor	
0.89	0.89	HealthCare	CH8
	0.42	Midwife	
	0.27	Doctor	
0.80	0.49	HealthCare	CH9
	0.53	Midwife	
	0.80	Doctor	
0.44	0.42	HealthCare	CH10
	0.30	Midwife	
	0.44	Doctor	
0.56	0.28	HealthCare	CH11
	0.62	Midwife	
	0.61	Doctor	

Table 3: Distance between centers (in km)

	CH1	CH2	CH3	CH4	CH5	CH6	CH7	CH8	CH9	CH10	CH11
CH1	0	3.5	2	4.3	2.1	2.3	3.6	2.7	1.7	2.2	2.7
CH2	3.5	0	3	2.6	4.8	4	6.4	1.8	3	5.2	2.7
CH3	2	3	0	4.6	3.6	3.7	4.1	1.7	2.8	3.1	3.4
CH4	4.3	2.6	4.6	0	4.9	3.7	7.4	3.7	3.3	6	2.4
CH5	2.1	4.8	3.6	4.9	0	1.8	3.6	4.1	2.3	2.2	3
CH6	2.3	4	3.7	3.7	1.8	0	4.8	3.8	1.5	3.3	1.9
CH7	3.6	6.4	4.1	7.4	3.6	4.8	0	5.3	4.7	2	5.6
CH8	2.7	1.8	1.7	3.7	4.1	3.8	5.3	0	2.7	4.2	2.9
CH9	1.7	3	2.8	3.3	2.3	1.5	4.7	2.7	0	3.2	1.4
CH10	2.2	5.2	3.1	6	2.2	3.3	2	4.2	3.2	0	4.1
CH11	2.6	2.7	3.4	2.4	3	1.9	5.6	2.9	1.4	4.1	0

lower transportation costs, making this assignment optimal within the *p*-median framework. Similarly, CH7 and CH10 are selected as individual medians, which can be explained by their relative spatial isolation from other centers; assigning them to alternative medians would increase the total transportation cost beyond the benefit gained from serving them through more efficient but distant centers.

Table 4: The results of solving the *p*-median problem

Centers covered by each solution	Optimal solutions of the *p*-median problem
CH4	CH4
CH1, CH5, CH6, CH9, CH11	CH6
CH7	CH7
CH8, CH2, CH3	CH8
CH10	CH10

Overall, the results demonstrate that the proposed model selects medians based on a trade-off dictated by the *p*-median formulation, where operational performance and transportation cost jointly determine the optimal solution. Efficient centers are favored when distance conditions are comparable, while geographically isolated centers may be selected as medians even if their efficiency levels are lower. This outcome directly reflects the structure of the model and confirms that the integration of efficiency assessment and location analysis operates as intended.

One the main components for providing good and acceptable services is the performance of providers. A unit which performs in the lower optimal condition may not provide an acceptable or even may not be able to do so. Thus considering the performance of healthcare and treatment centers seems critical when allocating patients. On the other hand, assessing performance of healthcare and treatment centers may be critical, especially when they have complex internal systems. This issue is handled by considering both internal and overall performance of healthcare and treatment centers using paralleled network DEA model in the current paper and our case study. decision makers are suggested to consider the production characteristics of production units when assessing their performance. Another important component for allocation of patients is of course geographical condition and transportation cost that should be taken into consideration by decision makers. This issue is handled by the location *p*-median problem in our cases study. It is important to point out that like previous step, namely, performance assessment policy maker should care about the nature of location problem in the process of patient allocation. This case was by done *p*-median location problem in our case study regarding the structure and process of healthcare and treatment centers. One of the main observation in our study is centralization suggestion to reach the optimal condition. Many patients are suggested to reallocate to other single healthcare and treatment centers. However, this reduce the choice of patients to recourse healthcare and treatment centers. We did not consider patient's preferences and associated social effects in our study. This can be a potential topic for further empirical investigation in future researches. However, an important issue in this regard that should be taken into consideration is that the healthcare and treatment centers are centralized running by government. This issue also supports our finding for a centralized service providing in healthcare and treatment centers that are working as sub-organization unit under the control by ministry of healthcare and central government.

5 Conclusion

Considering the importance of the optimal use of healthcare facilities and improving the efficiency of these centers in the country, particularly in Shahrood city, this study employed the p-median location problem to optimally allocate health facilities. In addition, a network data coverage analysis model was used to select the most efficient centers. In this model, by considering each center as a DMU that includes parallel and independent subunits, the efficiency of each subunit and then the DMU is calculated and these values are used to select more efficient centers in the p-median problem. Due to the effect of the efficiency of subunits on the overall efficiency of each unit, the network data coverage analysis model provides better information to the health organization. With this information, it is possible to check the reasons for the inefficiency of the sub-units in each center and take steps to improve it. Also, the p-median provides the best places to allocate advanced health facilities to reduce costs to the organization. Our approach relies on the data availability and we consider available crisp data in the proposed models which can be a limitation. For the future research, data uncertainty is a potential topic for this paper. We also considered a parallel network system in our paper and more complex network systems and their connection with location problems is an interesting theoretical future research line. More ever, we did not consider patient's preferences and associated social effects in our study. This can be a potential topic for further empirical investigation in future research.

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