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Effect of diffraction parameter on ultra relativistic dust-ion-acoustic waves in a quantum dusty plasma

R. Das [a,b], M. Das [c], K. Hosseini [d, e, f,*], F. Alizadeh [d], S. Kheybari [g], B. Kaymakamzade [d]

[a] Department of Mathematics, Arya Vidyapeeth College, Guwahati-781016, Assam, India
[b] Research Center of Applied Mathematics, Khazar University, Baku, Azerbaijan
[c] Department of Mathematics, Gauhati University, Guwahati-781014, Assam, India
[d] Department of Mathematics, Near East University TRNC, Mersin 10, Nicosia 99138, Turkey
[e] Research Center of Applied Mathematics, Khazar University, Baku, Azerbaijan
[f] Faculty of Engineering and Natural Sciences, Istanbul Okan University, Istanbul, Turkey
[g] Faculty of Art and Science, University of Kyrenia, TRNC, Mersin 10, Kyrenia 99320, Turkey

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ABSTRACT

The existence of both fast (compressive) and slow (rarefactive) solitons on the dust-ion-acoustic waves (DIAWs) is investigated considering quantum hydrodynamic (QHD) model with a three-components quantum dusty plasma and relativistic effects $\gamma_i \approx O(v_i^6/c^6)$ using the Korteweg–de Vries (KdV) equation. The impact of different plasma parameters like quantum diffraction parameter (H), ion to electron Fermi temperature ratio (σ_i), dust concentration (N_d), and the relativistic factor $(\nu = v_0/c)$ is also discussed. The relativistic effect $\gamma_i \approx O(v_i^6/c^6)$ demonstrates higher amplitude of the solitons than the relativistic effect $\gamma_i \approx O(v_i^4/c^4)$. From the results, dispersive property of quantum dusty plasma is strongly related to the quantum parameter H. Additionally, it is noticed that the ranges of quantum parameter H for the fast and slow modes, respectively, are significantly impacted by the dust concentration. Applications for the plasma model, which has inertialess electrons, very negatively charged dust grains, and ultrarelativistic positive ions, include fusion research and astrophysics.

1. Introduction

Dusty plasmas are a popular research issue in various fields such as space and current astrophysics, crystal physics, fusion devices, semiconductor technology, plasma chemistry, and biophysics [1]. Interplanetary and interstellar clouds, comet tails, and planetary rings are all

* Corresponding author.
E-mail addresses: kamyar_hosseini@yahoo.com (K. Hosseini)

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examples of dusty plasmas [1 – 6]. In scientific settings, dusty plasma can also be produced by methods like radio frequency discharges, direct current discharges, and the modified Q-machine, among others [7]. In a variety of applications, including plasma crystals [8] and thin-film coating and etching [9], dusty plasma is essential. Numerous researchers have been motivated to examine the novel aspects of the plasma dynamic system by the widespread existence of dusty plasmas. Negatively charged dust particles are typically maintained in the low temperature plasma environment [1]. DIAWs, where were initially studied theoretically in [10] and subsequently observed in laboratory experiments [11 – 13], have theoretically predicted the occurrence of the extremely low phase velocity in relation to the electron and ion thermal speeds DAWs in an unmagnetized dusty plasma. In the DAW, the dust mass supplies the inertia to primarily contain the wave, while the pressures of the inertia, less electrons and ions, supply the restoring forces. Numerous laboratory tests conducted at low temperatures have also revealed the DAW [14 – 17]. Many authors have covered a range of topics related to linear and nonlinear wave propagation in dusty plasmas in earlier years [18 – 23]. When waves have modest amplitudes, the linear theory is crucial to the study of waves and instabilities. Nonetheless, the nonlinearities cannot be ignored when the wave amplitudes are high enough. A few studies have been conducted using models of quantum dusty plasma [24 – 29]. Dusty plasma is referred to as the quantum model because of its extremely low temperature and great particle density. Examples of quantum effects include quantum dots, nanowires [30], intense laser-solid density plasma studies [31], ultracold plasmas [32], ultra-small electronic devices [33], and dense astrophysical settings [34 – 36]. The two primary methods used to study quantum plasmas are the quantum hydrodynamic approach and the quantum kinetic technique. To discuss the Landau damping [37] of waves in quantum plasmas, the kinetic approach is required. The QHD methodology is the most popular method for researching quantum plasmas. The QHD model was initially mathematically derived by Madelung [38]. Haas et al. [39] used the one-dimensional QHD model to investigate the existence of quantum ion acoustic (QIA) waves in unmagnetized quantum plasmas. Several linear and nonlinear aspects of QIA [40, 41], quantum electron acoustic [42 – 44], and quantum positron acoustic [45] solitary waves (SWs) in various plasma regimes have since been studied using the QHD model. The nonlinear propagation of DIA waves in quantum plasma was investigated by Zobaer et al. [46]. Rehman et al. [47] used the KdV equation to study IAWs in a homogenous quantum plasma that contained electrons, positive and negative ions. All of the above studies exclusively take into account non-relativistic scenarios and employ quantum hydrodynamic models. In the formation of solitary waves, the relativistic influence cannot be disregarded [48] when particle speeds are nearing to the speed of light. The relativistic effect in quantum plasma has only been the subject of a small number of published studies. In the weak relativistic limit, Sahu [49] has investigated the relativistic effect on IASWs in quantum plasma by taking into account relativistic electrons and non-relativistic ions. The quantum-weak relativistic influence on ion-acoustic shock waves in electron-positron-ion plasma has been studied by Gill et al. [50]. Ghosh et al. [51] have studied the weak relativistic effect on the properties of the electron plasma waves in quantum plasma. The effects of weak relativistic and quantum mechanics on the ion plasma wave in an unmagnetized dust-ion plasma were studied theoretically by Sahoo et al. [52]. Nevertheless, ions can potentially reach relativistic speed in some real-world settings. When compared to direct laser acceleration of ions, for instance, ions can achieve relativistic speed at comparatively lower light intensity in a laser-plasma interaction experiment due to their collective impact in plasma [53]. Consequently, it becomes crucial to incorporate relativistic ion motion into dust-ion plasma. These investigations encompass the incorporation of relativistic effects, achieved through $\gamma_i = (1-(v_i^2/c^2))^{-1/2} \approx 1+(v_i^2/2c^2)$ for ions or $\gamma_e = \left(1-(v_e^2/c^2)\right)^{-1/2} \approx 1+(v_e^2/2c^2)$. Recently, Madhukalya et al. [54] have used the QHD model to investigate the dynamics of IASWs in an unmagnetized, highly relativistic (considering $\gamma \approx 1+(v^2/2c^2)+(3v^4/8c^4)$) quantum plasma with positive and negative ions and electrons.

Table 1. The values of the expressions γ_0 for different values of v_0/c.

$\frac{v_0}{c}$	$\gamma_0 = 1+\frac{3v_0^2}{2c^2}$	$\gamma_0 = 1+\frac{3v_0^2}{2c^2}+\frac{15v_0^4}{8c^4}$	$\gamma_0 = 1+\frac{3v_0^2}{2c^2}+\frac{15v_0^4}{8c^4}+\frac{35v_0^6}{16c^6}$	$\gamma_0 = 1+\frac{3v_0^2}{2c^2}+\frac{15v_0^4}{8c^4}+\frac{35v_0^6}{16c^6}+\frac{315v_0^8}{128c^8}$
0.1	1.01	1.01	1.01	1.01
0.2	1.06	1.06	1.06	1.06
0.3	1.14	1.15	1.15	1.15
0.4	1.24	1.28	1.29	1.30
0.5	1.38	1.49	1.53	1.54
0.6	1.54	1.78	1.89	1.93
0.7	1.74	2.18	2.44	2.58
0.8	1.96	2.72	3.30	3.71
0.9	2.22	3.44	4.60	5.67
0.95	2.35	3.88	5.49	7.12

Table 1 indicates that the relativistic factor $(v_0/c) \leq 0.30$ (weak relativistic) yields a small discrepancy when higher-order terms are included. Moreover, ***Table 1*** shows that as the relativistic factor $(v_0/c) > 0.30$ (high relativistic) increases, the values of $(v_0/c)^6$ and $(v_0/c)^8$ are not negligible, necessitating the inclusion of higher-order terms in the expression of γ_0. Consequently, the expressions γ_0 involving higher-order terms are more suitable for the investigation of DIASWs within the plasma model. This paper is organized as follows: In Section 2, the fundamental set of QHD equations is presented. In Section 3, the KdV model is extracted using the reduction perturbation method (RPM). In Section 4, after presenting the KdV soliton, the criteria for its existence are provided. In Section 5, results and discussion are given, and finally outcomes are summarized in Section 6.

2. Governing equations

We consider the electrons to be inertialess and analyse a plasma system consisting of extremely negatively charged dust grains and inertial ions. This assumption holds true as long as the wave's phase velocity is substantially lesser than the electron's Fermi velocity (i.e., $v_{Fd} < v_{Fi} << \omega/k << v_{Fe}$). The fundamental equations regulating DIAWs governed by the following set of normalized equations:

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}\left(n_i v_i\right) = 0, \quad \textbf{(1)}$$

$$\left(\frac{\partial}{\partial t}+v_i\frac{\partial}{\partial x}\right)(\gamma_i v_i)+\frac{\partial\phi}{\partial x}+\sigma_i n_i\frac{\partial n_i}{\partial x}=0,\quad \gamma_i=\left(1-\frac{v_i^2}{c^2}\right)^{-\frac{1}{2}}\approx 1+\frac{v_i^2}{2c^2}+\frac{3v_i^4}{8c^4}+\frac{5v_i^6}{16c^6}. \tag{2}$$

The momentum of the inertialess electron is determined by

$$\frac{\partial\phi}{\partial x}-n_e\frac{\partial n_e}{\partial x}+\frac{H^2}{2}\frac{\partial}{\partial x}\left(\frac{1}{\sqrt{n_e}}\frac{\partial^2\sqrt{n_e}}{\partial x^2}\right)=0, \tag{3}$$

and the Poisson equation is as

$$\frac{\partial^2\phi}{\partial x^2}=\alpha n_e-n_i+N_d. \tag{4}$$

Here n_r for $r=i,e$ is the number density of positive ion and electron, respectively. The definitions for normalization $n_r\to n_r/n_{i0}$, $v_i\to v_i/C_{si}$, $\phi\to\phi/(2K_BT_{Fe}/e)$, $x\to x/\lambda_D$, $t\to 1/\omega_{pi}^{-1}$ have been defined. Additionally, $\alpha=n_{e0}/n_{i0}=1-N_d$ (electron to ion equilibrium number density ratio), $N_d=n_{d0}/n_{i0}$ (dust to ion equilibrium number density ration), Z_d is the charge number of the negatively charged dust, and $\sigma_i=T_{Fi}/T_{Fe}$ (ion to electron Fermi temperature ratio). The non-dimensional quantum diffraction parameter $H=\sqrt{(h^2\omega_{pi}^2)/(m_em_iC_{si}^4)}$. The plasma particles for a one-dimensional zero-temperature Fermi gas obey the pressure law, i.e. $p_r=(m_rv_{Fr}^2)/(3n_{r0}^2)n_r^3$ [27, 39].

3. Derivation of the KdV equation

To derive the KdV equation from the basic ***Eqs. (1) – (4)*** for the description of the propagation of DIAWs, we expand the flow variables asymptotically about the equilibrium state in terms of the smallness parameter ε as follows:

$$\begin{aligned}
n_i&=1+\varepsilon n_{i1}+\varepsilon^2 n_{i2}+\cdots,\\
n_e&=1+\varepsilon n_{e1}+\varepsilon^2 n_{e2}+\cdots,\\
v_i&=v_0+\varepsilon v_{i1}+\varepsilon^2 v_{i2}+\cdots,\\
\phi&=\varepsilon\phi_1+\varepsilon^2\phi_2+\varepsilon^3\phi_3+\cdots.
\end{aligned} \tag{5}$$

We use the stretched variables

$$\xi=\varepsilon^{\frac{1}{2}}(x-Ut),\quad \tau=\varepsilon^{\frac{3}{2}}t, \tag{6}$$

such that $\partial/\partial t=\varepsilon^{\frac{3}{2}}\partial/\partial\tau-U\varepsilon^{\frac{1}{2}}\partial/\partial\xi$, $\partial/\partial x=\varepsilon^{\frac{1}{2}}\partial/\partial\xi$, and U represents the phase speed. Substituting (5) and (6) into (1) – (4), first-order equations in the smallness parameter ε are obtained as

$$-(U-v_0)\frac{\partial n_{i1}}{\partial\xi}+\frac{\partial v_{i1}}{\partial\xi}=0, \tag{7}$$

$$-(U-v_0)\gamma_0\frac{\partial v_{i1}}{\partial\xi}+\sigma_i\frac{\partial n_{i1}}{\partial\xi}+\frac{\partial\phi_1}{\partial\xi}=0. \tag{8}$$

Integrating the equations of (7) and (8) subject to the boundary conditions $n_{i1}=n_{e1}=0$, $v_{i1}=v_0$, $\phi_1=0$ at $|\xi|\to\infty$, the following outcomes are constructed:

$$n_{i1} = \frac{\phi_1}{\left\{(U - v_0)^2 \gamma_0 - \sigma_i\right\}}, \quad v_{i1} = \frac{(U - v_0)\phi_1}{\left\{(U - v_0)^2 \gamma_0 - \sigma_i\right\}}, \quad n_{e1} = \phi_1, \quad \alpha n_{e1} - n_{i1} = 0, \tag{9}$$

where $\gamma_0 = 1 + \frac{3}{2}\nu^2 + \frac{15}{8}\nu^4 + \frac{35}{16}\nu^6$ and $\nu = \frac{v_0}{c}$.

By utilizing the values of n_{i1} and n_{e1} in the last equation of (9), we arrive at the equation for phase speed U as

$$U = v_0 \pm \sqrt{\frac{1}{\gamma_0}\left(\sigma_i + \frac{1}{1 - N_d}\right)}. \tag{10}$$

In the subsequent order exceeding ε (i.e. ε^2), second-order perturbation equations are found as

$$\frac{\partial n_{i1}}{\partial \tau} - (U - v_0)\frac{\partial n_{i2}}{\partial \xi} + \frac{\partial v_{i2}}{\partial \xi} + \frac{\partial}{\partial \xi}(n_{i1} v_{i1}) = 0, \tag{11}$$

$$\gamma_0 \frac{\partial v_{i1}}{\partial \tau} - (U - v_0)\gamma_0 \frac{\partial v_{i2}}{\partial \xi} - 2(U - v_0)\gamma_1 v_{i1} \frac{\partial v_{i1}}{\partial \xi} + \gamma_0 v_{i1} \frac{\partial v_{i1}}{\partial \xi}$$

$$+ \sigma_i \frac{\partial n_{i2}}{\partial \xi} + \sigma_i n_i \frac{\partial n_{i1}}{\partial \xi} + \frac{\partial \phi_1}{\partial \xi} = 0, \tag{12}$$

$$\frac{\partial \phi_2}{\partial \xi} - \frac{\partial n_{i2}}{\partial \xi} - n_{e1} \frac{\partial n_{e1}}{\partial \xi} + \frac{H^2}{4} \frac{\partial^3 n_{e1}}{\partial \xi} = 0, \tag{13}$$

$$\frac{\partial^2 \phi_1}{\partial \xi^2} = \alpha n_{e2} - n_{i2}, \tag{14}$$

where $\gamma_1 = \frac{3\nu}{2c} + \frac{15\nu^3}{4c} + \frac{105\nu^5}{16c}$.

Now, by differentiating ***Eq. (14)*** with respect to ξ partially, we have

$$\frac{\partial^3 \phi_1}{\partial \xi^3} = \alpha \frac{\partial n_{e2}}{\partial \xi} - \frac{\partial n_{i2}}{\partial \xi}. \tag{15}$$

From ***Eq. (13)***, we have

$$\frac{\partial n_{e2}}{\partial \xi} = \frac{\partial \phi_2}{\partial \xi} - \phi_1 \frac{\partial \phi_1}{\partial \xi} + \frac{H^2}{4} \frac{\partial^3 \phi_1}{\partial \xi^3} = 0. \tag{16}$$

Eliminating $\frac{\partial v_{i2}}{\partial \xi}$ from ***Eqs. (11)*** and ***(12)*** yields

$$(U - v_0)\gamma_0 \frac{\partial n_{i1}}{\partial \tau} + (U - v_0)\gamma_0 \frac{\partial}{\partial \xi}(n_{i1} v_{i1}) + \gamma_0 \frac{\partial v_{i1}}{\partial \tau} - 2(U - v_0)\gamma_1 v_{i1} \frac{\partial v_{i1}}{\partial \xi}$$

$$- \left\{(U - v_0)^2 \gamma_0 - \sigma_i\right\} \frac{\partial n_{i2}}{\partial \xi} + \gamma_0 v_{i1} \frac{\partial v_{i1}}{\partial \xi} + \sigma_i n_{i1} \frac{\partial n_{i1}}{\partial \xi} + \frac{\partial \phi_2}{\partial \xi} = 0,$$

or

$$\frac{\partial n_{i2}}{\partial \xi} = \frac{2(U - v_0)\gamma_0}{\left\{(U - v_0)^2 \gamma_0 - \sigma_i\right\}^2} \frac{\partial \phi_1}{\partial \tau} + \frac{3(U - v_0)^2 \gamma_0 - 2(U - v_0)^3 \gamma_1 + \sigma_i}{\left\{(U - v_0)^2 \gamma_0 - \sigma_i\right\}^3} \phi_1 \frac{\partial \phi_1}{\partial \xi}$$

$$+ \frac{1}{(U - v_0)^2 \gamma_0 - \sigma_i} \frac{\partial \phi_2}{\partial \xi}. \tag{17}$$

Using ***Eqs. (17)*** and ***(16)*** in ***Eq. (15)***, we derive

$$\frac{2(U-v_0)\gamma_0}{\left\{(U-v_0)^2\gamma_0-\sigma_i\right\}^2}\frac{\partial\phi_1}{\partial\tau}+\left\{\frac{3(U-v_0)^2\gamma_0-2(U-v_0)^3\gamma_1+\sigma_i}{\left\{(U-v_0)^2\gamma_0-\sigma_i\right\}^3}+(1-N_d)\right\}\phi_1\frac{\partial\phi_1}{\partial\xi}$$
$$+\left\{1-(1-N_d)\frac{H^2}{4}\right\}\frac{\partial^3\phi_1}{\partial\xi^3}=0, \quad \textbf{(18)}$$

that ***Eq. (10)*** is used in deducing ***Eq. (18)***. Finally, the authors can construct the KdV equation as

$$\frac{\partial\phi_1}{\partial\tau}+p\phi_1\frac{\partial\phi_1}{\partial\xi}+q\frac{\partial^3\phi_1}{\partial\xi^3}=0, \quad \textbf{(19)}$$

where $p=\frac{A}{B}$ and $q=\frac{C}{B}$ with

$$A=\frac{3(U-v_0)^2\gamma_0-2(U-v_0)^3\gamma_1+\sigma_i}{\left\{(U-v_0)^2\gamma_0-\sigma_i\right\}^3}+1-N_d,\quad \gamma_1=\frac{3v_0}{2c^2}+\frac{15v_0^3}{4c^4}+\frac{105v_0^5}{16c^6},$$
$$B=\frac{2(U-v_0)\gamma_0}{\left\{(U-v_0)^2\gamma_0-\sigma_i\right\}^2},$$
$$C=1-\frac{1}{4}(1-N_d)H^2.$$

4. KdV equation and its soliton wave

To obtain the soliton wave of ***Eq. (19)***, we apply the transformation $\eta=\xi-V\tau$ where V is the soliton velocity. After employing the above transformation, the KdV equation can be integrated to give

$$-V\phi_1+\frac{1}{2}p\phi_1^2+q\frac{d^2\phi_1}{d\eta^2}=0. \quad \textbf{(20)}$$

Now, using the ansatz method [55], we obtain the following soliton wave

$$\phi_1=\frac{3V}{p}\mathrm{sec}h^2\left(\sqrt{\frac{V}{4q}}\eta\right), \quad \textbf{(21)}$$

where $\phi_0=3V/p$ and $\Delta=2\sqrt{q/V}$ stand for the peak amplitude and width of the soliton wave, respectively. The criteria for the existence of KdV soliton follow the conditions

$$0<H<\frac{2}{\sqrt{1-N_d}},\quad U>v_0,\ \text{or}\ H>\frac{2}{\sqrt{1-N_d}},\quad U<v_0. \quad \textbf{(22)}$$

5. Result and discussion

In this section, the impact of several plasma parameters, including the quantum diffraction parameter (H), ion to electron Fermi temperature ratio (σ_i), dust concentration (N_d), and the relativistic factor $(v=v_0/c)$ is discussed. It is discovered that the dusty quantum plasma under consideration supports both compressive and rarefactive solitons in two different ranges of the diffraction parameter (H) for all N_d $(0<N_d<1)$. For $N_d=0$, i.e., when the immobile dust particle is absent, the existence ranges of the diffraction parameter H reduces to the ranges obtained by Sahu [49]. The impact of the relativistic effects, i.e.

$$\gamma_0 = 1+\frac{3v_0^{\,2}}{2c^2}+\frac{15v_0^4}{8c^4}+\frac{35v_0^6}{16c^6}, \text{ (Red Line)}$$

$$\gamma_0 = 1+\frac{3v_0^{\,2}}{2c^2}+\frac{15v_0^4}{8c^4}, \text{ (Blue Line)}$$

on ϕ_1 has been shown in ***Figure 1*** for $V=0.0075$, $\sigma_i=0.2$, $N_d=0.4$, and $\nu(=v_0/c)=0.8$ when **(a)** $H=2$, **(b)** $H=3$. ***Figure 1(a)*** displays the fast compressive soliton, while ***Figure 1(b)*** reflects the slow rarefactive soliton for both relativistic effects. It is observed that the higher-order relativistic effect (HORE) gives rise to a higher amplitude of the solitons as compared to the lower-order relativistic effect (LORE). Physically, higher-order relativistic effects strengthen nonlinear responses by increasing the effective inertia of particles, primarily electrons, which in turn increases the soliton amplitude. This leads to more energetic, higher-amplitude solitons by increasing the steepness and potential of solitary wave formations.

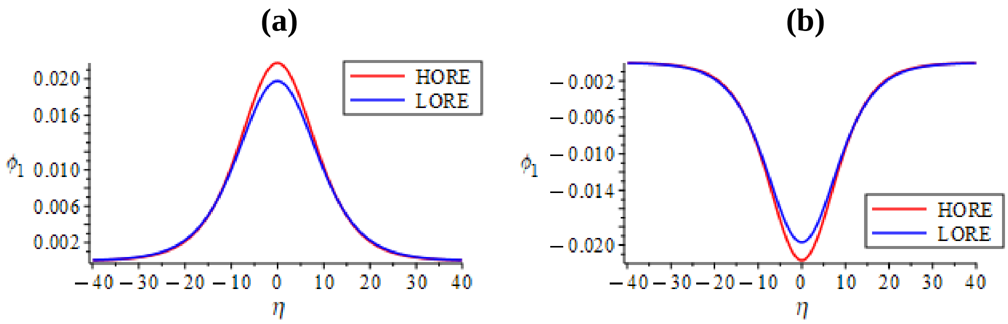

Figure 1. Variation of the KdV soliton involving higher- and lower-relativistic effects for **(a)** fast mode $(H=2)$ and **(b)** slow mode $(H=3)$ when $V=0.0075$, $\sigma_i=0.2$, $N_d=0.4$, and $\nu(=v_0/c)=0.8$.

In ***Figure 2***, the soliton width involving $\nu(=v_0/c)=0.8$ has been shown for fast and slow modes with the quantum diffraction parameter H for $\nu=0.6, 0.7, 0.8$ when $V=0.0075$, $\sigma_i=0.1$, and $N_d=0.2$. The width [***Figure 2(a)***] ([***Figure 2(b)***]) of the fast KdV soliton (slow KdV soliton) decreases rapidly (increases uniformly) with the increase of H. Further, the magnitude of the width of the soliton decreases with the increase of the relativistic factor ν for both the modes. From the computational works, it is revealed that as the dust concentration N_d increases, the ranges of the quantum diffraction parameter H for both fast and slow modes are expanded. For the higher value $N_d=0.9$, the existence range of the quantum diffraction parameter H for both fast and slow modes are $0\le H<6.4$ and $H\ge 6.4$, respectively. This kind of ranges is not seen in any of the previous investigations and this is really a new outcome of the present study.

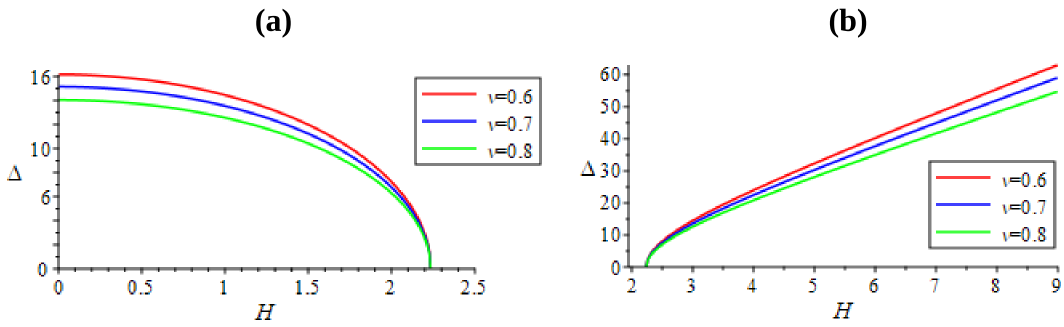

Figure 2. Variation of **(a)** the fast compressive KdV soliton width for $H \le 2.3$ and **(b)** the slow rarefactive KdV soliton width for $H > 2.3$ when $V = 0.0075$, $\sigma_i = 0.1$, $N_d = 0.2$, and $\nu = 0.6, 0.7, 0.8$.

To explore the effect of different values of the quantum diffraction parameter H in the presence of the higher-order relativistic effect, the graphical representation of the KdV solitary wave has been plotted against η in ***Figure 3*** when $V = 0.0075$, $\sigma_i = 0.2$, $N_d = 0.4$, and $\nu = 0.6$. It is seen that the amplitude of the fast compressive (slow rarefactive) solitary waves remains fixed, but the width of the solitons decreases (increases) with the increase of H in the range $0 \le H < 2.6$ (in the range $H \ge 2.6$). It is evident from ***Figures 3(a) and 3(b)*** that H have no impact on the amplitude of the soliton waves.

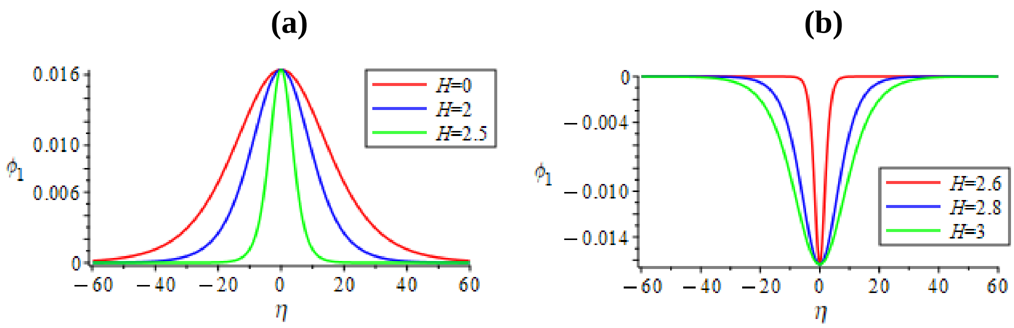

Figure 3. **(a)** Fast compressive ($H = 0, 2, 2.5$) and **(b)** slow rarefactive ($H = 2.6, 2.8, 3$) KdV solitons for $V = 0.0075$, $\sigma_i = 0.2$, $N_d = 0.4$, and $\nu = 0.6$.

Figure 4(a) depicts the structure of solitary waves for the fast mode in the presence of the higher-order relativistic effect, illustrating variations in temperature ratio σ_i for the quantum diffraction parameter $H = 2\,(0 \le H < 2.6)$ when $V = 0.0075$, $N_d = 0.4$, and $\nu = 0.6$. It is important to note that the amplitude as well as width of the compressive (fast) soliton decreases as σ_i increases, which is a congruent result with Rehman et al. [47], while maintaining the same parameter, the amplitude and width of the rarefactive (slow) soliton [***Figure 4(b)***] for the quantum diffraction parameter $H = 3$ (≥ 2.6) shows an opposite trend as σ_i increases.

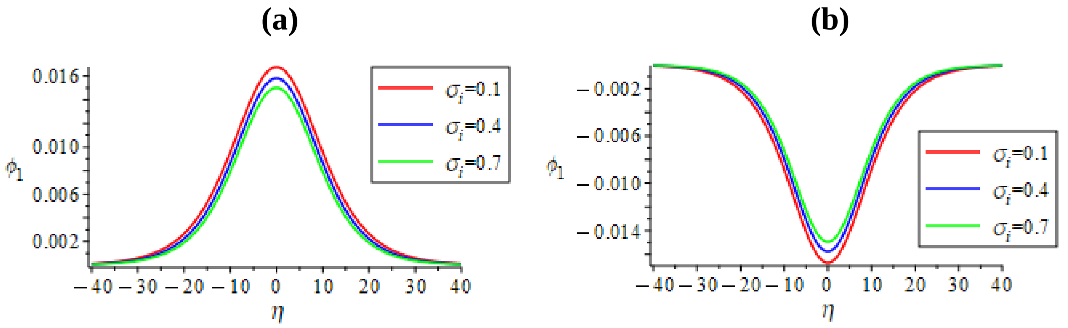

Figure 4. Variation of the KdV soliton as a function of η with $\sigma_i = 0.1, 0.4, 0.7$ for **(a)** Fast $(H = 2)$ and **(b)** slow $(H = 3)$ when $V = 0.0075$, $N_d = 0.4$, and $\nu = 0.6$.

To study the influence of the dust concentration N_d on the fast compressive KdV soliton in the presence of the higher-order relativistic effect with the quantum diffraction parameter $H = 2$, we represent graphically it for distinct values of $N_d = 0.1, 0.2, 0.3$ and $N_d = 0.7, 0.85, 0.9$ in ***Figure 5***. It is clearly evident that due to the impact of the relativistic factor, the dust concentration N_d plays a crucial role a crucial role in determining the soliton amplitude and width. From the results, we observe that the fast compressive solitary wave amplitude and width [***Figure 5(a)***] increase up to a certain value for $N_d \le 0.69$, and there after the amplitude (width) of the soliton [***Figure 5(b)***] decreases (increases) beyond that value of N_d. Moreover, at a higher value of N_d, the amplitude of the soliton does not exceed the value 0.0176.

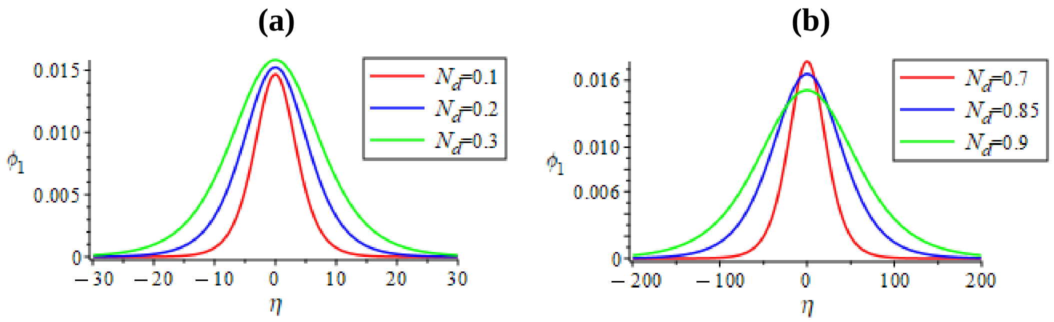

Figure 5. Fast compressive KdV soliton as a function of η for $H = 2$, $V = 0.0075$, $\sigma_i = 0.2$, and $\nu = 0.6$ with the distinct dust concentrations **(a)** $N_d = 0.1, 0.2, 0.3$ and **(b)** $N_d = 0.7, 0.85, 0.9$.

On the other hand, the slow rarefactive solitary wave involving $\nu(= v_0 / c) = 0.8$ has been shown in ***Figure 6*** for $H = 6.5$, $V = 0.0075$, $\sigma_i = 0.2$, and $\nu = 0.6$, when **(a)** $N_d = 0.1, 0.3, 0.5$ and **(b)** $N_d = 0.7, 0.83, 0.87$. From such figures, it is observed that the amplitude and width of the solitary wave respectively increase and decrease by increasing values of N_d. Physically, this is due to a larger background electrostatic environment that changes the balance required for the soliton formation by suppressing big perturbations and enhancing dispersion.

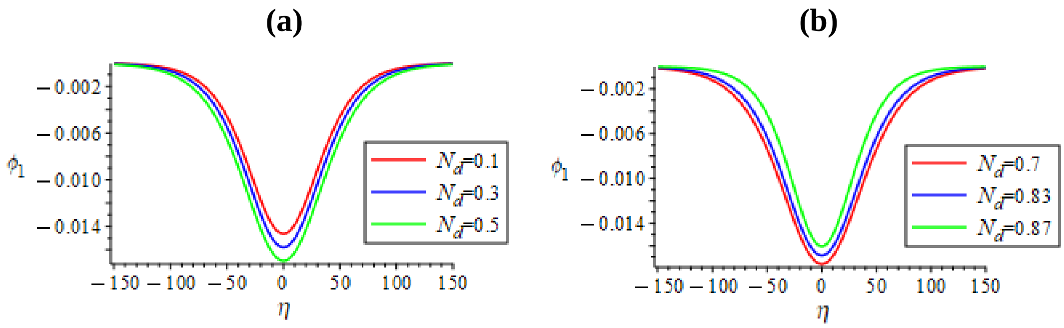

Figure 6. Slow compressive KdV soliton as a function of η for $H = 6.5$, $V = 0.0075$, $\sigma_i = 0.2$, and $\nu = 0.6$ with the distinct dust concentrations **(a)** $N_d = 0.1, 0.3, 0.5$ and **(b)** $N_d = 0.7, 0.83, 0.87$.

To examine the impact of the relativistic factor on the propagation of the solitary wave, we have presented both fast and slow modes in ***Figures 7(a)*** and ***(b)***, respectively, taking different values of the relativistic factor, i.e. $\nu = 0.6, 0.7, 0.8$. In ***Figure 7(a) (7(b))***, the amplitude and width of the fast compressive (slow rarefactive) soliton increases (decreases) as ν increases for $H = 2$ $(H = 3)$. This shows that the soliton wave is energized by the relativistic factor. Physically, when the relativistic factor rises, the particle velocity gets closer to the speed of light. Since solitary waves propagate locally while keeping their structure, their kinetic energy increases with increasing velocity. As a result of the increased energy, the soliton amplitude increases. This conclusion aligns with the findings reported by Madhukalya et al. [54].

6. Conclusion

In this paper, the propagation of DIAWs in an unmagnetized highly relativistic quantum dusty plasma consisting of relativistic positive ions, inertialess electrons, and negatively charged, immobile dust particles has been explored. The KdV equation has been successfully acquired using

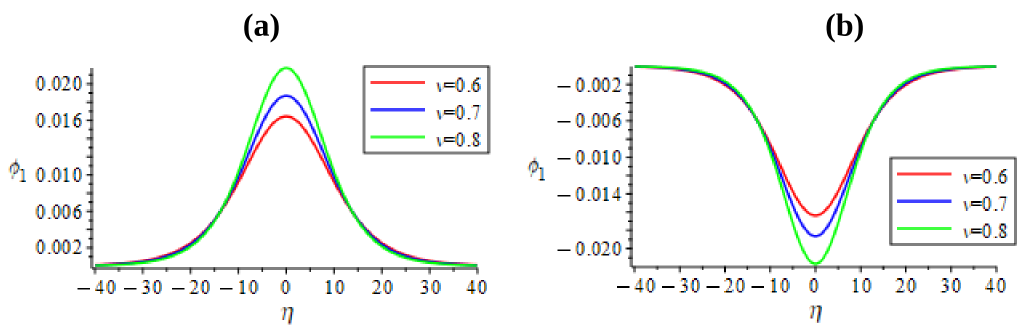

Figure 7. Variation of the KdV soliton as a function of η with distinct relativistic factors $\nu = 0.6, 0.7, 0.8$ for **(a)** fast $(H = 2)$ and **(b)** slow $(H = 3)$ modes when $V = 0.0075$, $\sigma_i = 0.2$, and $N_d = 0.4$.

the RPM, which allows for a comprehensive analysis of the impact of different plasma parameters on the solitary wave solution. The present problem has led us to observe the following conclusions:

1. Our findings demonstrate that the amplitude of the compressive KdV solitons exhibits in fast mode, while the rarefactive KdV solitons exhibits in slow mode.

2. The amplitude of the soliton is found to be larger for higher-order relativistic effects than for lower-order ones.

3. The existence of KdV solitons at the higher value $N_d = 0.9$, the ranges of quantum diffraction parameter H for both fast and slow modes are $0 \le H < 6.4$ and $H \ge 6.4$ respectively.

4. The increasing values of positive ion to electron temperature ratio σ_i reflect decreasing trends of soliton's amplitude for both fast and slow modes.

5. The soliton amplitude increases for a certain values of $N_d \le 0.6$, and thereafter, it shows an decreasing pattern for $N_d > 0.7$.

6. The increasing values of the relativistic factor $(v > 0.4)$ lead to an increase in the amplitude of the solitons for both modes.

We anticipate that such findings contribute to our understanding of the characteristics of nonlinear relativistic quantum structures in astrophysics, space, and lab environments. This paper may be extended by considering dust charge variation in the super relativistic quantum plasma with some critical plasma parameters. In the future, the authors will employ another well-organized method [56] to deal with the governing equation.

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