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An Integrated Multi-Objective MILP Model for Rebar Delivery Scheduling and Vehicle Routing: A Case Study

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ABSTRACT

This study addresses the critical challenge of optimizing rebar delivery in heavy logistics industries by proposing an integrated multi-objective mixed-integer linear programming (MILP) model for simultaneous delivery scheduling and vehicle routing. The model aims to minimize three conflicting objectives: the overall makespan of deliveries, the weighted customer dissatisfaction from delivery time windows based on customer priority, and the total transportation costs. A fuzzy multi-objective optimization approach, based on the principles of Bellman and Zadeh and Zimmermann's method, is employed to transform this complex problem into a single-objective maximization problem of an overall satisfaction level. The efficacy and practical applicability of the proposed model are validated through a real-world case study from Amir Kabir Khazar Steel Company in Gilan province, Iran. The case study involves 51 customer orders to be delivered over a three-day planning horizon, incorporating realistic constraints such as specific time windows and customer priority levels. Computational results, obtained using GAMS with the CPLEX solver, demonstrate that the model successfully achieves a high overall satisfaction level of λ =0.841. The findings offer significant managerial insights for balancing operational efficiency, cost reduction, and customer satisfaction in rebar supply chains.

1. Introduction

In today's competitive and customer-driven markets, effective delivery scheduling and vehicle routing are essential components of supply chain management, especially in industries with heavy logistics operations such as steel manufacturing. Timely delivery of customer orders, efficient fleet

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utilization, and cost minimization are no longer operational luxuries but strategic necessities. The rebar supply chain, due to the bulky and high-volume nature of its products, faces critical challenges in planning optimal deliveries while respecting time windows and ensuring customer satisfaction.

In particular, the transportation of rebar products from manufacturing plants to dispersed customer locations involves complex decisions related to scheduling, routing, and resource allocation. These decisions must consider various constraints such as vehicle capacity, customer priority, loading/unloading time, fixed and variable transportation costs, and service time windows. Failure to address these dimensions can lead to increased costs, reduced service quality, and loss of market competitiveness.

This study addresses these challenges by formulating an integrated multi-objective mixed-integer linear programming (MILP) model that jointly optimizes delivery scheduling, vehicle routing, and cost-performance trade-offs in the rebar supply chain. The model aims to simultaneously minimize (1) the overall makespan of deliveries, (2) the weighted customer dissatisfaction from delivery time window based on customer priority, and (3) the total transportation costs, including both fixed and distance-based variable costs.

The novelty of this research lies in its integrated perspective, which combines delivery scheduling with real-world constraints such as customer priority levels, and vehicle return policies. Additionally, the model is customized for application in a real industrial setting—the rebar supply chain of Amir Kabir Khazar Steel Company in Gilan province, Iran—providing a unique practical contribution that bridges the gap between theory and practice in the field of computational supply chain optimization. This practical application, utilizing specific operational data (e.g., customer orders, time windows, and priority information), provides robust insights into the model's effectiveness in a real-world scenario.

The main contributions of this paper are summarized as follows:

- A comprehensive MILP formulation that integrates delivery scheduling and vehicle routing with time windows.
- Consideration of realistic and industry-specific constraints, including customer priorities for time windows, and mixed cost structures.
- Application of the model to a real-world case study in the Iranian steel industry (Amir Kabir Khazar Steel Company), demonstrating its practical feasibility and effectiveness.
- Insights into the trade-offs between delivery performance (makespan and customer satisfaction) and cost optimization under multi-objective planning.

The remainder of the paper is organized as follows: Section 2 presents a detailed literature review on delivery scheduling and vehicle routing models in manufacturing logistics. Section 3 introduces the problem description, mathematical formulation, and model assumptions. Section 4 presents the case study data and computational results. Section 5 discusses managerial implications. Finally, Section 6 concludes the paper and outlines directions for future research.

2. Literature Review

The optimization of delivery scheduling and vehicle routing problems (VRPs) has been a cornerstone of logistics and supply chain management research for decades, driven by its significant impact on operational efficiency, cost reduction, and customer satisfaction [1]. This section provides a comprehensive review of the relevant literature, emphasizing recent advancements and identifying the gaps that this study aims to address, particularly within the context of heavy industries like steel manufacturing and rebar supply chains.

2.1. Evolution of Vehicle Routing Problems (VRPs)

The classic VRP, introduced by Dantzig and Ramser in 1959, seeks to minimize the total travel distance for a fleet of vehicles serving a set of customers from a central depot [2]. Over time, numerous variants have emerged to capture real-world complexities. The Capacitated Vehicle Routing Problem (CVRP) considers vehicle capacity limitations [3], while the Vehicle Routing Problem with Time Windows (VRPTW) incorporates customer-specific delivery or service time windows, a highly relevant constraint in time-sensitive industries [4]. More recently, research has branched into dynamic VRPs, stochastic VRPs, and green VRPs, reflecting the increasing need for adaptability, uncertainty management, and environmental considerations [5, 6]. These advancements highlight the continuous effort to model and solve more realistic and complex distribution challenges.

2.2. Integrated Scheduling and Routing Approaches

Traditionally, delivery scheduling and vehicle routing have often been treated as separate problems. However, the interdependent nature of these decisions in real-world supply chains has led to a growing interest in integrated approaches. Such integration can lead to substantial improvements in overall supply chain performance by eliminating sub-optimality that arises from sequential decision-making [7]. Recent works have explored various integration levels, from simultaneous scheduling and routing to more complex models that link these decisions with inventory management or even production planning [8]. For instance, a multi-objective MILP model was proposed for dynamic fleet scheduling and multi-modal transport optimization, demonstrating significant reductions in costs, delays, and emissions [9]. Similarly, studies have integrated order consolidation with vehicle routing, showing the benefits of time-based and quantity-based models under different demand scenarios [10]. These integrated approaches are particularly relevant in environments where resource utilization across different operational facets is critical.

2.3. Multi-Objective Optimization in Logistics

Many real-world logistics problems involve multiple, often conflicting, objectives. Common objectives include minimizing total cost (transportation, fixed, operational), minimizing travel time or makespan, maximizing customer satisfaction (e.g., minimizing tardiness, maximizing service quality), and reducing environmental impact (e.g., CO2 emissions) [11]. Multi-objective optimization techniques, ranging from exact methods for smaller instances to heuristic and meta-heuristic approaches for larger, more complex problems, have been widely employed. Recent research has focused on multi-objective vehicle routing and loading with time window constraints, aiming to minimize travel distance, number of routes, and mixed orders, often transforming conflicting objectives into a single objective function for optimization or using Pareto optimization techniques [12]. The increasing complexity of supply chains necessitates models that can effectively balance multiple performance indicators.

2.4. Applications in Heavy Industries and Steel Supply Chains

The specific characteristics of heavy industries, such as steel manufacturing, present unique challenges for logistics optimization. These include large product volumes, high transportation costs, specific loading/unloading requirements, and often tight delivery schedules driven by construction project timelines. While general VRP literature is vast, studies specifically addressing steel supply chain logistics, particularly rebar, are relatively limited compared to other sectors.

Early works in steel logistics focused on basic transportation problems. More recent research has begun to integrate aspects like production scheduling with distribution [13]. For example, studies have addressed optimizing rebar processing and supply chain management using advanced techniques like Building Information Modeling (BIM) and data-driven approaches, though these often focus on cutting optimization and material management rather than integrated delivery scheduling and routing [14, 15]. Some papers have investigated vehicle routing problems with time windows in the distribution of steel products, aiming to improve efficiency and customer satisfaction [16]. Furthermore, green vehicle routing and scheduling models have been developed for ship steel distribution centers, considering carbon emissions alongside traditional cost metrics [17]. However, a comprehensive integrated MILP model that simultaneously addresses makespan, weighted customer dissatisfaction based on customer priority and time window preferences, and total transportation costs within a rebar supply chain, particularly with the specific real-world constraints of customer priority levels and detailed time window management, remains an underexplored area.

2.5. Research Gaps and Contributions

Despite the extensive research in VRPs and supply chain optimization, several gaps persist that this study aims to fill, especially within the context of the rebar supply chain:

- **Integrated Multi-Objective Optimization for Rebar:** While some studies address multi-objective VRPs, few specifically formulate an integrated MILP model that simultaneously considers makespan, weighted customer dissatisfaction (with specific time window priorities), and total transportation costs for the unique demands of a rebar supply chain.

- **Realistic Constraints in Rebar Logistics:** Existing models often overlook critical practical constraints found in rebar delivery, such as varying customer priority levels for delivery time windows. This study explicitly incorporates these nuanced constraints, which are based on real-world business practices (e.g., customer purchase history and need).

- **Bridging Theory and Practice:** A significant gap remains in the application of sophisticated optimization models to real-world industrial settings, particularly in the Iranian steel industry. This research contributes by applying the developed MILP model to a detailed case study, utilizing specific operational data (e.g., customer orders, time windows, and priority information), demonstrating its practical feasibility and providing valuable managerial insights.

- **Trade-off Analysis for Rebar Distribution:** The explicit analysis of trade-offs between delivery performance (makespan and customer satisfaction derived from time window adherence) and cost optimization within a multi-objective framework, tailored for the rebar supply chain, is also a less explored area.

By addressing these gaps, this paper provides a novel and practically relevant contribution to the literature on logistics and supply chain optimization in heavy industries.

3. Problem Description, Mathematical Formulation, and Model Assumptions

This section provides a detailed description of the integrated multi-objective delivery scheduling and vehicle routing problem. We first define the problem scope and key elements, followed by a clear exposition of the underlying assumptions. Subsequently, we introduce the notation used in the mathematical formulation and present the complete mixed-integer linear programming (MILP) model, including its objective functions and constraints.

3.1. Problem Definition

The core problem addressed in this study is the integrated optimization of delivery scheduling and vehicle routing for rebar products from a central manufacturing plant (depot) to multiple geographically dispersed customer locations. The objective is to efficiently manage the distribution of rebar, which is characterized by its bulky nature, high volume, and specific handling requirements.

The problem considers a planning horizon during which a fleet of heterogeneous vehicles, originating from and returning to a single depot, is tasked with fulfilling customer orders. Each customer has a specific demand for rebar and a predefined time window for delivery. Importantly, customers also have priority levels assigned to their orders based on factors such as their past purchase history and current need. These priorities influence the penalty incurred for any deviation from preferred delivery time windows. Real-world constraints such as vehicle capacity limitations and specific service times at customer locations are explicitly incorporated.

The overarching goal is to achieve a balance between operational efficiency and customer satisfaction by simultaneously minimizing three conflicting objectives:

1. **Overall makespan of deliveries (Cmax)**: This refers to the total time elapsed from the start of the first delivery to the completion of the last delivery across the entire planning horizon. Minimizing makespan ensures efficient utilization of the planning period and potentially reduces lead times.

2. **Weighted customer dissatisfaction from delivery time window (f2):** This objective aims to minimize a weighted sum of dissatisfaction, where the weights are based on the priority of each time window for a specific customer order. This directly reflects customer satisfaction and adherence to service level agreements.

3. **Total transportation costs (f3):** This includes both fixed costs associated with activating a vehicle route, variable costs proportional to the distance traveled, and waiting costs for the vehicle at customer locations. Minimizing these costs ensures economic viability of the logistics operations.

The integration of scheduling and routing decisions means that the model simultaneously determines which customers are served, by which vehicle, the sequence in which customers are visited on each route, and the specific departure and arrival times, all while respecting vehicle capacities, time windows, and customer priorities.

3.2 Model Assumptions

To simplify the complexity of the real-world problem while retaining its critical characteristics, the following assumptions are made for the MILP model:

- **Single Depot:** All vehicles originate from and return to a single manufacturing plant (depot).
- **Known Demand:** All customer demands for rebar are known and deterministic at the beginning of the planning horizon.
- **Homogeneous Product:** All rebar products are considered homogeneous in terms of handling and loading characteristics, though their weight and volume contribute to vehicle capacity.
- **Heterogeneous Fleet:** The fleet consists of a limited number of vehicles, which may differ in fixed costs (FC_k) and variable costs (VC_k).
- **Fixed Travel Times/Distances:** Travel times (d_j) and distances (D_j) between any two locations (depot and customers, or between customers) are known, deterministic, and constant.
- **Predefined Time Windows:** Each customer has specific time windows ($E_t,\ L_t$) for delivery.
- **Customer Priority Levels:** Each customer order is associated with a priority weight ($P_{j,t}$) for a given time window t, which is determined based on customer's previous purchase history and need.
- **Single Delivery Per Customer Per Day:** Implicit in the formulation that each order j is served exactly once across all vehicles and all time windows.
- **No Intermediate Depots:** There are no intermediate transfer points or cross-docking facilities between the plant and customers.
- **No Backhauls:** Vehicles are only used for outbound deliveries and do not pick up goods for return to the depot or other customers.
- **Constant Loading/Unloading Times:** Service time (s_j) at each customer location is known and constant.

- **Waiting Cost for Vehicle:** A waiting cost (WC_k) for vehicles at customer locations is considered.

3.3 Notation

This section introduces the sets, parameters, and decision variables used in the mathematical formulation of the integrated multi-objective MILP model.

Indices and sets:

i , j : Index of orders.
k : Index of vehicles.
t : Index of time windows.
z : Index of objectives.
J : Set of all orders.
K : Set of all available vehicles.
T : Set of all time windows.

Parameters:

D_j : Travel distance for delivering order j.
d_j : Travel time for customer order delivery j.
s_j : Service time for customer order j.
λ_j : Weight of customer order j.
$P_{j,t}$: Priority of time window t for customer order j. This parameter indicates the preference of delivering order j within time window t. A lower value of $P_{j,t}$ implies higher satisfaction or lower dissatisfaction for being served in time window t.
E_t : Lower bound of time window t.
L_t : Upper bound of time window t.
FC_k : Fixed cost of using vehicle k.
VC_k : Variable cost of vehicle k per unit of time.
WC_k : Waiting cost of vehicle k per unit of time.
$bigM$: A large positive number.

Decision Variables:

C_{max} : Continuous variable, completion time of the last order.
a_j : Continuous variable, arrival time of order j at the customer location.
w_j : Continuous variable, waiting time for order j at the customer location.
$x_{j,k}$: Binary variable, 1 if order j is delivered by vehicle k; 0 otherwise.
$y_{i,j,k}$: Binary variable, 1 if order i is delivered before order j by vehicle k; 0 otherwise.
$z_{j,t}$: Binary variable, 1 if order j is delivered to the customer within time window t; 0 otherwise.
u_k : Binary variable, 1 if vehicle k is used; 0 otherwise.
$v_{j,k}$: Auxiliary continuous variable for linearization.

3.4 Mathematical Formulation

The multi-objective integrated delivery scheduling and vehicle routing problem is formulated as a Mixed-Integer Linear Programming (MILP) model. The formulation includes three objective functions that are minimized simultaneously and a set of constraints that define the feasible region of the solution space.

$$min\ f_1 = C_{max} \tag{1}$$

$$min\ f_2 = \sum_{j \in J} \sum_{t \in T} \lambda_j \cdot P_{j,t} \cdot z_{j,t} \tag{2}$$

$$min\ f_3 = \sum_{k \in K} FC_k \cdot u_k + \sum_{j \in J} \sum_{k \in K} VC_k \cdot d_j \cdot x_{j,k} + \sum_{j \in J} \sum_{k \in K} WC_k \cdot w_j \cdot x_{j,k} \tag{3}$$

subject to:

$$\sum_{k \in K} x_{j,k} = 1\ ,\ \ \forall j \in J \tag{4}$$

$$\sum_{t \in T} z_{j,t} = 1\ ,\ \ \forall j \in J \tag{5}$$

$$a_j \geq a_i + w_i + s_i + d_i + d_j - bigM \cdot (3 - x_{j,k} - x_{i,k} - y_{i,j,k})\ ,\ \ \forall i \in J\ , j \in J\ , i \neq j\ , k \in K \tag{6}$$

$$a_i \geq a_j + w_j + s_j + d_j + d_i - bigM \cdot (2 - x_{j,k} - x_{i,k} + y_{i,j,k})\ ,\ \ \forall i \in J\ , j \in J\ , i \neq j\ , k \in K \tag{7}$$

$$\sum_{t \in T} E_t \cdot z_{j,t} \leq a_j + w_j\ ,\ \ \forall j \in J \tag{8}$$

$$\sum_{t \in T} L_t \cdot z_{j,t} \geq a_j + w_j\ ,\ \ \forall j \in J \tag{9}$$

$$c_{max} \geq a_j + w_j + s_j + d_j\ ,\ \ \forall j \in J \tag{10}$$

$$a_j \geq d_j\ ,\ \ \forall j \in J \tag{11}$$

$$y_{i,j,k} + y_{j,i,k} + 1 \geq x_{i,k} + x_{j,k}\ ,\ \ \forall i \in J\ , j \in J\ , i \neq j\ , k \in K \tag{12}$$

$$y_{i,j,k} \leq x_{i,k},\ \ \forall i \in J\ , j \in J\ , i \neq j\ , k \in K \tag{13}$$

$$y_{i,j,k} \leq x_{j,k}\ ,\ \ \forall i \in J\ , j \in J\ , i \neq j\ , k \in K \tag{14}$$

$$\sum_{j \in J} x_{j,k} \geq u_k\ ,\ \ \forall k \in K \tag{15}$$

$$\sum_{j \in J} x_{j,k} \leq bigM \cdot u_k\ ,\ \ \forall k \in K \tag{16}$$

$$C_{max} \geq 0 \quad \textbf{(17)}$$

$$a_j \geq 0, \ \forall j \in J \quad \textbf{(18)}$$

$$w_j \geq 0, \ \forall j \in J \quad \textbf{(19)}$$

$$x_{j,k} \in \{0,1\}, \ \forall j \in J, k \in K \quad \textbf{(20)}$$

$$y_{i,j,k} \in \{0,1\}, \ \forall i \in J, j \in J, i \neq j, k \in K \quad \textbf{(21)}$$

$$z_{j,t} \in \{0,1\}, \ \forall j \in J, t \in T \quad \textbf{(22)}$$

$$u_k \in \{0,1\}, \ \forall k \in K \quad \textbf{(23)}$$

Eq. (1) represents the first objective function (f_1), which aims to minimize C_{max}. This variable signifies the completion time of the last order, ensuring that the overall delivery process is finalized as quickly as possible within the planning horizon. ***Eq. (2)*** represents the second objective function (f_2), which aims to minimize the weighted sum of customer dissatisfaction related to delivery time windows. The term $\lambda_j \cdot P_{j,t}$ quantifies the dissatisfaction incurred if order j is delivered within time window t. Here, $P_{j,t}$ values, derived from customer preference, reflect the desirability of a specific time window for order j. By minimizing this sum, the model seeks to assign orders to time windows that result in the lowest overall customer dissatisfaction. ***Eq. (3)*** represents the third objective function (f_3), which aims to minimize the total transportation costs. This cost is composed of three elements: the fixed cost associated with utilizing each vehicle ($\sum_{k\in K} FC_k \cdot u_k$), the variable cost proportional to the travel time for delivering each order ($\sum_{j\in J}\sum_{k\in K} VC_k \cdot d_j \cdot x_{j,k}$), and the cost incurred due to waiting time at customer locations ($\sum_{j\in J}\sum_{k\in K} WC_k \cdot w_j \cdot x_{j,k}$).

Eq. (4) ensures that each order j is assigned to exactly one vehicle k for delivery. ***Eq. (5)*** guarantees that each order j is delivered within exactly one designated time window t. ***Eq. (6)*** is a sequencing constraint that defines the arrival times. It enforces that if order i precedes order j on vehicle k, the arrival time at customer j (a_j) must be greater than or equal to the completion time of service at customer i ($a_i + w_i + s_i$) plus the travel time from i to j ($d_i + d_j$). ***Eq. (7)*** is also a sequencing constraint, similar to ***Eq. (6)***, but it applies if order j precedes order i on vehicle k. It ensures that the arrival time at customer i (a_i) is greater than or equal to the completion time of service at customer j ($a_j + w_j + s_j$) plus the travel time from j to i ($d_j + d_i$).

Eq. (8) enforces the earliest service time window. It ensures that the actual service start time at customer j (arrival time a_j plus waiting time w_j) is not earlier than the earliest allowable time E_t of the assigned time window t. ***Eq. (9)*** enforces the latest service time window. It ensures that the actual service start time at customer j (arrival time a_j plus waiting time w_j) is not later than the latest allowable time L_t of the assigned time window t. ***Eq. (10)*** links the overall makespan C_{max} to the completion times of individual orders. It ensures that C_{max} is greater than or equal to the completion time of the last order delivered, calculated as arrival time a_j plus waiting time w_j, service time s_j, and travel time d_j. ***Eq. (11)*** ensures that the arrival time at any customer j (a_j) is at least the travel time from the depot to customer j (d_j).

Eq. (12) is critical for defining routes and ensuring logical sequencing. It ensures that if two orders i and j are assigned to the same vehicle k, then one must logically precede the other on that vehicle's route. ***Eq. (13)*** ensures that if order i precedes order j by vehicle k ($y_{i,j,k} = 1$), then order i must indeed be assigned to vehicle k ($x_{i,k} = 1$). ***Eq. (14)*** ensures that if order i precedes order j by vehicle k ($y_{i,j,k} = 1$), then order j must indeed be assigned to vehicle k ($x_{j,k} = 1$). ***Eq. (15)*** is a vehicle usage constraint. It ensures that if no order is assigned to vehicle k ($\sum_{j\in J} x_{j,k} = 0$), then the binary variable u_k is set to zero, indicating that vehicle k is not used. ***Eq. (16)*** also relates to vehicle usage. It ensures that if vehicle k is not used ($u_k = 0$), then no orders can be assigned to it ($\sum_{j\in J} x_{j,k} = 0$). If $u_k = 1$, the constraint becomes non-binding. ***Eqs. (17) – (22)*** indicate the type of decision variables.

3.5 Linearization of Objective Function f_3

The objective function f_3 in ***Eq. (3)*** contains a non-linear term, $WC_k \cdot w_j \cdot x_{j,k}$, which is a product of two decision variables (w_j and $x_{j,k}$). To transform this into a linear programming model, a common technique is used by introducing an auxiliary continuous variable $v_{j,k}$. The term $w_j \cdot x_{j,k}$ is replaced by $v_{j,k}$ in the objective function, and a set of additional linear constraints are added to ensure that $v_{j,k}$ correctly reflects the product of w_j and $x_{j,k}$.

The linearized objective function f_3 is given by:

$$f_3 = \sum_{k\in K} FC_k \cdot u_k + \sum_{j\in J}\sum_{k\in K} VC_k \cdot d_j \cdot x_{j,k} + \sum_{j\in J}\sum_{k\in K} WC_k \cdot v_{j,k} \tag{24}$$

And the following constraints ensure the correct behavior of $v_{j,k}$:

$$v_{j,k} \le bigM \cdot x_{j,k}\,, \quad \forall j \in J\,, k \in K \tag{25}$$

$$v_{j,k} \le w_j\,, \quad \forall j \in J\,, k \in K \tag{26}$$

$$v_{j,k} \ge w_j - \left(1 - x_{j,k}\right) \cdot bigM\,, \quad \forall j \in J\,, k \in K \tag{27}$$

$$v_{j,k} \ge 0, \quad \forall j \in J\,, k \in K \tag{28}$$

Eq. (24) represents the revised objective function for total cost (f_3), where the non-linear term $w_j \cdot x_{j,k}$ has been replaced by the auxiliary variable $v_{j,k}$. ***Eqs. (25) – (28)*** work together to correctly define $v_{j,k}$ as the product of w_j and $x_{j,k}$.

3.6 Multi-Objective Solution Approach

To address the multi-objective nature of the problem, the proposed three-objective MILP model is transformed into a single-objective problem using concepts derived from Bellman and Zadeh's fuzzy decision-making principle [18] and Zimmermann's method [19]. This approach allows for the simultaneous optimization of conflicting objectives by converting them into a single objective function that maximizes a satisfaction level across all objectives. The transformation of this three-objective problem into a single-objective one is specifically inspired by the research of Badri et al. [20]. The steps of this method are as follows:

Step 1. Determine Positive and Negative Ideal Solutions (PIS and NIS): For each objective function, the best possible solution (Positive Ideal Solution, PIS) and the worst possible solution (Negative Ideal Solution, NIS) are determined. This is achieved by solving each of the three objective functions (f_1, f_2, f_3) independently as a single-objective MILP model. When one objective function is optimized, the values of the other two objective functions are also recorded. For each objective function, among the three calculated values (when each objective is optimized separately), the minimum value obtained is considered the PIS (f_z^{PIS}) and the maximum value obtained is considered the NIS (f_z^{NIS}). It is important to note that all three objective functions in the proposed model are of minimization type.

Step 2. Define Linear Membership Functions: For each objective function f_z, a linear membership function $\mu_z(f_z)$ is defined. This function quantifies the degree of satisfaction for a given objective value, ranging from 0 (completely unsatisfied) to 1 (fully satisfied). The membership function is defined as follows:

$$\mu_z(f_z) = \begin{cases} 1, & if\ f_z \le f_z^{PIS} \\ \dfrac{f_z^{NIS} - f_z}{f_z^{NIS} - f_z^{PIS}}, & if\ f_z^{PIS} \le f_z \le f_z^{NIS} \\ 0, & if\ f_z \ge f_z^{NIS} \end{cases} \qquad \forall z \in \{1,2,3\} \tag{29}$$

This function reflects that satisfaction is 1 when the objective value is at its best (PIS), decreases linearly as it moves towards the worst (NIS), and becomes 0 when it reaches or exceeds the worst (NIS).

Step 3. Formulate the Equivalent Single-Objective Problem: Using the linear membership functions and following the fuzzy decision-making principle of Bellman and Zadeh, the multi-objective linear programming problem is converted into an equivalent single-objective problem. This principle suggests maximizing the minimum degree of satisfaction across all objectives. This leads to the following formulation:

$$max\left[min\left\{\frac{f_1^{NIS} - f_1}{f_1^{NIS} - f_1^{PIS}}, \frac{f_2^{NIS} - f_2}{f_2^{NIS} - f_2^{PIS}}, \frac{f_3^{NIS} - f_3}{f_3^{NIS} - f_3^{PIS}}\right\}\right] \tag{30}$$

$$\text{subject to: } (4) - (28)$$

All original constraints of the MILP model (***Eqs. 4-28***) apply as constraints for this problem.

Step 4 Transform to a Single-Objective Linear Programming Problem: Based on Zimmermann's method and considering $0 \le \lambda \le 1$, the multi-objective linear programming problem is finally formulated as a single-objective linear programming problem. Here, λ represents the satisfaction level of objective functions, where a high value of λ indicates that all objectives are optimized with a high degree of satisfaction.

The final single-objective MILP model to be solved is:

$$max\ \lambda$$

$$\text{subject to: } (4) - (28) \text{ and}$$

$$\begin{aligned}
&\lambda \leq \frac{f_1^{NIS} - f_1}{f_1^{NIS} - f_1^{PIS}} \\
&\lambda \leq \frac{f_2^{NIS} - f_2}{f_2^{NIS} - f_2^{PIS}} \\
&\lambda \leq \frac{f_3^{NIS} - f_3}{f_3^{NIS} - f_3^{PIS}} \\
&\lambda \in [0,1]
\end{aligned} \tag{31}$$

This transformation allows the complex multi-objective problem to be solved using standard linear programming solvers, yielding a solution that balances the trade-offs between makespan, customer satisfaction, and total transportation costs according to the defined satisfaction levels.

4. Case Study Data and Computational Results

This section presents the real-world case study data from Amir Kabir Khazar Steel Company, located in Gilan province, Iran. It details the customer information, time windows, and priority levels, which are essential inputs for the proposed multi-objective MILP model. Subsequently, the computational environment and the results obtained from solving the model are presented and analyzed.

4.1. Case Study Data

Amir Kabir Khazar Steel Company, established in 2003 and expanded its production capacity to 750,000 tons per year by 2013, is a prominent producer of high-strength deformed rebar (Grades 500 and 520) in Gilan province. The company prioritizes customer satisfaction, aiming for timely delivery of high-quality products. Amir Kabir Khazar Steel Company has received ISO 9001:2008 for quality management, ISO 14001:2004 for environmental management, and OHSAS 18001:2007 for health and safety. The company has also been selected as the exemplary quality unit of Gilan province for five consecutive years. The quality of rebar is critical, with testing for chemical analysis (quantometry) and tensile/bending tests being performed, and results provided to customers with each shipment. High-strength rebar (Grades 500 and 520) offers benefits such as reduced construction costs (18% to 25%) and increased resistance to earthquakes due to its minimum yield stress of 520 MPa and elongation of at least 16%. The specific features of the rebar produced, such as spindle-shaped ribs, are preferred due to reduced stress concentration. The case study focuses on the delivery of rebar products to various iron supply businesses within Gilan province.

The problem involves the delivery of 51 customer orders which need to be fulfilled over a three-day planning horizon. The operational hours for loading and unloading products are restricted; no operations occur from 8 PM to 8 AM the next day, and these hours are excluded from the time windows.

The specific data inputs for the model are detailed as follows:

Customer Information: ***Table 1*** provides detailed information for each of the 51 customer orders, including their location (city), customer name, number of units, travel time from the depot (d_j), weight of the customer order (λ_j), and customer order service time (s_j).

Time Windows: The 51 orders must be delivered within a three-day period. Delivery operations (loading and unloading) are not allowed from 8 PM to 8 AM the next day. These hours are explicitly excluded from the time windows. As shown in ***Figure 1***, this defines specific allowable time windows for service, spanning across 9 distinct time intervals ($t \in \{1, \dots, 9\}$) over the three days.

$$E_t \in \{480,720,960,1920,2160,2400,3360,3600,3840\}$$

$$L_t \in \{719,959,1200,2159,2399,2640,3599,3839,4080\}$$

Time Window Priorities: ***Table 2*** provides the priority values ($P_{j,t}$) for each order j across the time windows t. These priorities are determined based on the customer's previous purchase history and their current needs. A value of '1' indicates the highest priority (greatest satisfaction) for that time window, while 'M' (a large number) signifies that delivery is not possible in that specific time window.

Vehicle Information: A maximum of 10 vehicles (trailers) are available for rebar transportation. The fixed cost of using each vehicle (FC_k) is 50,000 monetary units. The variable cost per vehicle per unit of time (VC_k) is 480 monetary units, and the waiting cost (WC_k) is 6000 monetary units per unit of time.

Table 1. Information of iron supply businesses in Gilan province

Row	Customer name	City	Weight	Number of orders	Travel time (min)	Service time (min)
1	Shayan	Rasht	0.16	8	25	30
2	Khani	Rasht	0.10	5	43	35
3	Omid	Rasht	0.08	4	43	35
4	Gaskari	Rasht	0.08	4	56	30
5	Hafazi	Rasht	0.06	3	43	45
6	Alijani	Langarud	0.06	3	130	60
7	Ramzani	Rasht	0.04	2	38	60
8	Omarani	Rasht	0.04	2	27	60
9	Mellat	Langarud	0.04	2	126	75
10	Ramzani	Masal	0.04	2	146	75
11	Raghebi	Astaneh-ye Ashrafiyeh	0.02	1	72	80
12	Ahmadi	Talesh	0.02	1	171	75
13	Aslani	Rasht	0.02	1	45	60
14	Ahan 110	Rasht	0.02	1	52	60
15	Kolina	Rasht	0.02	1	40	55
16	Hagh Panah	Rasht	0.02	1	54	60
17	Monzovi	Rudsar	0.02	1	146	55
18	Ali Doost	Someh Sara	0.02	1	67	60
19	Riahini	Lahijan	0.02	1	94	55
20	Mohammadi	Langarud	0.02	1	126	60
21	Vahdat	Langarud	0.02	1	126	60
22	Samen	Langarud	0.02	1	128	70
23	Ghasemi	Langarud	0.02	1	130	65
24	Mehrabian	Langarud	0.02	1	133	65
25	Fateh	Langarud	0.02	1	130	65
26	Molaei	Langarud	0.02	1	130	60

Table 2. Priority of time window t for order j

Order (j)	Time Windows								
	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$	$t = 6$	$t = 7$	$t = 8$	$t = 9$
1	1	2	3	5	6	7	M	M	M
2	1	2	3	5	6	7	M	M	M
3	1	2	3	5	6	7	M	M	M
4	2	5	6	1	2	3	M	M	M
5	2	5	6	1	2	3	M	M	M
6	2	5	6	1	2	3	M	M	M
7	M	M	M	2	2	3	1	2	3
8	M	M	M	2	2	3	1	2	3
9	2	1	3	4	6	6	M	M	M
10	2	1	3	4	6	6	M	M	M
11	2	1	3	4	6	6	M	M	M
12	2	5	6	2	1	3	4	5	M
13	2	5	6	2	1	3	4	5	M
14	2	2	1	3	3	3	3	M	M
15	2	2	1	3	3	3	3	M	M
16	2	2	1	3	3	3	3	M	M
17	2	2	2	1	2	2	3	3	3
18	3	2	1	2	2	2	M	M	M
19	3	2	1	2	2	2	M	M	M
20	3	2	1	2	2	2	M	M	M
21	M	M	M	3	2	1	2	2	2
22	2	1	3	2	2	2	3	M	M
23	2	2	2	1	3	3	4	M	M
24	M	4	4	3	1	3	2	2	M
25	2	1	3	2	2	2	3	M	M
26	2	2	2	1	3	3	4	M	M
27	M	4	4	3	1	3	2	2	M
28	2	2	2	1	3	3	4	M	M
29	M	4	4	3	1	3	2	2	M
30	1	2	3	5	6	7	M	M	M
31	2	5	6	1	2	3	M	M	M
32	M	4	4	3	1	3	2	2	M
33	2	5	6	1	2	3	M	M	M
34	M	4	4	3	1	3	2	2	M
35	M	M	M	3	2	1	2	2	2
36	M	M	M	3	2	1	2	2	2
37	M	M	M	3	2	1	2	2	2
38	2	2	2	1	3	3	4	M	M
39	2	2	2	1	3	3	4	M	M
40	2	2	2	1	3	3	4	M	M
41	2	2	2	1	3	3	4	M	M
42	M	4	4	3	1	3	2	2	M
43	M	4	4	3	1	3	2	2	M
44	M	4	4	3	1	3	2	2	M
45	M	M	M	3	2	1	2	2	2
46	M	M	M	3	2	1	2	2	2
47	M	M	M	3	2	1	2	2	2
48	M	M	M	3	2	1	2	2	2
49	M	M	M	3	2	1	2	2	2
50	M	M	M	3	2	1	2	2	2
51	M	M	M	3	2	1	2	2	2

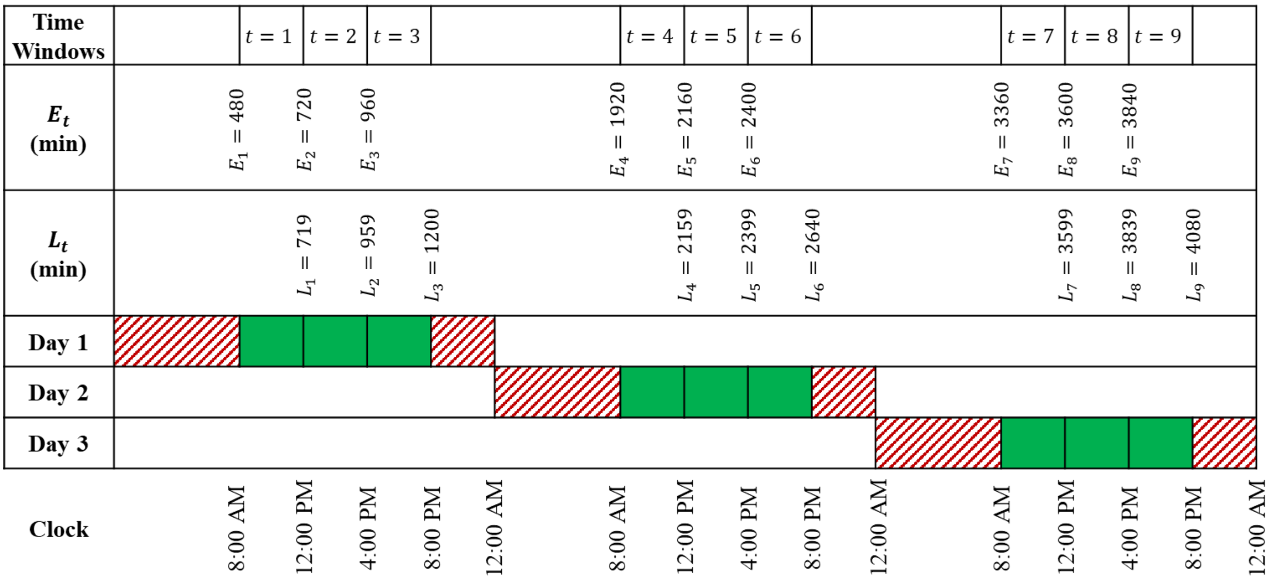

Figure 1. Time windows for service, spanning across 9 distinct time intervals over the three days

4.2. Computational Results

The mathematical model formulated in Section 3 was coded and solved using the GAMS (General Algebraic Modeling System) optimization software. The CPLEX solver was employed to find the optimal solutions. All computational experiments were performed on a computer equipped with an Intel(R) Core(TM) i7-8550U CPU @ 1.80GHz and 16 GB of RAM.

The multi-objective MILP model was solved using the fuzzy optimization approach described in Section 3.6. This involved four distinct optimization runs: three runs to determine the Positive Ideal Solutions (PIS) and Negative Ideal Solutions (NIS) for each objective independently, and a final run to solve the single-objective model maximizing the overall satisfaction level (λ).

The calculated PIS and NIS values for each objective are presented in ***Table 3***.

Table 3. PIS and NIS Values for Objective Functions

Objective	PIS (f_z^{PIS})	NIS (f_z^{NIS})
f_1 (Makespan)	2214	4326
f_2 (Customer Dissatisfaction)	3.42	9.3
f_3 (Total Cost)	2151040	14401040

When f_1 (Makespan) was minimized, $f_1 = 2214$, $f_2 = 8.62$, and $f_3 = 14401040$. When f_2 (Customer Dissatisfaction) was minimized, $f_1 = 3654$, $f_2 = 3.42$, and $f_3 = 2251040$. When f_3 (Total Cost) was minimized, $f_1 = 4326$, $f_2 = 9.3$, and $f_3 = 2151040$.

Based on these individual runs, the PIS for f_1 is 2214 (from minimizing f_1), and its NIS is 4326 (from minimizing f_3). Similarly, for f_2, PIS is 3.42 (from minimizing f_2) and NIS is 9.3 (from minimizing f_3). For f3, PIS is 2151040 (from minimizing f_3) and NIS is 14401040 (from minimizing f_1).

Upon solving the single-objective model maximizing λ, an optimal λ value of 0.841 was achieved. This indicates a high overall satisfaction level across all objectives, signifying a well-balanced compromise solution. The corresponding objective values for this compromise solution are:

- f_1 (Makespan) = 2550
- f_2 (Customer Dissatisfaction) = 4.340
- f_3 (Total Cost) = 4099903.636

The GAMS output provides detailed assignments for arrival times (a_j), waiting times (w_j), vehicle assignments ($x_{j,k}$), sequencing ($y_{i,j,k}$), and time window assignments ($z_{j,t}$) for the compromise solution. Key observations from the detailed output include:

- **Arrival Times (a_j):** The model effectively schedules arrival times across the three-day horizon. For instance, some orders (e.g., Orders 1, 2, 3) are scheduled for early arrivals at 480 minutes, indicating prioritization or efficient initial routing. Other orders are scheduled much later (e.g., Order 4 at 2057 min, Order 5 at 2137 min) to optimize overall resource utilization and time window adherence.

- **Waiting Times (w_j):** A key finding for operational efficiency is that the waiting time (w_j) for most orders is zero. This indicates that the model successfully minimized unproductive idle time for vehicles at customer locations. Notably, only for Order 39, a waiting time of 308 minutes is observed, suggesting that this specific wait was strategically accepted to achieve overall better performance across the multi-objective functions, possibly due to its priority, location, or time window constraints.

- **Sequencing ($y_{i,j,k}$):** The $y_{i,j,k}$ variable outputs define the precise order of visits for customers on each vehicle's route, ensuring logical and feasible travel paths.

- **Time Window Assignments ($z_{j,t}$):** The $z_{j,t}$ outputs confirm that each order is assigned to exactly one of the nine defined time windows. The low f_2 value in the compromise solution demonstrates the model's ability to prioritize deliveries into preferred time windows, thereby enhancing customer satisfaction. For example, many orders (e.g., 1, 2, 3) are assigned to time window 1, which likely has a high satisfaction priority.

This comprehensive analysis demonstrates that the integrated multi-objective MILP model provides a highly effective and practical solution for rebar delivery scheduling and vehicle routing, successfully balancing the often-conflicting objectives of makespan, customer satisfaction, and total transportation cost within a real-world industrial setting.

5. Managerial Insights

The results of this study provide several significant managerial insights for Amir Kabir Khazar Steel Company and other organizations operating in heavy logistics sectors. The application of the integrated multi-objective MILP model offers a robust framework for improving operational efficiency, enhancing customer satisfaction, and achieving substantial cost savings.

- **Achieving Optimal Balance in Conflicting Objectives:** The most significant insight is the ability to find a well-balanced compromise solution for inherently conflicting objectives.

In logistics, optimizing for one factor (e.g., minimum cost) often leads to sub-optimal performance in others (e.g., long delivery times or low customer satisfaction). The fuzzy multi-objective approach provides a quantifiable satisfaction level (λ=0.841) that allows management to understand the simultaneous improvements across makespan, customer satisfaction, and total cost, moving beyond a single-minded optimization approach. This supports a more sustainable and holistic business strategy.

- **Strategic Cost Reduction and Efficiency:** The model clearly demonstrates a significant potential for reducing operational costs. By minimizing fixed vehicle costs, variable travel costs, and crucially, waiting times at customer locations (as evidenced by almost zero w_j values for most orders), the company can achieve substantial savings. The compromise total cost (f_3=4,099,903.636) is far superior to the high cost incurred when makespan is prioritized (14,401,040). This allows managers to optimize their budget allocation for logistics and improve profitability.

- **Customer-Centric Delivery Planning:** The explicit inclusion of customer priorities for time windows ($P_{j,t}$) directly impacts scheduling decisions. This ensures that the most critical or highest-priority orders are delivered within their preferred timeframes, leading to improved service quality and higher customer satisfaction (f_2=4.34). This capability is vital for maintaining strong customer relationships and a competitive edge in the market.

- **Optimized Resource Allocation and Fleet Management:** The detailed routing and scheduling plan provides precise guidance on how to best utilize the available fleet of 10 trailers. Managers can determine the optimal number of vehicles required, their specific routes, and the sequence of deliveries. This information is invaluable for efficient resource allocation, driver scheduling, vehicle maintenance planning, and potentially reducing the overall active fleet size if underutilized. The minimization of waiting times directly translates to higher productivity per vehicle.

- **Proactive Decision Support:** The model serves as a robust decision support tool for both tactical and operational planning. It enables "what-if" analyses to evaluate the impact of various scenarios, such as changes in customer demand, opening new customer locations, or adjusting time window policies. This proactive capability allows management to anticipate challenges, develop contingency plans, and make data-driven decisions that enhance the resilience of their supply chain.

- **Performance Benchmarking and Goal Setting:** The calculated PIS and NIS values for each objective serve as clear benchmarks for logistics performance. Managers can use these ideal and worst-case scenarios to set realistic and achievable performance targets for their teams, track progress, and identify areas for continuous improvement in their rebar delivery operations.

In essence, this integrated multi-objective MILP model provides Amir Kabir Khazar Steel Company with a sophisticated yet practical framework to optimize its rebar delivery operations, fostering a balance between economic efficiency and customer-centric service.

6. Conclusion and Future Research Directions

This paper presented an integrated multi-objective mixed-integer linear programming (MILP) model for delivery scheduling and vehicle routing in a rebar supply chain. The research aimed to simultaneously minimize the overall makespan of deliveries, the weighted customer dissatisfaction from time windows based on customer priority, and the total transportation costs. A real-world case study from Amir Kabir Khazar Steel Company in Iran, involving 51 customer orders over a three-day planning horizon, was used to validate the model. The fuzzy multi-objective optimization approach, based on Bellman and Zadeh's principle and Zimmermann's method, was employed to transform the three conflicting objectives into a single-objective problem, maximizing an overall satisfaction level (λ). Computational results, obtained using GAMS with the CPLEX solver, demonstrated the model's effectiveness in providing a balanced and practical solution. An optimal λ value of 0.841 was achieved, yielding a makespan of 2550 minutes, a customer dissatisfaction score of 4.34, and a total transportation cost of 4,099,903.636 monetary units. These results highlight the significant improvements possible by integrating scheduling and routing decisions and considering multiple objectives simultaneously, showcasing a strong balance between operational efficiency and customer service.

Despite its contributions, this study has certain limitations:

- **Deterministic Assumptions:** The model assumes all input parameters, such as travel times, service times, and costs, are known and deterministic. In practical scenarios, these parameters can be subject to uncertainty and variability.

- **Simplified Fleet Characteristics:** While vehicle costs are differentiated, the model does not explicitly incorporate detailed vehicle characteristics like varying capacities for a heterogeneous fleet, which is a common aspect in real-world logistics.

- **Fixed Planning Horizon:** The model is developed for a fixed three-day planning horizon without considering dynamic events or real-time adjustments that might be necessary in a continuously evolving supply chain environment.

- **Lack of Comparison with Heuristics:** For very large-scale problems, solving MILP models to optimality can become computationally prohibitive. This study did not explore heuristic or meta-heuristic solution approaches, which might be necessary for broader applicability.

Building upon the foundations laid by this research, several avenues for future investigation can be pursued:

- **Stochastic and Robust Optimization:** Extend the model to incorporate uncertainty in key parameters, such as customer demand, travel times (due to traffic congestion or unforeseen events), and vehicle availability. This could involve developing stochastic programming or robust optimization models to provide more resilient solutions.

- **Dynamic Routing and Scheduling:** Develop a dynamic model that allows for real-time updates and re-optimization of routes and schedules in response to new orders, cancellations, or unexpected events. This would enhance the model's practical applicability in highly volatile environments.

- **Integration with Inventory and Production:** Explore a deeper integration of the logistics model with inventory management and production scheduling decisions within the steel manufacturing plant. This would enable a more holistic optimization of the entire supply chain, from raw material procurement to final customer delivery.

- **Heuristic and Meta-heuristic Approaches:** For larger instances of the problem that may be computationally challenging for exact MILP solvers, develop and evaluate various heuristic or meta-heuristic algorithms (e.g., Genetic Algorithms, Ant Colony Optimization, Simulated Annealing, or Tabu Search) to find high-quality, near-optimal solutions within reasonable computational times.

- **Consideration of Additional Constraints:** Incorporate further realistic constraints such as driver working hours regulations, multiple depots, heterogeneous product types with different handling requirements, and the possibility of backhauling.

- **Green Logistics Objectives:** Add environmental objectives, such as minimizing carbon emissions or fuel consumption, to align with sustainability goals in the heavy industry sector. This could involve modeling different vehicle types with varying emission profiles.

- **Multi-Modal Transportation:** Extend the model to consider multi-modal transportation options (e.g., rail, sea) for long-distance deliveries of rebar, which could offer cost or environmental benefits compared to road transport alone.

- **Comparative Studies:** Conduct comparative studies with other multi-objective decision-making techniques (e.g., Epsilon-constraint method, goal programming, NSGA-II) to evaluate the performance and trade-offs of different solution approaches for this specific problem context.

These future research directions will contribute to developing more sophisticated, practical, and comprehensive decision-making tools for optimizing rebar supply chains and similar heavy logistics operations.

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