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Evaluation of decision-making units using directional distance function with weak disposability in the presence of undesirable outputs

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Abstract. This study evaluates decision-making units (DMUs) that produce unavoidable undesirable outputs, which negatively affect performance. Previous studies by researchers have explored approaches for calculating the minimum undesirable outputs, presenting the abatement range for undesirable outputs and assessing DMU efficiency, yet challenges remain. To address this, we introduce a new method based on the directional distance function model with individual-proportion weak disposability. The proposed method calculates the minimum unavoidable undesirable outputs, evaluates DMU efficiency for both undesirable and minimum undesirable outputs, separates the effects of desirable and undesirable outputs on inefficiency, and resolves the limitations of earlier methods. Finally, the proposed method is applied to practical examples to demonstrate its superiority over existing approaches.

Keywords: Decision-making units, Data envelopment analysis, Efficiency, Weak disposability, Minimum undesirable outputs.
AMS Subject Classification 2010: 34A34, 65L05.

1 Introduction

Continuous improvements in performance ensure the growth and development of an organization. However, this requires a reliable and valid method for the performance evaluation. One of the most widely recognized methods is the data envelopment analysis (DEA) method, which has wide applications in various fields such as economics, industry, and management [7]. Researchers have leveraged DEA for diverse applications: Pourmahmoud and Arami [37] explored cost efficiency in uncertain environments; Soofizadeh and Fallahnejad [43] developed a bargaining game model for performance evaluation in network DEA; and Fakharzadeh Jahromi et al. [14] proposed a DEA model for optimal budget allocation in

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parallel networks. DEA assesses the efficiency of Decision-Making Units (DMUs) that consume inputs to produce outputs. However, these DMUs may produce undesirable outputs that detract from their efficiency, as observed in energy production from waste incineration, where environmental pollution is an undesirable output [25].

Researchers have examined various methods for evaluating DMUs that produce undesirable outputs by initially treating these outputs as inputs. However, this method has been criticized for contradicting the physical laws. Scholars like Seiford and Zhu [39] in 2002 and others highlighted the inadequacies of this approach in accurately representing production process and optimizing conflicting objectives. For example, in coal-fired power plants, SO_2 cannot substitute for coal in power generation. Additionally, considering undesirable outputs as inputs does not clarify the relationship between desirable and undesirable outputs. Alternative approaches for evaluating DMUs with undesirable outputs include the by-production and meta-frontier approach proposed by Lei et al. [31] and Hi and Jie [18] in 2025. The concept of weak disposability of outputs, which was introduced by Shephard [42] in 1997 is the other mothed for evaluating DMUs with undesirable outputs. Weak disposability of outputs means that desirable outputs should contract in proportion to the contraction of undesirable outputs. This concept has two perspectives on output contraction: the common-proportion, proposed by Fare and Grosskopf, [15] applies a single contraction factor across all DMUs, and the individual-proportion, proposed by Kuosmanen [27], which with separate abatement factor and contracts the outputs of each DMU. In 2009, Kuosmanen and Podinovski [29] demonstrated that common-proportion could lead to non-convexity and an excessive size. They proposed the Kuosmanen production possibility set as a more reliable and accurate framework for evaluating DMUs, adhering to the principles of the production possibility set.

In addition to the above approaches for evaluating DMUs with undesirable outputs, researchers have utilized the directional distance function (DDF) to expand desirable outputs while contracting undesirable outputs. Chambers et al. [6], in 1996, first introduced the DDF model based on the Shephard distance function and the Luenberger profit function. Chung et al. [9] added the constraints related to undesirable outputs to the Chambers et al. [6] model and generalized it in 1997. Fare and Grosskopf [16] in 2004 applied weak disposability to the DDF model in evaluating the efficiency of DMUs with undesirable outputs, where they assumed a common contraction and expansion factor for all inputs and outputs, respectively. Fare and Grosskopf [17] in 2010 criticized the uniform abatement factor proposed by Chambers et al. [6] and resolved this challenge by applying separate contraction and expansion factors for each input and output, respectively. Karagiannis and Kourtzidis [23], in 2024, advanced the model by introducing both common and individual weak disposability for outputs, and altering the constraints on undesirable outputs, thereby improving the identification of inefficient DMUs and resolving issues with positive shadow prices.

Some DMUs that produce both desirable and undesirable outputs can minimize undesirable outputs, but cannot fully eliminate them. Kuosmanen [27] proposed a model to calculate the minimum undesirable outputs by utilizing individual-proportion weak disposability. However, his model faces challenges such as applying a uniform contraction coefficient to all outputs and enforcing an equality constraint for undesirable outputs, which may lead to positive shadow prices [32]. Maghbouli et al. [34] explored this concept further, proposing an abatement range for undesirable outputs; nevertheless, their findings are unreliable and do not ascertain the minimum undesirable outputs. In 2021, Kao and Hwang [21] introduced the SBM model with common-proportion weak disposability to determine the minimum undesirable output levels and assess virtual DMU efficiency, focusing on a common abatement factor and ignoring the inputs. In 2023, they enhanced their model [22] by incorporating individual-proportion weak disposability, enabling each DMU to conduct undesirable outputs while considering input factors. However, their approach is challenged by nonlinearity (which complicates and lengthens the solving process) and the equality constraints inherent in the SBM model. Furthermore, Zhanxin Ma et al. [33] in 2022 highlighted additional challenges within the SBM model. Table 1 summarizes various models used to evaluate the DMUs with undesirable outputs.

Table 1: Various approaches for evaluating DMUs with undesirable outputs

Row	Authors	Features
1	Koopman [26] (1951), Berg [5] (1992), Dyson [12] (2001)	Considering undesirable output as input
2	Ayres [4] (1995), Ayres and Nice [3] (1996) , Seiford and Zhu [39] (2002), Fare and Grosskopf [15] (2003), Podinovski and Kuosmanen [36] (2011)	Rejecting the method of considering undesirable output as input
3	Shephard [42] (1997), Fare and Grosskopf [15] (2003), Kuosmanen [27] (2005), Kuosmanen and Podinovski [29] (2009), Cui [10] (2019), (2009), Kuosmanen and Matin [28] (2011), Cui [10] (2019), Karami Khorramabadi et al. [24] (2020), Asaniomoghadam [2] (2022), Salahi [38] (2022), Shakouri [41] (2022), Maghbouli et al. [34] (2024)	Weak disposability
4	Chamber et al. [6] (1996), Chung et al. [9] (1997), Seiford and Zhu [39] (2002), Fare and Grosskopf [16] (2004), Seiford and Zhu [40] (2005), Chen et al. [8] (2015), Tamaki et al. [44] (2016), Alfredsson et al. [1] (2016), Lee et al. [30] (2017), Kao [20] (2017), Karagiannis and Kourtzidis (2024)	DDF with undesirable output
5	Kao and Hwang [21] (2021), Kao and Hwang [22] (2023)	The minimum undesirable output

Various DDF models have been developed that incorporate weak disposability. However, these models have certain limitations. For example, Karagiannis and Kourtzidis [23] in 2024 introduced radial DDF models with weak disposability for both common and individual proportions. However, their models are only radial and consider common abatement and expansion factors for all inputs and outputs, respectively. This study aims to overcome these limitations by employing a DDF model with individual-proportion weak disposability, where constraints for undesirable outputs are treated as less than or equal to. The benefits of the DDF model include a linear objective function, flexibility in using arbitrary default directions, and separate contraction and expansion factors for each input and output, enabling both radial and non-radial analyses. Furthermore, this study explores the influence of desirable and undesirable outputs on DMU inefficiency.

The second section of this study describes the primary concepts, including the minimum undesirable outputs, the impact of both desirable and undesirable outputs on the inefficiency of DMUs, and the evaluation of DMUs using the DDF model with weak disposability. The third section presents a proposed method based on the DDF model with individual-proportion weak disposability, aiming to measure the minimum unavoidable undesirable outputs in production and calculate efficiency with undesirable outputs and the minimum undesirable outputs. The validity of the proposed model will be demonstrated through examples in the fourth section, and finally, conclusions will be provided in the fifth section. Figure 1 presents an outline of our study.

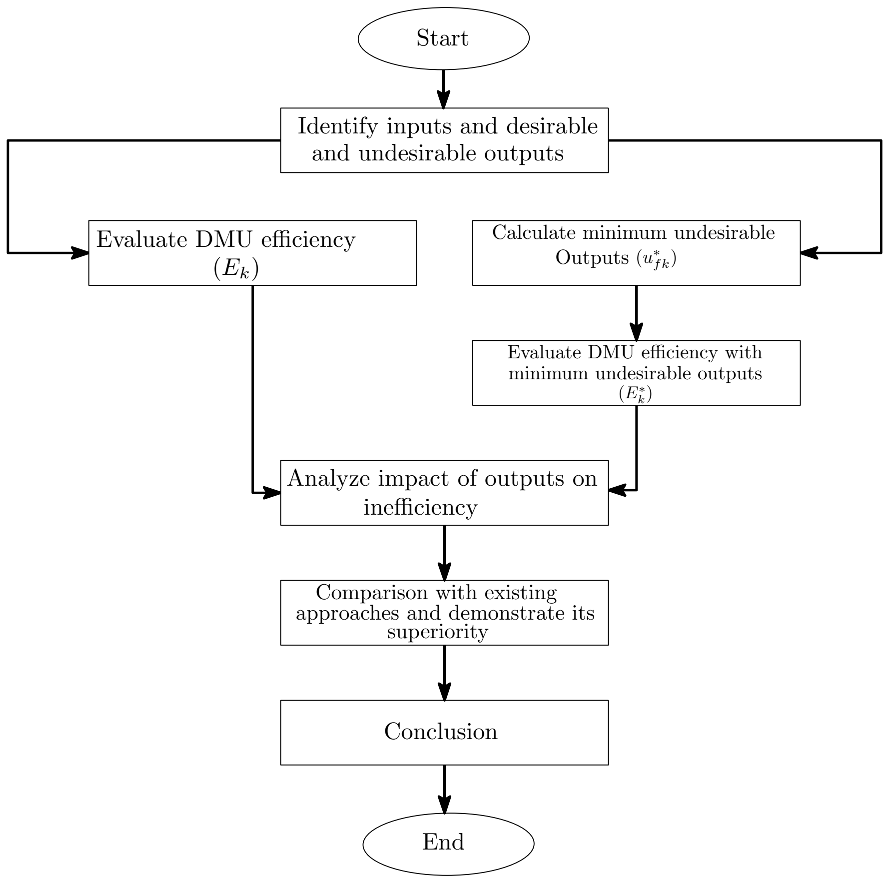

Figure 1: Research framework

2 Preliminary concepts

2.1 Minimum undesirable outputs

A significant challenge for managers is the simultaneous production of undesirable outputs with desirable ones during organizational growth. The complete elimination of these undesirable outputs is often impractical, leading to a focus on their minimization. Consequently, DEA scholars have proposed various methods for evaluating DMUs in this context, as referenced in the introduction. Assume that $\boldsymbol{x} = (x_1, x_2, \ldots, x_m)$, $\boldsymbol{y} = (y_1, y_2, \ldots, y_s)$, and $\boldsymbol{u} = (u_1, u_2, \ldots, u_h)$ represent inputs, desirable outputs, and undesirable outputs, respectively. In this case, the production possibility set can be represented as: $T = \{(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{u}) \,|(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{u})\ \boldsymbol{x} \text{ can produce } (\boldsymbol{y}, \boldsymbol{u})\}$. Following Kuosmanen [27] in 2005, weak disposability is defined such that if $(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{u}) \in T$, $0 \leq \tau_j \leq 1$, then $(\boldsymbol{x}, \tau_j \boldsymbol{y},\ \tau_j \boldsymbol{u}) \in T$ where, τ_j is the contraction factor for each desirable and undesirable output individually. Kuosmanen [27] defined the following individual-proportion weak disposability production possibility set $\left(T^{IP}\right)$ under variable returns to scale, satisfying the individual-proportion weak disposability assumption:

$$T^{IP} = \left\{ (\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{u}) \left| \begin{array}{ll} \sum_{j=1}^{n} \lambda_j x_{ij} \leq x_i, & i = 1, \ldots, m, \\ \sum_{j=1}^{n} \tau_j \lambda_j y_{rj} \geq y_r, & r = 1, \ldots, s, \\ \sum_{j=1}^{n} \tau_j \lambda_j u_{fj} = u_f, & f = 1, \ldots, h, \\ \sum_{j=1}^{n} \lambda_j = 1, & \\ 0 \leq \tau_j \leq 1, \quad \lambda_j \geq 0, & j = 1, \ldots, n. \end{array} \right. \right\}. \tag{1}$$

By change of variables $\tau_j \lambda_j = \mu_j$ and $(1 - \tau_j)\lambda_j = \theta_j$, T^{IP} can be rewritten as:

$$T^{IP} = \left\{ (\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{u}) \left| \begin{array}{ll} \sum_{j=1}^{n} (\mu_j + \theta_j) x_{ij} \leq x_i, & i = 1, \ldots, m, \\ \sum_{j=1}^{n} \mu_j y_{rj} \geq y_r, & r = 1, \ldots, s, \\ \sum_{j=1}^{n} \mu_j u_{fj} = u_f, & f = 1, \ldots, h, \\ \sum_{j=1}^{n} (\mu_j + \theta_j) = 1, & \\ \mu_j \geq 0, \ \theta_j \geq 0, & j = 1, \ldots, n. \end{array} \right. \right\}. \tag{2}$$

Kuosmanen [27] utilized the linearly formulated individual-proportion weak disposability production possibility set to find the minimum amount of undesirable outputs and presented model (3) to calculate the contraction coefficient of undesirable outputs at the same level of inputs and desirable outputs:

$$\begin{aligned} \min \quad & \phi \\ \text{s.t.} \quad & \sum_{j=1}^{n} (\mu_j + \theta_j) x_{ij} \leq x_{ik}, && i = 1, \ldots, m, \\ & \sum_{j=1}^{n} \mu_j y_{rj} \geq y_{rk}, && r = 1, \ldots, s, \\ & \sum_{j=1}^{n} \mu_j u_{fj} = \phi u_{fk}, && f = 1, \ldots, h, \\ & \sum_{j=1}^{n} (\mu_j + \theta_j) = 1, && \\ & \mu_j \geq 0, \quad \theta_j \geq 0, \quad \phi \geq 0, && j = 1, \ldots, n. \end{aligned} \tag{3}$$

Let ϕ^* be the optimal value of model (3). It reflects the maximum contraction coefficient for undesirable outputs, while maintaining a constant level of inputs and desirable outputs. The minimum unavoidable level of undesirable outputs produced alongside desirable outputs is expressed as: $u^*_{fk} = \phi^* \mathrm{u}_{fk}$; $f = 1, \ldots, h$. It is obvious that $u_{fk} \geq u^*_{fk}$.

Model (3) does not focus on desirable inputs and outputs, as these factors remain at the previous level for DMUs. Additionally, based on this model, it is impossible to determine the efficiency of DMUs and their improvement points. On the other hand, most managers prefer evaluations of DMUs to provide improvement points for each inefficient DMU so that they can make better decisions based on the given data pattern. To address managers' requests, new models using the DDF and the principle of individual-proportion weak disposability will be introduced in section three of the article, aiming to resolve the mentioned challenges while assessing the efficiency of DMUs and the improvement points of the units.

2.2 Impact of desirable and undesirable outputs on DMU inefficiency

The evaluation of DMUs reveals that outputs significantly affect their performance, as higher desirable outputs improve efficiency, whereas higher undesirable outputs decrease it. To examine the impact of desirable and undesirable outputs on the inefficiency of DMUs $(1-E_k)$, Kao and Hwang [22] in 2023 decomposed the inefficiency expression $1-E_k$ as follows to separate the effects of desirable and undesirable outputs on the inefficiency of DMUs:

$$(1-E_k) = (1-E_k^*) + (E_k^* - E_k)\,, \tag{4}$$

where E_k and E_k^* denote the efficiencies of the evaluated DMU_k with undesirable outputs and minimum undesirable outputs, respectively. They divided both sides of relation (4) by $(1-E_k)$ to obtain the following relation:

$$1 = \frac{\left(1-E_k^*\right)}{\left(1-E_k\right)} + \frac{\left(E_k^* - E_k\right)}{\left(1-E_k\right)} \tag{5}$$

The expression $\frac{\left(1-E_k^*\right)}{\left(1-E_k\right)}$ and $\frac{\left(E_k^*-E_k\right)}{\left(1-E_k\right)}$ represent the impact of inefficiency created by desirable outputs and the impact of the additional production of undesirable outputs, respectively. Relation (5) helps decision-makers identify the causes of inefficiency and further improve the performance of that part.

2.3 Evaluation of DMUs using the DDF model with weak disposability

The DDF is a method used by some scholars to evaluate DMUs, allowing the calculation of DMU efficiency and improvement points. It enables changes in input and output factors by selecting a default direction that expands desirable outputs while contracting undesirable ones. Combining this DDF feature with individual-proportion weak disposability, Karagiannis and Kourtzidis [23] in 2024 proposed the following model for the output-oriented case:

$$\begin{aligned}
\max \quad & \delta_k \\
\text{s.t.} \quad & \sum_{j=1}^{n} (\mu_j + \theta_j) x_{ij} \le x_{ik}, && i = 1, \ldots, m, \\
& \sum_{j=1}^{n} \mu_j y_{rj} \ge y_{rk} + \delta_k d_r^y, && r = 1, \ldots, s, \\
& \sum_{j=1}^{n} \mu_j u_{fj} \le u_{rk} - \delta_k d_f^u, && f = 1, \ldots, h, \\
& \sum_{j=1}^{n} (\mu_j + \theta_j) = 1, \\
& \mu_j \ge 0 \quad \theta_j \ge 0, && j = 1, \ldots, n,
\end{aligned} \tag{6}$$

where δ_k is unrestricted in sign, and the default direction for outputs is considered as $\boldsymbol{d^y} = \left[d_1^y, d_2^y, \ldots, d_s^y\right]^T$ and $\boldsymbol{d^u} = \left[d_1^u, d_2^u, \ldots, d_h^u\right]^T$. In addition, to ensure that not all shadow prices are positive, a "less than or equal to" constraint is used instead of an equality constraint in the third constraint. Karagiannis and

Kourtzidis [23], in their article, in addition to model (6), have similarly presented a model for the input-oriented case.

Model (6) is only radial and includes only expansion for all outputs, whereas the similar model for inputs includes only abatement for all inputs. These models do not calculate the expansion of each output or abatement of each input separately. Therefore, in the next section, we propose models that, in addition to addressing the aforementioned challenges, calculate the minimum undesirable outputs and the efficiency of DMUs with the minimum undesirable outputs using the DDF with individual-proportion weak disposability in both radial and non-radial cases.

3 Proposed models

3.1 Calculation of DMU efficiency with undesirable outputs

Some managers or decision-makers in an organization may request their consultants to provide a plan in which the separate expansion of each output occurs alongside the separate contraction of each input. Non-radial DDF models, due to their flexibility and the advantages mentioned in the previous section, are among the models that can address such challenges. Furthermore, DMUs considered in this study involve undesirable outputs, and one of the approaches for evaluating such DMUs is individual-proportion weak disposability. Therefore, to tackle the challenges , a new model based on the DDF model with individual-proportion weak disposability under variable returns to scale is proposed as follows:

$$
\begin{aligned}
\max \quad & \frac{1}{m+s+h}\left(\sum_{i=1}^{m}\zeta_i+\sum_{r=1}^{s}\beta_r+\sum_{f=1}^{h}\delta_f\right) & & \\
\text{s.t.} \quad & \sum_{j=1}^{n}(\mu_j+\theta_j)x_{ij}\leq x_{ik}-\zeta_i d_i^x, & i=1,\dots,m, & \qquad (7)\\
& \sum_{j=1}^{n}\mu_j y_{rj}\geq y_{rk}+\beta_r d_r^y, & r=1,\dots,s, & \\
& \sum_{f=1}^{h}\mu_j u_{fj}\leq u_{fk}-\delta_f d_f^u, & f=1,\dots,h, & \\
& \sum_{j=1}^{n}(\mu_j+\theta_j)=1, & & \\
& \mu_j\geq 0,\quad \theta_j\geq 0, & j=1,\dots,n, &
\end{aligned}
$$

where $\boldsymbol{d^x}=[d_1^x,d_2^x,\dots,d_m^x]^T$, $\boldsymbol{d^y}=\left[d_1^y,d_2^y,\dots,d_s^y\right]^T$, and $\boldsymbol{d^u}=\left[d_1^u,d_2^u,\dots,d_h^u\right]^T$ represent the pre-determined directions for input, desirable, and undesirable outputs of the evaluated DMU, respectively. In this model, due to the presence of variables $\zeta_i; i=1,\dots,m$, $\beta_r; r=1,\dots,s$ and $\delta_f;\ f=1,\dots,h$, the expansion of outputs occurs separately alongside the abatement of each input.

Model (7) is a non-radial model. In some cases, it may not be possible to change both input and output factors simultaneously. In such instances, the model must be applied either as input-oriented or output-oriented. When the system manager aims to evaluate a system based on inputs, an input-oriented model must be used. In this case, it is sufficient to set d_r^y and d_f^u to zero in model (7), resulting in the following input-oriented model:

$$\begin{aligned}
\max \quad & \frac{1}{m}\sum_{i=1}^{m}\zeta_i \\
\text{s.t.} \quad & \sum_{j=1}^{n}(\mu_j+\theta_j)x_{ij}\leq x_{ik}-\zeta_i d_i^x, && i=1,\dots,m, \\
& \sum_{j=1}^{n}\mu_j y_{rj}\geq y_{rk}, && r=1,\dots,s, \\
& \sum_{f=1}^{h}\mu_j u_{fj}\leq u_{fk}; && f=1,\dots,h, \\
& \sum_{j=1}^{n}(\mu_j+\theta_j)=1, \\
& \mu_j\geq 0,\quad \theta_j\geq 0, && j=1,\dots,n.
\end{aligned} \tag{8}$$

If in model (8), instead of using separate abatement coefficients for each input $(\zeta_i, i=1,\dots,m)$, a common abatement coefficient (ζ) for all inputs is used, then all inputs will contract at the same rate and model (8) transforms into the input-oriented model proposed by Karagiannis and Kourtzidis [23].

In some situations, it is impossible to reduce inputs during the evaluation of DMUs, such as by reducing the age of employees or decreasing wages, as these actions may lead to cultural or social tensions. Therefore, model (8) may not be suitable for all managerial objectives. Additionally, if the system manager aims to evaluate the system based on outputs, a radial output-oriented model (9) is proposed to address these challenges. By setting d_i^x to zero in model (7), the following model is obtained:

$$\begin{aligned}
\max \quad & \frac{1}{s+h}\left(\sum_{r=1}^{s}\beta_r+\sum_{f=1}^{h}\delta_f\right) \\
\text{s.t.} \quad & \sum_{j=1}^{n}(\mu_j+\theta_j)x_{ij}\leq x_{ik}, && i=1,\dots,m, \\
& \sum_{j=1}^{n}\mu_j y_{rj}\geq y_{rk}+\beta_r d_r^y, && r=1,\dots,s, \\
& \sum_{f=1}^{h}\mu_j u_{fj}\leq u_{fk}-\delta_f d_f^u, && f=1,\dots,h, \\
& \sum_{j=1}^{n}(\mu_j+\theta_j)=1, \\
& \mu_j\geq 0,\quad \theta_j\geq 0, && j=1,\dots,n.
\end{aligned} \tag{9}$$

Similarly, in model (9), rather than employing separate expansion coefficients for desirable outputs $(\beta_r; r=1,\dots,s)$ and separate abatement coefficients for undesirable outputs $(\delta_f; f=1,\dots,h)$, a single common coefficient (δ) is applied to all outputs. This modification transforms model (9) into the output-oriented model proposed by Karagiannis and Kourtzidis [23].

Evaluating DMUs using any proposed model requires determining the values of the variables from the solution of the model. These variable values are crucial indicators for the evaluation or impact of DMUs. The proposed model must be feasible and bounded to determine the value of each variable. The following theorem demonstrates these two characteristics of the proposed model model (7).

Theorem 1. *Model* (7) *is feasible and bounded.*

Proof. **Proof of feasibility**: Assume that the DMU under evaluation is DMU_k, and the improvement direction is the hypothetical direction $\vec{d} = \left(d_i^x, d_r^y, d_f^u\right)$, where $d_i^x,\ d_r^y,\ and\ d_f^u$ are arbitrary directions. To prove the feasibility of model (7), it is sufficient to provide a solution for the variables that satisfy all constraints of model (7). For this purpose, consider the following values for the variables:

$$\zeta_i = \beta_r = \delta_f = 0, \quad \theta_k = 0,\ \mu_j = 0,\ \ j \neq k, \quad \mu_k = 1.$$

These values satisfy the constraints of model (7), confirming the feasibility of model (7).

Proof of boundedness: To prove the boundedness of model (7), it is sufficient to demonstrate the boundedness of its feasible region. Because DMU_k is one of the observed DMUs, the first constraint of model (7), for this DMU is expressed as:

$$\sum_{j=1}^{n}(\mu_j+\theta_j)x_{ij} - x_{ik} + \zeta_i d_i^x \leq 0, \quad i = 1,\ldots,m \Rightarrow \zeta_i d_i^x + \sum_{j=1}^{n}(\mu_j+\theta_j)x_{ij} \leq x_{ik}.$$

In this equation, the left-hand side is a weighted sum of variables that is less than or equal to a known value, x_{ik}.Therefore, each of the variables, including ζ_i, is bounded. Similarly, it can be proven from the second and third constraints of model (7) that β_r and δ_f are also bounded. Thus, the feasible region of model (7) is closed and bounded. Hence, the statement holds true. □

Models (8) and (9) are special cases of the non-radial model (7), which are input-oriented and output-oriented, respectively. By applying the proof for model (7) similar to models (8) and (9), the following result is obtained.

Result 1. *Models* (8) *and* (9) *are feasible and bounded.*

To determine the efficiency of DMUs, the optimal solution of model (7) must be calculated. For this purpose, the following lemma is introduced.

Lemma 1. *Model* (7) *has an optimal solution.*

Proof. For an optimal solution to exist, the model must be feasible and bounded, as proven in Theorem 1. Therefore, the statement holds true. □

In every organization, identifying efficient DMUs is necessary to provide feedback and interpretations based on these efficient DMUs. Therefore, the proposed model must introduce efficient DMUs to offer improved solutions. However, model (7) does not explicitly determine the efficiency values of the DMUs. To compute the efficiency in various non-radial and radial cases, the following definitions are introduced.

Definition 1: Efficiency

(a): DMU_k is considered efficient in all three cases, non-radial, input-oriented, and output-oriented, if the optimal solution of models (7), (8), and (9), respectively, equals zero. The efficiency value of DMU_k in each case will be equal to one.

(b) DMU_k is inefficient in non-radial, input-oriented, and output-oriented cases if the optimal solutions of models (7), (8), and (9), respectively, are non-zero. The efficiency for each case can be calculated as follows:

(b-1) The non-radial efficiency of DMU_k is given by $E_{nonradial} = \frac{1-\frac{1}{m+h}\left(\sum_{i=1}^{m}\zeta_i+\sum_{f=1}^{h}\delta_f\right)}{1+\frac{1}{s}\sum_{r=1}^{s}\beta_r}$.

(b-2) The radial input-oriented efficiency is given by $E_{input} = 1-\frac{1}{m}\sum_{i=1}^{m}\zeta_i$.

(b-3) The radial output-oriented efficiency is given by $E_{output} = \frac{1-\frac{1}{h}\sum_{f=1}^{h}\delta_f}{1+\frac{1}{s}\sum_{r=1}^{s}\beta_r}$.

3.2 Calculation of DMU efficiency with minimum undesirable outputs

Undesirable outputs can significantly weaken organizational performance. Although it is impossible to eliminate these outputs entirely, they can be minimized. Various methods have been developed to address this issue in DMU evaluations. This study proposes a new model based on the DDF model specifically designed to evaluate DMUs by individually contracting undesirable outputs with variable returns to scale as follows:

$$\begin{aligned}
\min \quad & \frac{1}{h}\sum_{f=1}^{h}\delta_f \\
\text{s.t.} \quad & \sum_{j=1}^{n}(\mu_j+\theta_j)x_{ij}\le x_{ik}, && i=1,\dots,m, \\
& \sum_{j=1}^{n}\mu_j y_{rj}\ge y_{rk}, && r=1,\dots,s, \\
& \sum_{f=1}^{h}\mu_j u_{fj}\le u_{fk}-\delta_f d_f^u, && f=1,\dots,h, \\
& \sum_{j=1}^{n}(\mu_j+\theta_j)=1, \\
& \mu_j\ge 0,\quad \theta_j\ge 0,\quad j=1,\dots,n,\ \ \delta_f\ge 0, && f=1,\dots,h.
\end{aligned} \tag{10}$$

The optimal solution of model (10) is represented by $\delta^{k*}=\left[\delta_1^*,\delta_2^*,\dots,\delta_h^*\right]^T$, where $\delta_f^*; f=1,\dots,h$ indicates the optimal solution for each variable. Based on to this solution, the minimum amount of undesirable output of DMU_k is calculated using relation $(u_{fj}^*=u_{fj}-\delta_f d_f^u)$, where $\delta_f d_f^u$ represents the additional undesirable output of DMU_k. To calculate the efficiency of DMU_k with the minimum undesirable outputs, undesirable output $u_{fj};\ f=1,\dots,h,\ j=1,\dots,n$ is replaced with $u^*_{fj};\ f=1,\dots,h,\ j=1,\dots,n$ in the third constraint of model (7). This results in the following proposed model:

$$
\begin{aligned}
\max\quad & \frac{1}{m+s+h}\Big(\sum_{i=1}^{m}\zeta_i+\sum_{r=1}^{s}\beta_r+\sum_{f=1}^{h}\delta_f\Big) && \\
\text{s.t.}\quad & \sum_{j=1}^{n}(\mu_j+\theta_j)x_{ij}\le x_{ik}-\zeta_i d_i^x, && i=1,\dots,m, \\
& \sum_{j=1}^{n}\mu_j y_{rj}\ge y_{rk}+\beta_r d_r^y, && r=1,\dots,s, \\
& \sum_{f=1}^{h}\mu_j u^*_{fj}\le u^*_{fk}-\delta_f d_f^u, && f=1,\dots,h, \\
& \sum_{j=1}^{n}(\mu_j+\theta_j)=1, && \\
& \mu_j\ge 0,\ \ \theta_j\ge 0, && j=1,\dots,n.
\end{aligned}
\tag{11}
$$

The efficiency of model (11) is denoted as E_k^*, where it is clear that $E_k^*\ge E_k$. Using relation (5) along with models (7), and (11), the impact of undesirable outputs on the inefficiency of DMU_k can be analyzed.

To derive optimal solutions and ultimately calculate the efficiency with minimum undesirable outputs, the following theorem must be proved.

Theorem 2. *Models* (10) *and* (11) *are feasible and bounded.*

Proof. The proof is similar to the proof of Theorem 1. □

Lemma 2. *Model* (11) *has an optimal solution.*

Proof. The proof follows similarly to Lemma 1. □

To calculate the efficiency and inefficiency values for each DMU with minimum undesirable outputs, Definition 1 is applied using the optimal values from models (10) and (11). □

When simultaneous changes in the inputs and outputs are impossible, the non-radial model (11) cannot be used. In this case, input-oriented or output-oriented models should be applied. Therefore, similar to the transformation of the non-radial model (7) into input-oriented (8) and output-oriented (9) models, the corresponding models can be derived.

4 Examples

In this section, to demonstrate the superiority of the proposed models over existing models, they are applied to two examples. Both examples are case studies that have been examined by other researchers and involve undesirable outputs.

4.1 Example 1

Consider a dataset collected from 30 paper factories by Kao and Hwang along the Huai River in Anhui Province, China [22]. The dataset in Table 2 includes 30 DMUs with two inputs $(x_1,\ x_2)$, two desirable outputs (y_1, y_2), and one undesirable output (u).

Table 2: Input and output values for Example 1.

	Inputs		Desirable outputs		Undesirable output
DMU	x_1	x_2	y_1	y_2	u
1	437	1438	2015	14667	665
2	884	1061	3452	2822	491
3	1160	9171	2276	2484	417
4	626	10151	953	16434	302
5	374	8416	2578	19715	229
6	597	3038	3003	20743	1083
7	870	3342	1860	20494	1053
8	685	9984	3338	17126	740
9	582	8877	2859	9548	845
10	763	2829	1889	18683	517
11	689	6057	2583	15732	664
12	355	1609	1096	13104	313
13	851	2352	3924	3723	1206
14	926	1222	1107	13095	377
15	203	9698	2440	15588	792
16	1109	7141	4366	10550	524
17	861	4391	2601	5258	307
18	249	7856	1788	15869	1449
19	652	3173	793	12383	1131
20	364	3314	3456	18010	826
21	670	5422	3336	17568	1357
22	1023	4338	3791	20560	1089
23	1049	3665	4797	16524	652
24	1164	8549	2161	3907	999
25	1012	5162	812	10985	526
26	464	10504	4403	21532	218
27	406	9365	1825	21378	1339
28	1132	9958	2990	14905	231
29	593	3552	4019	3854	1431
30	262	6211	815	17440	965

Kao and Hwang [22] applied the SBM model in two stages in 2023 to evaluate the DMUs based on the dataset in Table 2. In the first stage, they calculated the minimum undesirable output DMU_k (u_k^*), and in the second stage, they used u_k^* to calculate the efficiency with the minimum undesirable output. The efficiency results in undesirable outputs $(E_{k,H})$, the efficiency results in the minimum undesirable output $(E_{k,H}^*)$, and the ratios $\frac{1-E_k^*}{1-E_k}$ and $\frac{E_k^*-E_k}{1-E_k}$ are presented in columns two, three, seven and eight of Table 3, respectively.

To compare the results with the proposed models, the dataset from Table 2 is applied using model (7) in MATLAB. The evaluation results of the DMUs with their undesirable outputs (E_k) are presented in column five of Table 3. To calculate the efficiency of DMUs with the minimum undesirable output (E_k^*), model (10) is first used to calculate the contraction coefficient of the undesirable outputs (δ_f). Then, using relation $u_{fj}^* = u_{fj} - \delta_f d_f^u$, the minimum undesirable output for each DMU is calculated. Finally, the minimum undesirable outputs obtained from model (10), are substituted into the proposed model (11) to obtain E_k^*. The values of u_{fj}^* and E_k^* are listed in columns four and six of Table 3, respectively. The last two columns of Table 3 show the ratios $\frac{1-E_k^*}{1-E_k}$ and $\frac{E_k^*-E_k}{1-E_k}$ for the results obtained from the proposed models. Finally, the last row of Table 3 presents the average values for each column.

By comparing the results from both methods, it is observed that 15 DMUs are efficient, with undesirable outputs in both methods. However, when considering the minimum undesirable output, 20 DMUs are efficient in the Kao and Huang method, whereas the number of efficient DMUs remains unchanged in the proposed method. Specifically, DMU_{25}, DMU_{17}, DMU_{11}, DMU_9, DMU_8 are evaluated as efficient in the Kao and Huang method but inefficient in the proposed method. Therefore, improvement strategies can be suggested for inefficient DMUs. Additionally, comparing the average values of E_k^*, E_k and the average ratios in the last row of Table 3 reveals that these values are lower in the proposed method than the Kao and Huang method. Hence, the proposed method is more accurate and ultimately more reliable. Additional advantages of the proposed method include the ability to select a desired default direction, flexibility in choosing between radial and non-radial models, and the ability to select individual output expansion and input contraction factors.

To analyze the impact of desirable and undesirable outputs on the inefficiency of DMUs, relations (4) and (5) can be utilized. For example, referring to Table 2 and 3 for DMU_3, the following values are obtained:

$$\begin{aligned}
&u_3 = 417, \quad u_3^* = 112.68, \quad u_3 - u_3^* = 417 - 112.68 = 304.32,\\
&E_3 = 0.1012, \quad 1 - E_3 = 1 - 0.1012 = 0.8988,\\
&E_3^* = 0.24709, \quad 1 - E_3^* = 1 - 0.24709 = 0.75291,\\
&E_3^* - E_3 = 0.24709 - 0.1012 = 0.14589.
\end{aligned}$$

The value $u_3^* = 112.68$ indicates the minimum unavoidable undesirable output required for production activities of DMU_3. If DMU_3 can reduce its undesirable outputs by 304.32 units (i.e., decrease undesirable outputs to 112.68), its efficiency will increase by 0.14589, improving from $E_3 = 0.1012$ to $E_3^* = 0.24709$. Consequently, the impact of desirable and undesirable outputs on inefficiency can be calculated using relation (5), as follows:

$$\begin{aligned}
\frac{(1-E_3^*)}{(1-E_3)} &= \left(\frac{0.75291}{0.8988}\right) \times 100 = 83.77\%,\\
\frac{(E_3^*-E_3)}{(1-E_3)} &= \left(\frac{0.14589}{0.8988}\right) \times 100 = 16.22\%.
\end{aligned}$$

Table 3: Numerical results for Example 1.

DMU	$E_{K,H}$	$E^*_{k,H}$	u^*_{fj}	E_k	E^*_k	$\frac{1-E^*_{k,H}}{1-E_{K,H}}$	$\frac{E^*_{k,H}-E_{K,H}}{1-E_{K,H}}$	$\frac{1-E^*_k}{1-E_k}$	$\frac{E^*_k-E_k}{1-E_k}$
1	1	1	665	1	1	0	0	0	0
2	1	1	491	1	1	0	0	0	0
3	0.1309	0.3215	112.688	0.1012	0.2470	0.7807	0.2193	0.8377	0.1622
4	0.2842	0.3722	166.385	0.2842	0.3722	0.8771	0.1229	0.8770	0.1229
5	1	1	229	1	1	0	0	0	0
6	1	1	1083	1	1	0	0	0	0
7	0.6541	0.7031	966.36	0.6540	0.7031	0.8553	0.1417	0.8582	0.1417
8	0.5276	1	173.391	0.47	0.8180	0	1	0.3432	0.6567
9	0.3977	1	141.553	0.1239	0.6934	0	1	0.3498	0.6501
10	1	1	517	1	1	0	0	0	0
11	0.6101	1	224.512	0.56101	0.8960	0	1	0.2368	0.7631
12	1	1	313	1	1	0	0	0	0
13	0.5029	0.5913	664.997	0.5028	0.5912	0.8222	0.1778	0.8221	0.1778
14	1	1	377	1	1	0	0	0	0
15	1	1	792	1	1	0	0	0	0
16	0.572	0.6638	381.06	0.5720	0.6638	0.7855	0.2145	0.7854	0.2145
17	0.4843	1	225.984	0.2474	0.7299	0	1	0.3587	0.6412
18	0.6696	0.9447	702.023	0.6696	0.9446	0.1674	0.8326	0.16751	0.83249
19	0.2695	0.6934	236.726	0.2846	0.6934	0.4197	0.5803	0.4285	0.5714
20	1	1	826	1	1	0	0	0	0
21	0.5703	0.9759	391.457	0.5703	0.9759	0.0561	0.9439	0.05602	0.9439
22	1	1	1089	1	1	0	0	0	0
23	1	1	652	1	1	0	0	0	0
24	0.1507	0.3689	106.994	0.13048	0.3688	0.7431	0.2569	0.7115	0.2884
25	0.2685	1	127.531	0.1148	0.6233	0	1	0.4254	0.5745
26	1	1	218	1	1	0	0	0	0
27	1	1	1339	1	1	0	0	0	0
28	0.5575	0.7132	150.905	0.5575	0.7131	0.6481	0.3519	0.6482	0.3517
29	1	1	1431	1	1	0	0	0	0
30	1	1	965	1	1	0	0	0	0
AVE	0.7217	0.8783	525.319	0.6942	0.8344	0.4373	0.5627	0.2635	0.2364

These relations indicate that the inefficiency resulting from desirable outputs is 83.77%, which can be reduced by increasing the production of desirable outputs. On the other hand, 16.22%, of inefficiency arises from excess undesirable outputs, which can be improved by reducing the production of excess undesirable outputs. The significant difference between 83.77 compared to 16.22 indicates that the primary source of inefficiency for DMU_3 is the low production of desirable ooutputs. Consequently, decision-makers should prioritize increasing desirable outputs to enhance the performance of this DMU. This conclusion can be similarly applied to other DMUs. Finally, the averages in the last two columns of Table 3 reveal that across the 30 DMUs, 26.35% of inefficiency is attributed to desirable outputs, while 23.64% is due to excess undesirable outputs.

4.2 Example 2: Case study in OECD Countries

Since 1960, 38 countries have been registered with the OECD [35]. However, due to limited access to data and unavailability, we decided to evaluate only 20 OECD members in this study. This study selects the inputs and outputs from Wangs et al'.s study [46] to evaluate the environmental efficiency of renewable energy utilization in 20 OECD Countries for 2020. The inputs are the labor force, gross capital formation, total renewable energy capacity, and share of renewable energy. The desirable output is the gross domestic product (GDP), and the undesirable output is CO2 emissions.

Table 4: Input and output values and country names.

DMU	countries	I1	I2	I3	I4	Y	U
1	Australia	13587001	319789	34536	22.6	1350534	370.48
2	Belgium	5167188	122655	11277	27.4	462150	87.24
3	Canada	20482633	388410	100582	67.1	1556509	521.00
4	Czech Republic	5375292	57656	10151	14.3	188033	89.09
5	France	30379167	606702	55365	24.4	2438208	273.59
6	Germany	43501190	771775	131739	44.8	3356236	619.29
7	Italy	25126337	353309	55299	42.4	1835899	280.47
8	Japan	68898380	1243302	186259	20.3	4444931	1029.82
9	Korea, Rep.	28597159	524326	27405	6.4	1465773	609.20
10	Mexico	53137902	265449	28358	19.8	171868	369.00
11	Netherlands	9502134	171292	17678	26.6	765265	147.04
12	New Zealand	2832047	49384	7425	80.4	178064	33.00
13	Norway	2893601	110890	37212	98.2	385802	37.82
14	Poland	18245536	104835	12220	18.4	477812	286.17
15	Portugal	5166305	38039	14274	59.6	199314	37.87
16	Spain	22838137	254150	59108	44.5	1195119	200.80
17	Sweden	5569519	139176	32883	68.5	505104	34.21
18	Turkey	31361351	263662	49398	41.8	864317	405.11
19	United Kingdom	34633314	492004	47387	43.9	2956574	306.50
20	United States	165649358	4049754	291680	19.9	18238301	4686.08

The definitions of the utilized variables are provided below, and their values along with the corresponding country names are listed in Table 4 [46].

- **I1 (Input) Labor force**: This includes people aged 15 and older who provide labor for the production of goods and services over a specified period [45] (this unit's input is person).

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- **I2 (Input) Gross capital formation**: Formerly known gross domestic investment consists of outlays on additions to the fixed assets of the economy plus net changes in the level of inventories (this unit's input is in million USD) [45].

- **I3 (Input) Total renewable energy capacity**: Represents the total net generating capacity of power plants and other installations that generate electricity from renewable energy sources (this unit's input is MW (megawatt)) [19].

- **I4 (Input) Share of renewable energy**: Represents the ratio of renewable energy production to total electricity production (i.e., hydro, wind, geothermal and solar (this unit's input is %) [13].

- **Y (Desirable output) Gross domestic product (GDP)**: GDP is the amount of all resident producers' gross value-added plus any commodity tax in the economy (this unit's output is million USD) [45].

- **U (Undesirable output) CO2 emissions**: CO2 emissions include only emissions from fossil fuel combustion (coal, oil, and gas) (this unit's output is million MtCO2 or million tons of CO2) [13].

Table 5: Numerical result of Example 2.

DMU	u^*_{fj}	E_k	E^*_k	$\frac{1-E^*_k}{1-E_k}$	$\frac{E^*_k-E_k}{1-E_k}$
1	370.48	1	1	0	0
2	87.24	1	1	0	0
3	150.99	0.55086	0.7761	0.49838	0.5016
4	89.09	1	1	0	0
5	273.59	1	1	0	0
6	421.04	0.7373	0.8014	0.7562	0.24379
7	186.58	0.7829	0.9250	0.34547	0.6545
8	910.89	0.7687	0.8234	0.7633	0.2366
9	609.20	1	1	0	0
10	15.29	0.10718	0.8083	0.2146	0.7853
11	118.60	0.9438	0.9910	0.1595	0.8404
12	33.00	1	1	0	0
13	37.82	1	1	0	0
14	286.17	1	1	0	0
15	37.87	1	1	0	0
16	117.05	0.6609	0.8511	0.439014	0.56098
17	34.21	1	1	0	0
18	82.17	0.45522	0.82411	0.3228	0.677135
19	306.50	1	1	0	0
20	4686.08	1	1	0	0
AVE	442.6945	0.850353	0.940037	0.17497	0.22503

Similar to the previous example, models (10), (7), and (11) were executed on the dataset from Table 4 to obtain the values of u^*_{fj}, E_k, and E^*_k. The values of u^*_{fj}, E_k, and E^*_k are presented in the second, third, and fourth columns of Table 5, respectively. The values in the last two columns of Table 5 represent ratios $\frac{1-E_k^*}{1-E_k}$ and $\frac{E_k^*-E_k}{1-E_k}$, which reflect the results obtained from the proposed models. Finally, the last row of Table 5 displays the average values for each column.

Comparing the results from the proposed models (third and fourth columns), the number of efficient DMUs remains constant, but the average efficiency increases by 0.08968 (0.94003 - 0.8505 = 0.0869). Additionally, undesirable outputs decreased by 15.06% as shown in Table 3 (fifth column) and Table 4 (second column).

Similarly, tthe inefficiency causes for each DMU can be identified using relations (4) and (5). Finally, the last two columns of Table 5 reveal that, on average, 17.49% of the inefficiency of the twenty DMUs is due to inefficiencies in desirable outputs, while 22.50% results from the production of excess undesirable outputs.

5 Conclusion

Evaluation and decision-making regarding the performance of units with undesirable outputs have always been significant for senior decision-makers. Considering the negative impact of undesirable outputs on the performance evaluation of DMUs and the practical impossibility of completely eliminating undesirable outputs, it is possible to reduce the amount of undesirable outputs to the minimum unavoidable level and calculate the minimum amount. Various researchers, including Kuosmanen [27], Kao and Hwang [22], Karagiannis and Kourtzeidis [23], and Maghbouli et al. [34], have proposed models for evaluating the efficiency of such DMUs, each of which has faced challenges. In this study, models were proposed to calculate the minimum undesirable outputs and efficiency with undesirable outputs and the minimum undesirable outputs using the DDF model with individual-proportion weak disposability.With these proposed models, the existing challenges were resolved. Using the proposed model, the inefficiency of a DMU can be decomposed into the total inefficiency of the shortfall in desirable outputs and the excess undesirable outputs. Decomposing inefficiency helps decision-makers identify inefficiency factors, enabling them to make more effective efforts to improve their DMU performance. Additionally, decomposing efficiency assists influential institutions in identifying inefficiency factors to enhance DMU performance and reduce undesirable outputs by using more suitable technologies or higher-quality materials such as better fuel. Finally, the superiority of the proposed method over the previous methods is demonstrated through the results obtained from two practical examples. This study, in addition to the aforementioned advantages, has limitations, which are outlined below and can present opportunities for further research:

- In this article, we assume that all data is positive. The exploration of alternative data types, including negative or zero valuse, present a promising avenue for future research.
- The fact that our study did not evaluate the models in the presence of constant or increasing returns to scale may present an opportunity for future research in this area.
- Using models that consider several factors, such as cost and environmental impact, can improve optimization results. These topics provide insights and interesting directions for future studies.

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