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# Fractal complex analysis

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**Abstract.** In this paper, we begin by providing a concise overview of fractal calculus. We then explore the concepts of fractal complex numbers and functions, define the fractal complex derivative, and derive the fractal Cauchy-Riemann equations. Additionally, we introduce fractal contour integrals, offer illustrative examples, and present their visualizations. Finally, we examine and visualize the transformations of circles under fractal complex functions.

*Keywords*: Fractal calculus, fractal complex number, fractal complex function, fractal complex derivative, fractal contour integrals.
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## 1 Introduction

Fractals, with their intricate patterns and self-similarity, are widely observed in nature, such as in coastlines, blood vessels, and clouds [21]. These structures are characterized by fractional dimensions exceeding their topological dimensions and are analyzed using tools from fractal geometry, measure theory, and stochastic processes [4, 5, 20, 24, 26, 28].

Fractional calculus generalizes the order of differentiation and integration but remains fundamentally non-local. However, most physical measurements are local. In an attempt to define local formulations of

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fractional calculus, Kolwankar and Gangal introduced the concept of local fractional derivatives in [19]. They discovered that enforcing locality leads to a change in the underlying mathematical framework, where the measure of the space aligns with that of a fractal set. This observation gave rise to fractal calculus [23], which redefines differential and integral operators based on the measure theory of fractals [2,3].

It is important to note that while some researchers attempt to analyze fractals using fractional derivatives, this approach has a fundamental limitation [27]. Fractional calculus changes only the order of differentiation and does not alter the underlying measure space, which remains based on the real number line. In contrast, fractals require a measure that captures their non-integer dimensional structure. Therefore, fractal calculus, which is grounded in the appropriate fractal measure, is more suitable for studying dynamics on fractal geometries.

To address the mathematical challenges posed by fractals, classical calculus has been extended to fractal settings, leading to fractal derivatives, integrals, and novel frameworks for studying complex geometries and dynamics [6, 22, 23, 25]. Fractal calculus has found significant applications in physics, engineering, and beyond, enabling models for phenomena such as sub- and super-diffusion [7, 10, 11], fractal time dynamics [12, 14, 17], and fractal space analysis [15, 18]. Recent advancements include fractal Laplace and Fourier transforms [9, 12], stability analyses of fractal differential equations [13], and generalizations to mean square calculus and nonstandard analysis [8, 16].

It is well known that the Mandelbrot set is one of the most famous and visually captivating objects in mathematics and fractal geometry. Defined by a deceptively simple iterative process, the Mandelbrot set exhibits a boundary of infinite complexity and self-similarity. Specifically, the Mandelbrot set $M \subset \mathbb{C}$ consists of all complex numbers $c$ for which the sequence defined by [21]:

$$z_{n+1} = z_n^2 + c, \quad z_0 = 0,$$

remains bounded as $n \to \infty$.

Closely related is the Julia set, which, for a fixed complex number $c \in \mathbb{C}$, is defined as the set of initial points $z_0 \in \mathbb{C}$ for which the iteration [1]:

$$z_{n+1} = z_n^2 + c$$

does not escape to infinity. In other words, the Julia set captures the boundary between stable and unstable behavior under iteration for a given $c$, whereas the Mandelbrot set captures the set of all such $c$ values that lead to bounded orbits starting from zero.

Motivated by the intricate and fractal nature of these sets, this paper explores the extension of complex analysis into the realm of fractal geometry through fractal complex analysis. Since fractal calculus adapts classical calculus to sets with non-integer dimensions, it allows for the formulation of integrals and derivatives on fractal supports. By incorporating the concept of fractal dimension and the appropriate measure, fractal complex analysis enables us to investigate transformations and properties of fractal objects such as the Mandelbrot and Julia sets in a more mathematically rigorous way.

This paper introduces a novel framework termed Fractal Complex Analysis, extending fractal calculus to the realm of complex functions. The structure of the paper is as follows: foundational definitions of fractal calculus are presented in Section 2, followed by the introduction of fractal complex numbers and functions in Section 3. Fractal derivatives for complex functions are explored in Section 4, and Section 5 defines fractal contour integrals along with their properties. Visualizations of transformations under fractal complex functions are discussed in Section 6, with Section 7 concluding the study by summarizing key findings and implications.

## 2 Basic definitions of fractal calculus

This section summarizes the definitions of fractal calculus for the Cantor set $F \subset [a,b] \subset \mathbb{R}$, as discussed in [6, 23].

**Definition 1.** *The indicator function of $F$ is given by*

$$\mathbb{I}_F(J) = \begin{cases} 1, & \text{if } F \cap J \neq \emptyset, \\ 0, & \text{otherwise}, \end{cases}$$

*where $J = [a,b] \subset \mathbb{R}$.*

**Definition 2.** *The coarse-grained fractal measure of $F \cap [a,b]$ is*

$$\mathscr{H}_\delta^\alpha(F,a,b) = \inf_{|\mathscr{P}| \leq \delta} \sum_{i=0}^{n-1} \Gamma(\alpha+1)(x_{i+1}-x_i)^\alpha \mathbb{I}_F([x_i,x_{i+1}]),$$

*where $\mathscr{P} = \{x_0 = a, x_1, \dots, x_n = b\}$ and $|\mathscr{P}| = \max_i(x_{i+1}-x_i)$.*

**Definition 3.** *The fractal measure is defined as*

$$\mathscr{H}^\alpha(F,a,b) = \lim_{\delta \to 0} \mathscr{H}_\delta^\alpha(F,a,b).$$

**Definition 4.** *The fractal dimension of $F \cap [a,b]$ is*

$$\begin{aligned} \dim_\gamma(F) &= \inf\{\alpha : \mathscr{H}^\alpha(F,a,b) = 0\} \\ &= \sup\{\alpha : \mathscr{H}^\alpha(F,a,b) = \infty\}. \end{aligned} \tag{1}$$

**Definition 5.** *The integral staircase function is*

$$S_F^\alpha(x) = \begin{cases} \mathscr{H}^\alpha(F,a_0,x), & x \geq a_0, \\ -\mathscr{H}^\alpha(F,x,a_0), & x < a_0, \end{cases}$$

*where $a_0 \in \mathbb{R}$.*

**Definition 6.** *A function $g : F \to \mathbb{R}$ is $F$-continuous at $x \in F$ if*

$$g(x) = \lim_{y \to x} g(y), \quad y \in F,$$

*whenever the limit exists.*

**Definition 7.** *The $F^{\alpha}$-derivative of $g$ at $x \in F$ is*

$$D_F^{\alpha} g(x) = \begin{cases} \lim\limits_{y \to x} \dfrac{g(y) - g(x)}{S_F^{\alpha}(y) - S_F^{\alpha}(x)}, & x \in F, \\ 0, & x \notin F, \end{cases}$$

*if the limit exists.*

**Definition 8.** *The $F^{\alpha}$-integral of $g \in B(F)$ (bounded on $F$) is*

$$\int_a^b g(x)\, d_F^{\alpha} x = \sup_{\mathscr{P}} \sum_{i=0}^{n-1} \inf_{x \in F \cap [x_i, x_{i+1}]} g(x) \Delta S_F^{\alpha} = \inf_{\mathscr{P}} \sum_{i=0}^{n-1} \sup_{x \in F \cap [x_i, x_{i+1}]} g(x) \Delta S_F^{\alpha},$$

*where $\Delta S_F^{\alpha} = S_F^{\alpha}(x_{i+1}) - S_F^{\alpha}(x_i)$, and the infimum/supremum is over subdivisions $\mathscr{P}$.*

## 3 Fractal complex numbers and functions

Here, we introduce and explain the ideas of fractal functions and fractal complex numbers.

**Definition 9.** *A fractal complex number is defined as $\zeta = a + ib$, $a, b \in \mathscr{F}$, where $i = \sqrt{-1}$.*

**Example 1.** Let $\mathscr{F}$ be the ternary Cantor set. Choose $a$ and $b$ from $\mathscr{F}$. Then the fractal complex number $\zeta$ is: $\zeta = \frac{1}{3} + i\frac{2}{9}$. This represents a fractal complex number where both real and imaginary parts belong to the ternary Cantor set.

**Definition 10.** *A complex function $f(w)$ is a mapping that associates complex numbers with other complex numbers. Specifically, if $w = u + iv$ where $u, v \in \mathbb{R}$ and $i = \sqrt{-1}$, then a complex function $f$ is formally defined as:*

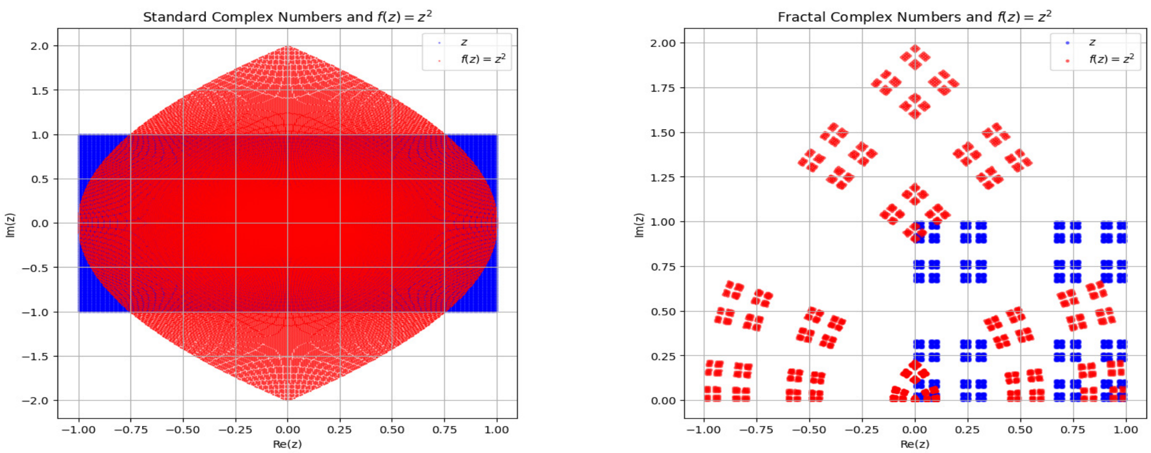
$$f : \mathbb{C} \to \mathbb{C}, \quad w \mapsto f(w).$$

**Definition 11.** *A fractal complex function $f(\zeta)$ is a mapping that associates fractal complex numbers with fractal complex numbers. If $\zeta = a + ib$ where $a, b \in \mathscr{F}$ and $i = \sqrt{-1}$, then a fractal complex function $f$ is defined as:*

$$f : \mathscr{F} \times \mathscr{F} \to \mathbb{C}, \quad \zeta \mapsto f(\zeta),$$

*where $\mathscr{F}$ is a fractal set.*

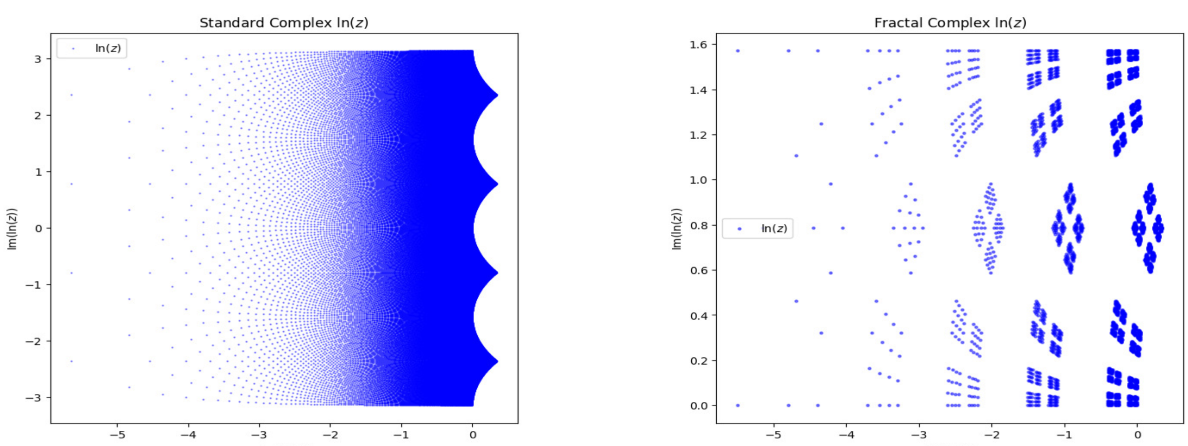
**Example 2.** Let $\mathscr{F}$ be the ternary Cantor set. Consider the fractal complex function $f(\zeta) = \zeta^2$. If we choose $\zeta = \frac{1}{3} + i\frac{2}{9}$, then $f\left(\frac{1}{3} + i\frac{2}{9}\right) = \frac{5}{81} + i \cdot \frac{4}{27}$. This illustrates a fractal complex function applied to a fractal complex number. In Figure 1, we compare the standard complex function $f(w) = w^2$ with its fractal counterpart.




**(a)** Standard Complex Numbers and $f(w) = w^2$ **(b)** Fractal Complex Numbers and $f(\zeta) = \zeta^2$

**Figure 1:** Comparison of $f(w) = w^2$ for standard complex numbers and fractal complex numbers from the ternary Cantor set. 1a shows the transformation of standard complex numbers under quadratic mapping. 1b illustrates the transformation of fractal complex numbers derived from the ternary Cantor set, demonstrating the unique behavior and distribution of the fractal set under quadratic mapping.

**Example 3.** Let $F$ be the ternary Cantor set. Consider the fractal complex function $f(\zeta) = \ln(\zeta)$. In Figure 2, we compare the standard complex logarithm $\ln(w)$ with its fractal counterpart derived from the ternary Cantor set.




**(a)** Standard complex numbers and $\ln(w)$ **(b)** Fractal complex numbers and $\ln(\zeta)$

**Figure 2:** Comparison of $\ln(w)$ for standard complex numbers and $\ln(\zeta)$ for fractal complex numbers from the ternary Cantor set. 2a shows the transformation of standard complex numbers under the logarithmic mapping. 2b illustrates the transformation of fractal complex numbers derived from the ternary Cantor set, revealing the unique behavior and distribution of the fractal set under the logarithmic mapping.

## 4 Fractal derivative of fractal complex functions

The notion of the fractal derivative for fractal complex functions is defined in this section.

**Definition 12.** *The fractal derivative of a fractal complex function generalizes the concept of the derivative for complex functions. For a complex function $f(\zeta): F \times F \to \mathbb{C}$, where $f(\zeta) = u(x,y) + iv(x,y)$, the derivative at a point $\zeta_0$ is defined as:*

$$\begin{aligned} D_F^{\alpha} f(\zeta_0) &= F\_\lim_{\zeta \to \zeta_0} \frac{f(\zeta) - f(\zeta_0)}{S_F^{\alpha}(\zeta) - S_F^{\alpha}(\zeta_0)} \\ &= F\_\lim_{\Delta^{\alpha} S_F^{\alpha}(\zeta) \to 0} \frac{f(\zeta) - f(\zeta_0)}{\Delta^{\alpha} S_F^{\alpha}(\zeta)} \\ &= D_{F,x}^{\alpha} u + i D_{F,x}^{\alpha} v, \quad if \quad \Delta^{\alpha} S_F^{\alpha}(y) = 0, \\ &= D_{F,y}^{\alpha} u + i D_{F,y}^{\alpha} v, \quad if \quad \Delta^{\alpha} S_F^{\alpha}(x) = 0, \end{aligned}$$

*if it exists, where $\Delta^{\alpha} S_F^{\alpha}(\zeta) = \Delta^{\alpha} S_F^{\alpha}(x) + i\Delta^{\alpha} S_F^{\alpha}(y)$.*

**Definition 13.** *For a complex function $f(\zeta) = u(x,y) + iv(x,y)$ defined on a fractal domain $F \times F$, where $\zeta = x + iy$, the fractal version of the Cauchy-Riemann equations are given by:*

$$\begin{aligned} D_{F,x}^{\alpha} u &= D_{F,y}^{\alpha} v, \\ D_{F,y}^{\alpha} u &= -D_{F,x}^{\alpha} v, \end{aligned} \tag{2}$$

*where $D_{F,x}^{\alpha}$ and $D_{F,y}^{\alpha}$ denote the fractal derivatives with respect to $x$ and $y$, respectively. These equations ensure that the function $f(\zeta)$ is fractal differentiable, analogous to the classical Cauchy-Riemann equations, which ensure holomorphy (complex differentiability) in classical complex analysis.*

**Definition 14.** *A fractal complex function $f(\zeta) = u(x,y) + iv(x,y)$ is differentiable at a point $\zeta = \zeta_0$ if we say that $f(\zeta)$ is fractal analytic at $\zeta = \zeta_0$. In other words, $f(\zeta)$ is fractal analytic if it satisfies the fractal Cauchy-Riemann equations and the fractal derivative exists at $\zeta_0$.*
*If $f(\zeta)$ is fractal analytic everywhere in the fractal domain (finite or infinite), we call it a fractal entire function. If the fractal derivative $D_F^{\alpha} f(\zeta)$ does not exist at $\zeta = \zeta_0$, then $\zeta_0$ is labeled a fractal singular point.*

**Example 4.** Let $f(\zeta) = \zeta^2$. Then the real part $u(x,y) = x^2 - y^2$, $x, y \in F$ and the imaginary part $v(x,y) = 2xy$. Using Eq.(2) we have:

$$D_{F,x}^{\alpha} u = 2x = D_{F,y}^{\alpha} v, \quad D_{F,y}^{\alpha} u = -2y = -D_{F,x}^{\alpha} v, \quad x, y \in F.$$

We see that $f(\zeta) = \zeta^2$ satisfies the fractal Cauchy-Riemann conditions throughout the fractal complex plane. Since the partial fractal derivatives are clearly $F$-continuous, we conclude that $f(\zeta) = \zeta^2$ is fractal analytic.

**Example 5.** Let $f(\zeta) = \zeta^*$. Now $u = x$ and $v = -y$. By applying the fractal Cauchy-Riemann conditions, we obtain:

$$D_{F,x}^{\alpha} u = 1, \quad \text{whereas} \quad D_{F,y}^{\alpha} v = -1.$$

The fractal Cauchy-Riemann conditions are not satisfied, and $f(\zeta) = \zeta^*$ is not a fractal analytic function of $\zeta$. It is interesting to note that $f(\zeta) = \zeta^*$ is $F$-continuous but nowhere fractal differentiable. In Figure 3, we illustrate $f(\zeta) = \zeta^*$ on fractal complex numbers derived from the ternary Cantor set.

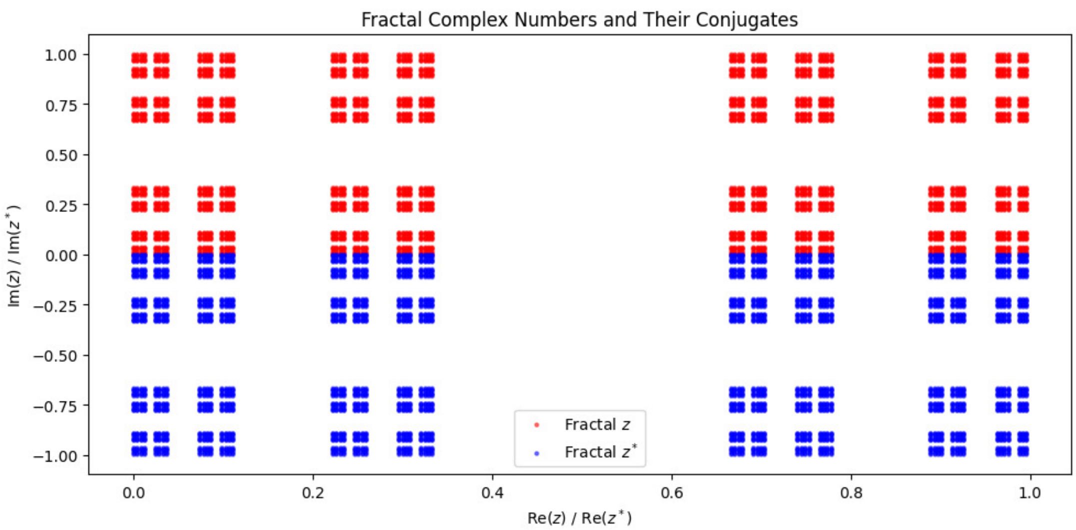



**Figure 3:** Plot of fractal complex numbers and their conjugates. The red points represent the original fractal complex numbers $\zeta$, while the blue points represent the fractal complex conjugates $\zeta^*$. This plot demonstrates the unique distribution and transformation of fractal complex numbers under the complex conjugate mapping.

# 5 Fractal contour integrals

In this section, we introduce and define fractal contour integrals.

**Definition 15.** *Given a complex function $f(\xi): F \times F \to \mathbb{C}$ defined in a region of the complex plane, the contour integral of $f(\xi)$ along a path $\Gamma$ from $\xi_0$ to $\xi_1$ is defined by dividing the contour from $\xi_0$ to $\xi_1$ into $n$ intervals with $n-1$ intermediate points $\xi_1, \xi_2, \ldots, \xi_{n-1}$ on the contour. This contour integral is given by:*

$$\int_\Gamma f(\xi)\, d_F^\alpha \xi = \underset{\Delta^\alpha S_F^\alpha(\xi_j)\to 0}{F\_lim} \sum_{j=1}^{n} f(\zeta_j)\,(S_F^\alpha(\xi_j) - S_F^\alpha(\xi_{j-1})),$$

*where $\zeta_j$ is a point on the curve between $\xi_j$ and $\xi_{j-1}$. Alternatively, for a complex function $f(\xi) = u(x,y) + iv(x,y)$, the contour integral along a path $\Gamma$ from $\xi_0$ to $\xi_1$ is defined by:*

$$\begin{aligned}\int_{\xi_0}^{\xi_1} f(\xi)\, d_F^\alpha \xi &= \int_{x_0,y_0}^{x_1,y_1} [u(x,y) + iv(x,y)][d_F^\alpha x + i\, d_F^\alpha y] \\ &= \int_{x_0,y_0}^{x_1,y_1} [u(x,y)\, d_F^\alpha x - v(x,y)\, d_F^\alpha y] + i \int_{x_0,y_0}^{x_1,y_1} [v(x,y)\, d_F^\alpha x + u(x,y)\, d_F^\alpha y].\end{aligned} \tag{3}$$

**Example 6.** Consider the contour integral of the fractal complex function $f(\xi) = \xi$ along a path $\Gamma$ from

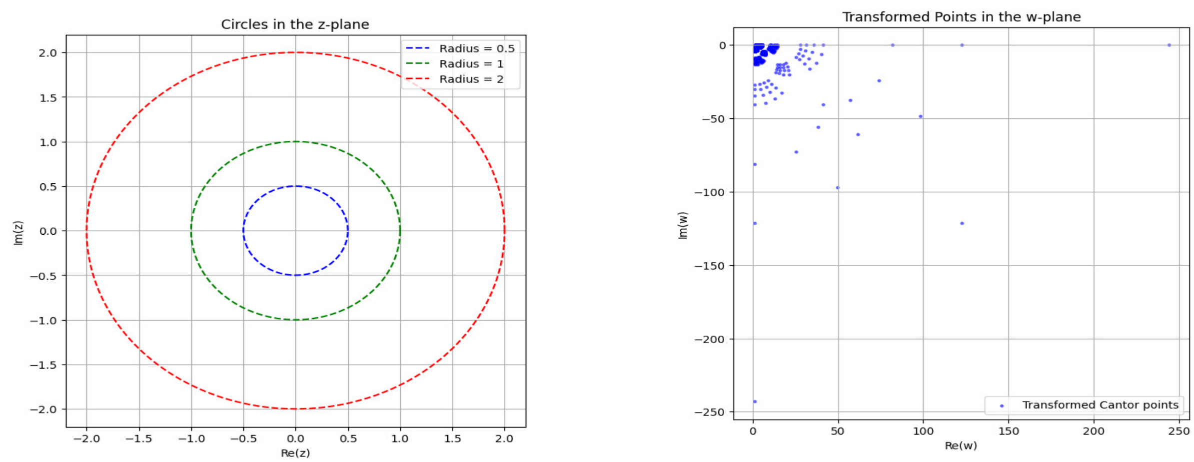
$\xi_0 = 0$ to $\xi_1 = 1+i$. Using the fractal integral definition in Eq. (3), we have

$$\int_{\xi_0}^{\xi_1} f(\xi)\, d_F^{\alpha}\xi = i\int_0^1 2t\, d_F^{\alpha} t = iS_F^{\alpha}(t)^2\Big|_0^1 = iS_F^{\alpha}(1)^2 = i(\Gamma(\alpha+1))^2,$$

here we use $x = t, y = t$.

## 6 Visualization of circle transformations

This section examines how circles centered at the origin in the $\xi$-plane are transformed under the mapping $\omega(\xi) = \frac{\xi+1}{\xi}$. Both complex numbers and fractal supports are considered for this transformation.




**(a)** Circles centered at the origin in the $\xi$-plane with various radii.

**(b)** Transformed points in the $\omega$-plane after applying the mapping $\omega(\xi) = \frac{\xi+1}{\xi}$ to the Cantor set points.

**Figure 4:** Comparison of the effects of the mapping $\omega(\xi) = \frac{\xi+1}{\xi}$ on circles in the $\xi$-plane and the fractal Cantor set in the $\omega$-plane.

As shown in Figure 4, the effects of the mapping $\omega(\xi) = \frac{\xi+1}{\xi}$ are observed in two different configurations. Figure 4a presents the transformation of circles centered at the origin in the $\xi$-plane, demonstrating how various radii are mapped. In contrast, Figure 4b illustrates the transformation of points from the Cantor set under the same mapping in the $\omega$-plane, highlighting the impact of the mapping on the fractal structure.

## 7 Conclusion

In this paper, we have laid the groundwork for a theory of fractal complex analysis by introducing fractal complex numbers and defining corresponding functions, derivatives, and integrals within fractal spaces. We derived the fractal Cauchy-Riemann equations, proposed the concept of fractal contour integration, and explored the behavior of functions over fractal domains. In particular, we visualized the transformation of geometric objects, such as circles, under fractal complex functions, uncovering intricate and often self-similar distortions. These investigations offer new insights into the interplay between complex analysis and fractal geometry.

These results provide a foundational framework for extending classical complex analysis into the setting of fractal spaces. They open promising avenues for further research in both theoretical mathematics and applied fields such as mathematical physics, where systems governed by fractal structures frequently arise.

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