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ORDER SUM SIGNED GRAPH OF A GROUP

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ABSTRACT. The order sum graph associated with the group G, denoted by Γ_{os}, is a graph with vertex set consisting of elements of G and two vertices say a,$b \in \Gamma_{os}$ are adjacent if $o(a) + o(b) > o(G)$, where $o(*)$ denotes the order of a group or an element of a group. In this paper, we introduce a signed graph called order sum signed graph where the underlying graph is a complete graph of order n and the edges receive positive and negative signs based on the order sum graph. We characterise the balanced negated order sum signed graphs. We also characterise the positive and negative homogeneous order sum signed graphs. Further, we study the properties such as clusterability, sign-compatibility, consistency and switching of signed graphs. Further, we obtain the adjacency spectra, Laplacian spectra and signless Laplacian spectra of the order sum signed graphs associated with cyclic groups.

1. INTRODUCTION

Study on graphs based on different algebraic structures has been of interest to many researchers. Some of the research articles which relate the concepts of groups and rings to graph theory are subgroup graphs (see [8]), Cayley graphs (see [17]), coprime graphs (see [26]), distance coprime graphs (see [34]), power graphs (see [11]). zero divisor graphs of semigroups (see [16]), zero divisor graphs of commutative rings (see [9]), intersection graphs (see [14]) and so on. Motivated by the literature stated above, we have introduced order sum graph of a group in [6].

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A signed graph is a graph in which the edges are either assigned positive or negative sign (see [20]). Characterisation of balanced signed graphs, complete signed graphs and subgraphs of signed graphs was determined in this paper.

A *marking* of a signed graph S is a function $\mu : V(S) \to \{+, -\}$, which assigns each vertex v a sign, which is the product of signs of the edges incident at v. A *marked signed graph* is a signed graph in which every vertex has a sign. A graph is called *marked* if each vertex is assigned either a positive or negative sign. Further research has been done by extending the concept of algebraic graphs to signed graphs. The notion of total graphs was extended in the realm of signed graphs for commutative rings and characterisation of the rings for which signed total graph and its line signed total graph to be balanced was determined in [31]. The notion of signed zero divisor graph was introduced and some of the properties like balancing, clusterability, sign-compatibility and consistency were investigated in [32]. A signed graph S is said to be *sign-compatible* if there exists a marking μ of its vertices such that the end vertices of every negative edge receive the sign $-$ in μ and no positive edge holds this property (see [32]).

In this paper, we extend the concept of order sum graphs to signed graphs by defining a new signed graph called order sum signed graph of a group whose definition is given in the next section.

For the terms and definitions in graph theory, we refer to [37] and for those in group theory, refer to [15].

2. Order Sum Signed Graph of a Group

Definition 2.1. [6] Let G be a finite group of order $o(G)$. Then, the *order sum graph* associated with the group G, denoted by Γ_{os}, is a graph with vertex set consisting of elements of G and two vertices say a,$b \in \Gamma_{os}$ are adjacent if $o(a) + o(b) > o(G)$.

For example, let U_{18} be a group of elements less than 18 and relatively prime to 18 under the operation of multiplication modulo 18. That is, $U_{18} = \{1, 5, 7, 11, 13, 17\}$. Here, $o(U_{18}) = 6$, $o(1) = 1, o(5) = 6, o(7) = 3, o(11) = 6, o(13) = 3, o(17) = 2$. The order sum graph associated with the group U_{18} is given in Figure 1:

Theorem 2.2. [6] *The order sum graph of a group G is connected if and only if G is cyclic with at least one generator, that is, there is at least one element in G whose order is equal to order of G.*

Definition 2.3. An *order sum signed graph* of a group G of order n is an ordered pair $\Gamma^{\Sigma}_{os}(G) = (K_n, \sigma)$, where K_n is a complete graph of

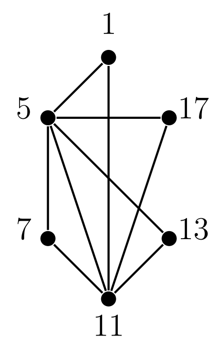

FIGURE 1. $\Gamma_{os}(U_{18})$

order n and for an edge ab of K_n, the signature function σ is defined as

$$\sigma(ab)=\begin{cases} +, & \text{if } ab \in E(\Gamma_{os}(G) \\ -, & otherwise \end{cases}$$

For example, consider the same group $U_{18} = \{1, 5, 7, 11, 13, 17\}$ under the operation of multiplication modulo 18. The order sum signed graph associated with the group U_{18} is given in Figure 2:

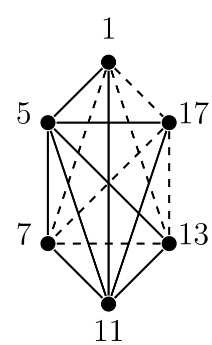

FIGURE 2. $\Gamma_{os}^{\Sigma}(U_{18})$

Theorem 2.4. [6] *The Γ_{os} associated with a group G of order n is a null graph if and only if G is non-cyclic group, that is, there is no element in G whose order is equal to n.*

The following results discuss necessary and sufficient conditions for the order sum signed graph $\Gamma_{os}^{\Sigma}(G)$ to be homogeneous.

Proposition 2.5. *Let G be a group of order n, then the $\Gamma_{os}^{\Sigma}(G)$ is a negative homogeneous signed graph if and only if G is not a cyclic group.*

Proof. Let $\Gamma_{os}^{\Sigma}(G)$ be a negative homogeneous signed graph associated with a group G of order n. Then, by Definition 2.1 and Definition 2.3, Γ_{os} must be a null graph. Therefore, by Theorem 2.4, the associated group G has no such element whose order is equal to n. Therefore, G is not a cyclic group.

Conversely, assume that G is a non-cyclic group, then there is no element in G whose order is equal to n. Then, by Definition 2.1, Γ_{os} must be a null graph. Therefore, by Definition 2.3, $\Gamma_{os}^{\Sigma}(G)$ is a negative homogeneous signed graph. □

Theorem 2.6. [6] *Let G be a group of order n, $n \geq 3$. Then, the order sum graph associated with G is complete if and only if all the elements of G other than the identity element have order equal to n.*

Proposition 2.7. *$\Gamma_{os}^{\Sigma}(G)$ is a positive homogeneous signed graph if and only if all the elements of G other than the identity element are generators of G.*

Proof. Let $\Gamma_{os}^{\Sigma}(G)$ be a positive homogeneous signed graph associated with a group G of order n. Then, by Definition 2.1 and Definition 2.3, Γ_{os} must be a complete graph. Therefore, by Theorem 2.6, all the elements in G other than the identity element have order equal to n. Hence, all the elements of G other than the identity element are generators of G.

Conversely, assume that all the elements of G other than the identity element are generators of G. Then, all the elements in G other than the identity element have order equal to n. Then, by Definition 2.1, Γ_{os} must be a complete graph. Therefore, by Definition 2.3, $\Gamma_{os}^{\Sigma}(G)$ is a positive homogeneous signed graph. □

Note that the order sum signed graph of a group G is never balanced. Therefore, we consider the negated order sum signed graph in the next theorem.

Theorem 2.8. [13] *(Structure Theorem) A signed graph S is balanced if and only if its vertex set can be partitioned into two subsets V_1 and V_2, one of them may be empty, such that any edge joining two vertices within the same subset is positive, while any edge joining two vertices in different subsets is negative.*

Lemma 2.9. [20] *A signed graph S is balanced if each of its cycles includes an even number of negative edges.*

Theorem 2.10. *The negated order sum signed graph $\sim \Gamma_{os}^{\Sigma}(G)$ associated with a group G is balanced if and only if G contains exactly one generator of G*

Proof. Let $\sim \Gamma_{os}^{\Sigma}(G)$ associated with a group G of order n be balanced. Assume that G contains more than one generator, say $a \in G$ and $b \in G$. Let $c \in G$ be an element which does not generate G. Then, by Definition 2.1, the vertices a, b and c are mutually adjacent in $\Gamma_o s(G)$ which forms a C_3. Therefore, By Definition 2.3, the $\sim \Gamma_{os}^{\Sigma}(G)$ consists of a cycle with odd number of negative edges, which follows from Lemma 2.9 that the $\sim \Gamma_{os}^{\Sigma}(G)$ is unbalanced. This is a contradiction to the fact that $\sim \Gamma_{os}^{\Sigma}(G)$ is balanced. Therefore, G must contain exactly one generator.

Conversely, let G contain exactly one generator, say x. Then, we can partition the vertex set of $\sim \Gamma_{os}^{\Sigma}(G)$ into two subsets say A and B where A is a singleton set containing x and B is a set containing all the remaining vertices such that all the negative edges lie across the partitions A and B and all the positive edges lie within the partition B. Therefore, by Theorem 2.8 $\sim \Gamma_{os}^{\Sigma}(G)$ is balanced. □

Definition 2.11. [30] A signed graph is k-clusterable for some $k \geq 2$, if its vertex set can be partitioned into k subsets $V_1, V_2, \ldots, V_k$ called clusters such that every positive edge joins two vertices in the same cluster and every negative edge joins two vertices in different clusters.

Theorem 2.12. *The order sum signed graph $\Gamma_{os}^{\Sigma}(G)$ associated with a group G of order n is n-clusterable if and only if G is not a cyclic group.*

Proof. Let $\Gamma_{os}^{\Sigma}(G)$ associated with a group G of order n be n-clusterable. Assume that G is a cyclic group. Then, G contains at least one generator say a. Then by Definition 2.1, a is adjacent to all the remaining vertices in $\Gamma_o s(G)$. Then by Definition 2.3, an edge between a and any other vertex in $\Gamma_{os}^{\Sigma}(G)$ is positive and the remaining edges are all negative. Therefore, we cannot partition the vertex set of $\Gamma_{os}^{\Sigma}(G)$ such that the negative edges lie across the clusters and positive edges lie within the clusters, which is a contradiction that $\Gamma_{os}^{\Sigma}(G)$ is n-clusterable. Therefore, G must not be a cyclic group.

Conversely, let G be a non-cyclic group. Then, G contains no generator, which means that there is no element in G whose order is equal to n. Then, by Theorem 3.2, $\Gamma_{os}^{\Sigma}(G)$ is a negative homogeneous signed graph. Therefore, we can partition the vertex set of $\Gamma_{os}^{\Sigma}(G)$ into n clusters such that all the negative edges lie across the clusters. Hence, $\Gamma_{os}^{\Sigma}(G)$ is n-clusterable. □

Theorem 2.13. *Any negated order sum signed graph $\sim \Gamma_{os}^{\Sigma}(G)$ associated with a group G of order n is $(\vartheta + 1)$-clusterable, where, ϑ is the number of generators in G.*

Proof. Let $\sim \Gamma^{\Sigma}_{os}(G)$ be a negated order sum signed graph associated with a group G of order n. Let ϑ be the number of generators in G. We can partition the vertex set of $\sim \Gamma^{\Sigma}_{os}(G)$ into ϑ singleton sets each containing one generator and one set containing all the remaining vertices such that all the negative edges lie across the clusters and positive edges lie within the cluster. Therefore, $\sim \Gamma^{\Sigma}_{os}(G)$ is $(\vartheta + 1)$-clusterable. □

Definition 2.14. [32] A signed graph S is sign-compatible if there exists a marking μ of its vertices such that the end vertices of every negative edge receive '$-$' sign in μ and no positive edge in S has both of its ends '$-$' sign in μ.

Proposition 2.15. *The order sum signed graph associated with a group G of order n is sign-compatible.*

Proof. Let $\Gamma^{\Sigma}_{os}(G)$ be associated with a group G of order n. We can always mark the vertices of $\Gamma^{\Sigma}_{os}(G)$ such that the end vertices of every negative edge receive '$-$' sign and no positive edge has both its ends assigned '$-$' sign by marking all the generators of G as '$+$' and all the remaining vertices '$-$'. Therefore, by Definition 2.14, $\Gamma^{\Sigma}_{os}(G)$ is sign-compatible. □

A switching function for a signed graph S is a function $\zeta : V \to \{+, -\}$. The switched signature is $\sigma^{\zeta}(e) = \zeta(v)\sigma(e)\zeta(w)$, where e has endpoints v, w. The *switched signed graph* is $S^{\zeta} := (G, \sigma^{\zeta})$. We say $\sum$ is switched by ζ (see [38]).

Theorem 2.16. *The order sum signed graph associated with a group G of order n with ϑ generators is switching invariant under canonical marking if n is odd and ϑ is even or n is even and ϑ is odd.*

Proof. Let G be a group of order n and $\Gamma^{\Sigma}_{os}(G)$ be its order sum signed graph with ϑ generators. Consider the canonical marking of the vertices in $\Gamma^{\Sigma}_{os}(G)$.

For the switched graph to be invariant, the canonical marking of the vertices in $\Gamma^{\Sigma}_{os}(G)$ must be $'+'$ for every vertex.

Since there are ϑ generators in $\Gamma^{\Sigma}_{os}(G)$, the canonical marking of these generators is $'+'$ as there are no negative edges incident to it. The vertices other than generators have positive degree ϑ and therefore the negative degree must be $n - 1 - \vartheta$. For the canonical marking of the vertices other than the generators to be $'+'$, $n - 1 - \vartheta$ must be even. Consider the following two cases:

Case-1: Let n be odd. Then, $n - 1 - \vartheta$ is even only if ϑ is even. Therefore, $\Gamma^{\Sigma}_{os}(G)$ is switching invariant if n is odd and ϑ is even.

Case-2: Let n be even. Then, $n-1-\vartheta$ is even only if ϑ is odd. Therefore, $\Gamma_{os}^{\Sigma}(G)$ is switching invariant if n is even and ϑ is odd.
This completes the proof. □

Theorem 2.17. *The order sum signed graph associated with a group G of order n with ϑ generators is either sign-compatible or consistent and both simultaneously if and only if $\vartheta = n-1$.*

Proof. Consider the order sum signed graph $\Gamma_{os}^{\Sigma}(G)$ associated with a group G of order n with ϑ generators. Consider the canonical marking of the vertices in $\Gamma_{os}^{\Sigma}(G)$. Since there are ϑ generators in $\Gamma_{os}^{\Sigma}(G)$, the canonical marking of these generators is $'+'$ as there are no negative edges incident to it. For the vertices other than the generators, the positive degree is ϑ and negative degree is $n-1-\vartheta$. Now, the following cases arise:

Case-1: n and ϑ are of same parity Since either n is even and ϑ is even or n is odd and ϑ is odd, $n-1-\vartheta$ is odd. Therefore, there are odd number of negative edges incident to the vertices other than generators. Hence, their canonical marking will be $'-'$. Hence, every negative edge receive $'-'$ sign and no positive edge has both its end vertices $'-'$. Therefore, $\Gamma_{os}^{\Sigma}(G)$ is sign compatible. But, it is not consistent since there exists C_3 with odd number of negative vertices.

Case-2: n and ϑ are of opposite parity. Since, either n is even and ϑ is odd or n is odd and ϑ is even, $n-1-\vartheta$ is even. Therefore, there are even number of negative edges incident to the vertices other than generators. So, their canonical marking will be $'+'$. Hence, all the vertices in $\Gamma_{os}^{\Sigma}(G)$ have $'+'$ sign. Clearly, it is consistent as there are no cycles with *odd* number of negative vertices. Since end vertices of every negative edge receive $'+'$ signs , it is not sign-compatible except for the case when $\vartheta = n-1$ where there are no negative edges.
This completes the proof. □

3. Spectra of Order Sum Signed Graphs

Theorem 3.1. *The spectra of $\Gamma_{os}^{\Sigma}(G)$ associated with a cyclic group G of order n and ϑ generators is*

$$\begin{pmatrix} \frac{1}{2}(\eta+\sqrt{\zeta}) & \frac{1}{2}(\eta-\sqrt{\zeta}) & 1 & -1 \\ 1 & 1 & n-(\vartheta+1) & \vartheta-1 \end{pmatrix},$$

where $\eta = +2\vartheta - n$ and $\zeta = -4\vartheta^2 + 4n\vartheta + n^2 - 4n + 4$.

Proof. Let G be a cyclic group of order n with ϑ generators. Then, the adjacency matrix of the order sum signed graph denoted by $A(\Gamma_{os}^{\Sigma}(G))$ is given by

$$A(\Gamma_{os}^{\Sigma}(G)) = \big[a_{ij}\big]_{n\times n} = \big[a_{ji}\big]_{n\times n}$$

where,

$$a_{ij} = \begin{cases} 1, & \sigma(v_iv_j) = +1 \ ; \\ -1, & \sigma(v_iv_j) = -1; \\ 0, & \text{otherwise.} \end{cases}$$

Consider $\det(\lambda I - A(\Gamma_{os}^{\Sigma}(G)))$. Here, we perform the following steps:

Step 1: Let $R_i \to R_i - R_\vartheta$, for $i = 1, 2, \ldots, \vartheta - 1$, then we get $det(\lambda I - A(\Gamma_{os}^{\Sigma}(G)))$ of the form $[\lambda + 1]^{\vartheta-1}\det(A)$.

Step 2 : In $\det(A)$, let $R_i \to R_i - R_{\vartheta+1}$, where $i = \vartheta+2, \vartheta+3, \ldots, \vartheta + (n-\vartheta)$ to obtain $\det(A)$ of the form $[\lambda - 1]^{n-(\vartheta+1)}\det(B)$. Here, we need to consider the following cases.

Step 3: On expansion and simplification of $\det(B)$ we get, $\det(B) = \left(\lambda - (\frac{1}{2}(\eta + \sqrt{\zeta}))\right)\left(\lambda - (\frac{1}{2}(\eta - \sqrt{\zeta}))\right)$, where $\eta = -n + 2\vartheta$ and $\zeta = -4\vartheta^2 + 4n\vartheta + n^2 - 4n + 4$.

Therefore, the characteristic polynomial of $A(\Gamma_{os}^{\Sigma}(G))$ is $\phi(A(\Gamma_{os}^{\Sigma}(G))) =$

$$(\lambda+1)^{\vartheta-1}(\lambda-1)^{n-(\vartheta+1)}\left[\lambda - \left(\frac{(\eta+\sqrt{\zeta})}{2}\right)\right]\left[\lambda - \left(\frac{(\eta-\sqrt{\zeta})}{2}\right)\right],$$

This completes the proof. □

The degree matrix of the order sum signed graph of a group G of order n is a diagonal matrix in which all the diagonal elements are $n-1$. It is denoted by $D(\Gamma_{os}^{\Sigma}(G))$.

Theorem 3.2. *The Laplacian spectra of $\Gamma_{os}^{\Sigma}(G)$ associated with a cyclic group G of order n and ϑ generators is*

$$\begin{pmatrix} n-2 & n & \frac{1}{2}(\eta_1 + \sqrt{\zeta_1}) & \frac{1}{2}(\eta_1 - \sqrt{\zeta_1}) \\ n-(\vartheta+1) & \vartheta - 1 & 1 & 1 \end{pmatrix},$$

where $\eta_1 = 3n - 2\vartheta - 2$ and $\zeta_1 = -4\vartheta^2 + 4n\vartheta + n^2 - 4n + 4$.

Proof. Let G be a cyclic group of order n with ϑ generators. Then, the Laplacian matrix of the order sum signed graph denoted by $L(\Gamma_{os}^{\Sigma}(G))$ is given by

$$L(\Gamma_{os}^{\Sigma}(G)) = D(\Gamma_{os}^{\Sigma}(G)) - A(\Gamma_{os}^{\Sigma}(G))$$

where, $A(\Gamma_{os}^{\Sigma}(G))$ is the adjacency matrix and $D(\Gamma_{os}^{\Sigma}(G))$ is the degree matrix of the order sum signed graph.

Consider $\det(\lambda I - L(\Gamma_{os}^{\Sigma}(G)))$. Here, we perform the following steps:

Step 1: Let $R_i \to R_i - R_1$, for $i = 2, 3, \ldots, \vartheta$, then we get $det(\lambda I - L(\Gamma_{os}^{\Sigma}(G)))$ of the form $[\lambda - n]^{\vartheta-1}\det(A)$.

Step 2 : In $\det(A)$, let $R_i \to R_i - R_n$, where $i = \vartheta+1, \vartheta+2, \ldots, \vartheta + (n-(\vartheta+1))$ to obtain $\det(A)$ of the form $[\lambda-(n-2)]^{n-(\vartheta+1)}\det(B)$.

Step 3: On expansion and simplification of $\det(B)$ we get,

$$\det(B) = \left(\lambda - (\frac{1}{2}(\eta_1+\sqrt{\zeta_1}))\right)\left(\lambda - (\frac{1}{2}(\eta_1-\sqrt{\zeta_1}))\right),$$

where $\eta_1 = 3n - 2\vartheta - 2$ and $\zeta_1 = -4\vartheta^2 + 4n\vartheta + n^2 - 4n + 4$.

Therefore, the characteristic polynomial of $L(\Gamma^{\Sigma}_{os}(G))$ is $\phi(L(\Gamma^{\Sigma}_{os}(G))) =$

$$(\lambda-n)^{\vartheta-1}(\lambda-(\vartheta-1))^{n-(\vartheta+1)}\left[\lambda - \left(\frac{(\eta_1+\sqrt{\zeta_1})}{2}\right)\right]\left[\lambda - \left(\frac{(\eta_1-\sqrt{\zeta_1})}{2}\right)\right],$$

completing the proof. □

Theorem 3.3. *The signless Laplacian spectra of $\Gamma^{\Sigma}_{os}(G)$ associated with a cyclic group G of order n and ϑ generators is*

$$\begin{pmatrix} n & n-2 & \frac{1}{2}(\eta_2+\sqrt{\zeta_2}) & \frac{1}{2}(\eta_2-\sqrt{\zeta_2}) \\ n-(\vartheta+1) & \vartheta-1 & 1 & 1 \end{pmatrix},$$

where $\eta_2 = n + 2\vartheta - 2$ and $\zeta_2 = -4\vartheta^2 + 4n\vartheta + n^2 - 4n + 4$.

Proof. Let G be a cyclic group of order n with ϑ generators. Then, the signless Laplacian matrix of the order sum signed graph denoted by $L^+(\Gamma^{\Sigma}_{os}(G))$ is given by

$$L^+(\Gamma^{\Sigma}_{os}(G)) = D(\Gamma^{\Sigma}_{os}(G)) + A(\Gamma^{\Sigma}_{os}(G))$$

where, $A(\Gamma^{\Sigma}_{os}(G))$ is the adjacency matrix and $D(\Gamma^{\Sigma}_{os}(G))$ is the degree matrix of the order sum signed graph.
Consider $\det(\lambda I - L^+(\Gamma^{\Sigma}_{os}(G)))$. Here, we perform the following steps:

Step 1: Let $R_i \to R_i - R_1$, for $i = 2, 3, \ldots, \vartheta$, then we get $det(\lambda I - L^+(\Gamma^{\Sigma}_{os}(G)))$ of the form $[\lambda-(n-2)]^{\vartheta-1}\det(A)$.

Step 2 : In $\det(A)$, let $R_i \to R_i - R_n$, where $i = \vartheta+1, \vartheta+2, \ldots, \vartheta + (n-(\vartheta+1))$ to obtain $\det(A)$ of the form $[\lambda - n]^{n-(\vartheta+1)}\det(B)$.

Step 3: On expansion and simplification of $\det(B)$ we get,

$$\det(B) = \left(\lambda - \frac{1}{2}(\eta_2+\sqrt{\zeta_2})\right)\left(\lambda - \frac{1}{2}(\eta_2-\sqrt{\zeta_2})\right),$$

where $\eta_2 = n + 2\vartheta - 2$ and $\zeta_2 = -4\vartheta^2 + 4n\vartheta + n^2 - 4n + 4,$.

Thus, the characteristic polynomial of $L^+(\Gamma^{\Sigma}_{os}(G))$ is $\phi(L^+(\Gamma^{\Sigma}_{os}(G))) =$

$$(\lambda-(n-2))^{\vartheta-1}(\lambda-n)^{n-(\vartheta+1)}\left[\lambda-\left(\frac{(\eta_2+\sqrt{\zeta_2})}{2}\right)\right]\left[\lambda-\left(\frac{(\eta_2-\sqrt{\zeta_2})}{2}\right)\right],$$

This completes the proof. □

4. Conclusion

In this paper, we defined a signed graph called order sum signed graph associated with a group and characterised the balanced negated order sum signed graphs, positive and negative homogeneous order sum signed graphs. We further investigated the properties such as sign-compatibility, clusterability and switching of order sum signed graphs. We also obtained adjacency spectra, Laplacian spectra and signless Laplacian spectra of the order sum signed graphs associated with cyclic groups.

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