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A study of American options under stochastic volatility and double exponential jumps

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Abstract. In this study, we introduce and validate a novel approach for pricing American-style options. Our model integrates stochastic volatility with a double exponential jump-diffusion process and incorporates a memory feature in the volatility component. We analyze the structure of the proposed model and demonstrate its accuracy and precision using real data from the *S&P* 500 index. Our results show that the model effectively captures market dynamics and provides a more accurate pricing of American options compared to traditional models.

Keywords: Fractional Brownian motion, rough stochastic volatility, double exponential jump, American option pricing.
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1 Introduction

The first commonly used mathematical formula to determine the theoretical value of European call and put options is the Black–Scholes option pricing model. The ability of the model to calculate fair option pricing based on a variety of parameters such as the current stock price, the strike price of the option, the time to expiration, the risk–free interest rate, and the volatility of the underlying stock, is what makes it significant. However, the assumptions upon which this model is created do not accurately represent the actual financial landscape. The primary premise is that volatility stayed constant during the option lifetime, which ignores empirical data that shows varying levels of market volatility. Stochastic volatility models let volatility change over time in an effort to address this issue with the Black–Scholes model [3, 5, 33]. The idea behind stochastic volatility models is that, contrary to what the Black–Scholes options pricing model assumes, the volatility of asset prices varies and is not constant. The first and

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of course the most popular option pricing model with stochastic volatility is the Heston model [18]. A stochastic volatility process based on the Cox–Ingersoll–Ross (CIR) model is incorporated into this model. It captures the observed volatility clustering and mean reversion by allowing the volatility to vary over time. Furthermore, a crucial aspect of options markets known as the volatility smile is captured by the Heston model. The model may provide volatility smiles, which indicate the market expectation of future volatility and offer a more accurate depiction of option pricing, by integrating stochastic volatility and correlation. In the Heston model, S_t, an asset price at time t, is determined by a stochastic process as

$$\mathrm{d}S_t = rS_t\mathrm{d}t + \sqrt{V_t}S_t\mathrm{d}W_t,$$

where r is a constant risk–free interest rate of the asset and V_t, the volatility of the asset price, is given by

$$\mathrm{d}V_t = \kappa(\theta - V_t)\,\mathrm{d}t + \sigma\sqrt{V_t}\mathrm{d}B_t,$$

where parameters σ, the volatility of volatility process, θ, the long–term price variance, and κ, the rate of reversion to the long–term price variance are constant and also, W_t and B_t are two Brownian motions corresponding to price and volatility processes, respectively, with correlation ρ.

In a parallel development, numerous research have proposed using a jump–diffusion modification of the price process to price financial instruments in order to increase model flexibility and improve market calibration. Jump–diffusion processes have been employed in finance to capture discontinuous behavior in asset pricing, in contrast to fundamental insights into continuous time asset pricing models that have been driven by stochastic diffusion processes with continuous sample paths. The primary goal of the jump–diffusion models was to expand upon the Black–Scholes model by incorporating more logical assumptions and addressing the reality that empirical research on market returns does not conform to a log–normal distribution with constant variance. The ability of the jump–diffusion models to generate the volatility smile that is seen in all options markets makes them intriguing as well. The first model to utilize jump–diffusion processes in finance was introduced by a noted American economist named Robert C. Merton in 1976 [32]. The Kou model proposed by Steven Kou in 2002 [19], as another prominent jump–diffusion model, is very similar to the Merton model. Under this model, an asset price is described by the equation

$$\frac{\mathrm{d}S_t}{S_t} = \mu\mathrm{d}t + \sigma\mathrm{d}W_t + \mathrm{d}\left(\sum_{i=1}^{N_t}(X_i - 1)\right),$$

where W_t is a standard Brownian motion, N_t is a Poisson process with rate λ, and $\{X_i\}$ is a sequence of independent identically distributed non–negative random variables such that $Y = \log(X)$ has an asymmetric double exponential distribution with the density function as

$$f_Y(y) = p\eta_1 e^{-\eta_1 y}\mathbb{1}_{y\geq 0} + q\eta_2 e^{\eta_2 y}\mathbb{1}_{y\leq 0},$$

where $\eta_1 > 1$, $\eta_2 > 0$, $p,q \geq 0$, $p+q = 1$, p is the probability of the upward jumps and q is the probability of the downward jumps. The double exponential jump–diffusion model actually matches the asset price process better than the normal jump-diffusion model, according to empirical research. In the last decade, many studies have examined the pricing of financial securities using the various jump–diffusion models [6, 21, 26, 30, 31] and, moreover, research professionals in the financial industry have begun applying jump–diffusion models as an evaluation tool [1, 20, 22, 34].

A jump process alongside stochastic volatility, as another step towards improving the pricing issue, has first been proposed by Bates in 1996 [2]. The model is a Heston process for stochastic volatility with an added Merton log–normal jump. Stochastic volatility jump–diffusion model is a type of model commonly used for equity returns that includes both stochastic volatility and jumps. The advantage of the model is that it is possible to replicate stylized facts such as heavy tails and volatility clustering and mean reversion, negative correlation between returns and volatility, and sudden large movements in the price of the asset. Studies such as [17, 27, 35, 36], provide more comprehensive knowledge about this framework.

The purpose of this work is to verify the problem of American option pricing under a fractional stochastic volatility double exponential jump–diffusion model. As we previously discussed, the goal of modeling volatility using a stochastic process driven by Brownian motion is to more closely approximate the distribution of stock returns, resulting in implied volatility surfaces with smiles. Nonetheless, the stochastic volatility models still fail to account for a few other features of the volatility (often referred to as the stylized facts). The autocorrelation of the realized volatility time series is one of them. Efforts have been undertaken to simulate this phenomenon in terms of a memory-based method for volatility. To incorporate the dependence of volatility on its past values into a model, processes driven by the fractional Brownian motion suggested [7–10,13,14,30]. A fractional Brownian motion (fBm) with Hurst parameter $H \in (0,1)$ is a Gaussian process $B^H = \{B_t^H,\ 0 \le t \le T\}$ with $\mathbb{E}[B_t^H] = 0$ and the covariance function

$$R_H(t,s) = \frac{1}{2}(t^{2H} + s^{2H} - (t-s)^{2H}), \quad t,s \in [0,T].$$

If $H = \frac{1}{2}$, then $R_H(t,s) = \min(t,s)$ and B_t^H is the standard Brownian motion. In the case $H \in (\frac{1}{2},1)$, the increments of the process are positively correlated; however, in the case $H \in (0,\frac{1}{2})$, the increments of the process are negatively correlated.

The rough volatility model is a novel kind of stochastic volatility model that was recently proposed in [4,16] and has attracted a lot of interest from academics and industry. The salient feature of this model is its assumption that a fractional Brownian motion, rather than a conventional Brownian motion, is what drives the latent stochastic volatility process (e.g., in traditional stochastic (local) volatility models). Because of the fractional Brownian motion driver, the volatility process trajectories are continuous but have irregular route characteristics. According to empirical research shown in [16], at any appropriate time scale, the log–volatility effectively behaves like a fractional Brownian motion with a Hurst exponent H of order 0.1. In [15], more empirical evidence is presented. The authors developed a quasi–likelihood estimator and applied it to realized volatility time series. They verified that the Hurst parameter is much less than half, indicating that volatility is, in fact, rough.

The rough volatility model has been incredibly successful in simulating implied volatility smiles for SPX options as well as numerous stylized facts of historical volatility time series. Rough volatility models, especially for at-the-money skew curves, offer a very precise fit to the implied volatility smile shape. They also reproduce stylized facts for realized volatility [12,23]. On the other hand, it is consistent with economic micro–structural models and naturally emerges from economic agents behaviors, as shown in [11]. In this work, we exploit this process and present a new stochastic volatility jump–diffusion model with the intention of pricing an exotic option.

The reminder of this paper is organized as follows. Section 2 makes known the structure of our model as “rough stochastic volatility double exponential jump–diffusion model”. Section 3 proves the existence and uniqueness of solution related to this model. Section 4 reports numerical experiments referring to a $S\&P500$ index as data set. Finally, Section 5 concludes the paper.

2 The model

We consider the asset price $\{S_t, t \in [0,T]\}$ which is defined on a filtered probability space $(\Omega, \mathscr{F}, \mathbb{F}, \mathbb{P})$, where $\mathbb{F} = \{\mathscr{F}_t\}_{t\geq 0}$ denotes the standard filtration generated by a two–dimensional $\mathbb{F}$–Brownian motion $(B_t, B_t^{\perp})$ and $W_t = \rho B_t + \sqrt{1-\rho^2} B_t^{\perp}$ with constant correlation $\rho \in (-1,1)$. The Heston stochastic volatility model with double exponential jumps is defined as

$$\begin{cases} \mathrm{d}S_t &= rS_t \mathrm{d}t + \sqrt{V_t} S_t \mathrm{d}W_t + \mathrm{d}\left(\sum_{i=1}^{N_t}(X_i - 1)\right), \\ \mathrm{d}V_t &= \kappa(\theta - V_t)\,\mathrm{d}t + \sigma\sqrt{V_t}\mathrm{d}B_t. \end{cases}$$

A fractional Brownian motion B_t^H with Hurst parameter $H \in (0,1)$ can be built through the Mandelbrot-van Ness representation [28] as follows

$$B_t^H = \frac{1}{\Gamma(H+1/2)} \int_{-\infty}^{0} \left((t-s)^{H-\frac{1}{2}} - (-s)^{H-\frac{1}{2}}\right) \mathrm{d}W_s + \frac{1}{\Gamma(H+1/2)} \int_0^t (t-s)^{H-\frac{1}{2}} \mathrm{d}W_s,$$

where the kernel $(t-s)^{H-\frac{1}{2}}$ plays a crucial role in the rough dynamic of the fractional Brownian motion for $H < \frac{1}{2}$. In order to allow for a rough behavior of the volatility in a Heston–type model with double exponential jumps, the kernel $(t-s)^{\alpha-1}$ is introduced as

$$\begin{cases} \mathrm{d}S_t &= rS_t \mathrm{d}t + \sqrt{V_t} S_t \mathrm{d}W_t + \mathrm{d}\left(\sum_{i=1}^{N_t}(X_i - 1)\right), \\ V_t &= V_0 + \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} \kappa(\theta - V_s)\,\mathrm{d}s + \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} \sigma\sqrt{V_s}\mathrm{d}B_s, \end{cases} \tag{1}$$

where the parameters κ, θ and σ are positive and play the same role as in the classical Heston model, and here also W and B are two Brownian motions with correlation ρ. The additional parameter $\alpha := H + \frac{1}{2}$ belongs to $(\frac{1}{2}, 1)$ and governs the smoothness of the volatility sample paths.

But the existence of the fractional kernel forces the variance process to leave both the semi–martingale and Markovian properties. To deal with such a problem in our rough stochastic volatility jump–diffusion model, encouraged by Cui et al. [25], who performs a semi–martingale approximation to the rough stochastic volatility models, we approximate the fractional kernel $(t-s)^{H-\frac{1}{2}}$ by a perturbed kernel $(t+\varepsilon-s)^{H-\frac{1}{2}}$ with $0 < \varepsilon << 1$. Based on this approximation, the volatility process V_t in system (1) would be approximated by the semi–martingale process

$$V_t^{\varepsilon} = V_0^{\varepsilon} + \frac{1}{\Gamma(\alpha)} \int_0^t (t+\varepsilon-s)^{\alpha-1} \kappa(\theta - V_s)\,\mathrm{d}s + \frac{1}{\Gamma(\alpha)} \int_0^t (t+\varepsilon-s)^{\alpha-1} \sigma\sqrt{V_s}\mathrm{d}B_s.$$

So, according to what was mentioned, our goal model to verify an American style option price process is defined as

$$\begin{cases} \mathrm{d}S_t &= rS_t \mathrm{d}t + \sqrt{V_t} S_t \mathrm{d}W_t + \mathrm{d}\left(\sum_{i=1}^{N_t}(X_i - 1)\right), \\ V_t^{\varepsilon} &= V_0^{\varepsilon} + \frac{1}{\Gamma(\alpha)} \int_0^t (t+\varepsilon-s)^{\alpha-1} \kappa(\theta - V_s)\,\mathrm{d}s + \frac{1}{\Gamma(\alpha)} \int_0^t (t+\varepsilon-s)^{\alpha-1} \sigma\sqrt{V_s}\mathrm{d}B_s. \end{cases} \tag{2}$$

Hereinafter referred to study the existence and uniqueness solutions of the price S_t and the volatility V_t^{ε} processes in our proposed system (2).

3 Examination of the model

In this section, we verify the existence and uniqueness of the solution to the price and volatility equations of rough Heston jump–diffusion model.

Theorem 1. *There exists a unique solution to jump diffusion process*

$$S_t = \int_0^t f(S_s, V_s^{\varepsilon})\, ds + \int_0^t g(S_s, V_s^{\varepsilon})\, dW_s + \int_0^t \int_{\mathbb{R}-0} \gamma(S_s, y)\tilde{N}(\mathrm{d}s, \mathrm{d}y)\,, \tag{3}$$

where $\tilde{N}(\mathrm{d}t, \mathrm{d}y)$ is the compensated Poisson random measure given by $\tilde{N}(\mathrm{d}t, \mathrm{d}y) = N(\mathrm{d}t, \mathrm{d}y) - \mathrm{d}t\,\nu(\mathrm{d}y)$. Here, $\nu(\mathrm{d}y)$ is the Lévy measure associated to N.

We first state the following assumptions that guarantee the existence and uniqueness of the solution to Equation (3).

Assumption 1. ***Locally Lipschitz condition***. *For every integer $n \geq 1$, there exists a positive constant C_n such that for all $x, \bar{x}, v, \bar{v} \in \mathbb{R}^d$ with $|x| \vee |\bar{x}| \vee |v| \vee |\bar{v}| \leq n$, we have*

$$|f(x,v) - f(\bar{x},\bar{v})| + |g(x,v) - g(\bar{x},\bar{v})| + \int_{\{|y|<C_n\}} |\gamma(x,y) - \gamma(\bar{x},y)|\nu(\mathrm{d}y) \leq C_n\left(|x-\bar{x}|^2 + |v-\bar{v}|^2\right).$$

Assumption 2. ***Linear growth condition***. *There exists a positive constant C such that for all $(x,v) \in \mathbb{R}^d \times \mathbb{R}^d$,*

$$|f(x,v)|^2 + |g(x,v)|^2 + \int_{\{|y|<C\}} |\gamma(x,y)|^2 \nu(\mathrm{d}y) \leq C\left(1 + |x|^2 + |v|^2\right).$$

Proof. To prove the uniqueness of the solution, let $\hat{S}_t$ and $\bar{S}_t$ be two solutions to Eq. (3). For each $n \in \mathbb{N}$, define $\tau_n = \inf\{t \in [0,T] \;:\; |V_t^{\varepsilon}| \geq n\}$. Then for any $t \leq T$, we have

$$\begin{aligned}
\mathbb{E}\left[\sup_{u \leq t\wedge\tau_n} \left|\hat{S}_u - \bar{S}_u\right|^2\right] &= \mathbb{E}\Bigg[\bigg|\int_0^{t\wedge\tau_n} \left(f(\hat{S}_s, V_s^{\varepsilon}) - f(\bar{S}_s, V_s^{\varepsilon})\right)\mathrm{d}s \\
&\quad + \sup_{u\leq t\wedge\tau_n} \int_0^u \left(g(\hat{S}_s, V_s^{\varepsilon}) - g(\bar{S}_s, V_s^{\varepsilon})\right)\mathrm{d}W_s \\
&\quad + \sup_{u\leq t\wedge\tau_n} \int_0^u \int_{\mathbb{R}-\{0\}} \left(\gamma(\hat{S}_s, y) - \gamma(\bar{S}_s, y)\right)\tilde{N}(\mathrm{d}s, \mathrm{d}y)\bigg|^2\Bigg] \\
&\leq \mathbb{E}\Bigg[3\left|\int_0^{t\wedge\tau_n} \left(f(\hat{S}_s, V_s^{\varepsilon}) - f(\bar{S}_s, V_s^{\varepsilon})\right)\mathrm{d}s\right|^2 \\
&\quad + 3\sup_{u\leq t\wedge\tau_n} \left|\int_0^u \left(g(\hat{S}_s, V_s^{\varepsilon}) - g(\bar{S}_s, V_s^{\varepsilon})\right)\mathrm{d}W_s\right|^2 \\
&\quad + 3\sup_{u\leq t\wedge\tau_n} \left|\int_0^u \int_{\mathbb{R}-\{0\}} \left(\gamma(\hat{S}_s, y) - \gamma(\bar{S}_s, y)\right)\tilde{N}(\mathrm{d}s, \mathrm{d}y)\right|^2\Bigg].
\end{aligned}$$

Using Jensen inequality, we get

$$\begin{aligned}\mathbb{E}\left[\sup_{u\le t\wedge\tau_n}\left|\hat{S}_u-\bar{S}_u\right|^2\right] &\le 3\mathbb{E}\left[\left|\int_0^{t\wedge\tau_n}\left(f(\hat{S}_s,V_s^{\varepsilon})-f(\bar{S}_s,V_s^{\varepsilon})\right)\mathrm{d}s\right|^2\right]\\ &+3\mathbb{E}\left[\sup_{u\le t\wedge\tau_n}\left|\int_0^{u\wedge\tau_n}\left(g(\hat{S}_s,V_s^{\varepsilon})\ -g(\bar{S}_s,V_s^{\varepsilon})\right)\mathrm{d}W_s\right|^2\right]\\ &+3\mathbb{E}\left[\sup_{u\le t\wedge\tau_n}\left|\int_0^u\int_{\mathbb{R}-\{0\}}\left(\gamma(\hat{S}_s,y)-\gamma(\bar{S}_s,y)\right)\tilde{N}(\mathrm{d}s,\mathrm{d}y)\right|^2\right]\end{aligned}$$

According to L^2–norm, Doob's inequality and Iˆto's isometry, we obtain

$$\begin{aligned}\mathbb{E}\left[\sup_{u\le t\wedge\tau_n}\left|\hat{S}_u-\bar{S}_u\right|^2\right] &\le 3L^2t\ \mathbb{E}\left[\int_0^{t\wedge\tau_n}\left|\hat{f}-\bar{f}\right|^2\mathrm{d}u\right]+3L^2\ \mathbb{E}\left[\int_0^{t\wedge\tau_n}\left|\hat{g}-\bar{g}\right|^2\mathrm{d}u\right]\\ &+3L^2\ \mathbb{E}\left[\int_0^{t\wedge\tau_n}\int_{\mathbb{R}-\{0\}}\left|\gamma(\hat{S}_u,y)-\gamma(\bar{S}_u,y)\right|^2\nu(\mathrm{d}y)\mathrm{d}u\right],\end{aligned}$$

and by *Assumption 1*,

$$\mathbb{E}\left[\sup_{u\le t\wedge\tau_n}\left|\hat{S}_u-\bar{S}_u\right|^2\right]\le 3C_n^2L^2\,(t+2)\int_0^t\mathbb{E}\left[\sup_{u\le s\wedge\tau_n}\left|\hat{S}_u-\bar{S}_u\right|^2\right]\mathrm{d}s.$$

Now, defining $\varphi(t)=\mathbb{E}\left[\sup_{u\le t\wedge\tau_n}\left|\hat{S}_u-\bar{S}_u\right|^2\right]$ results in

$$\varphi(t)\le 3C_n^2L^2(t+2)\int_0^t\varphi(s)\mathrm{d}s.$$

The required assertion now follows from the Gronwell inequality.

Now, we verify the existence of the solution to Equation (3). Denote,

$$S_t^{(n+1)}=\int_0^t f\left(S_u^{(n)},V_u^{\varepsilon}\right)\mathrm{d}u+\int_0^t g\left(S_u^{(n)},V_u^{\varepsilon}\right)\mathrm{d}W_u+\int_0^t\int_{\mathbb{R}-0}\gamma\left(S_u^{(n)},y\right)\tilde{N}\left(\mathrm{d}u,\mathrm{d}y\right)$$

Applying the Picard iteration method and $S_t^{(0)}=0$, for each $n\in\mathbb{N}$ and $t\in[0,T]$, we have

$$\begin{aligned}S_t^{(n+1)}-S_t^{(n)} &= \int_0^t\left(f(S_u^{(n)},V_u^{\varepsilon})-f(S_u^{(n-1)},V_u^{\varepsilon})\right)\mathrm{d}u\\ &+\int_0^t\left(g(S_u^{(n)},V_u^{\varepsilon})-g(S_u^{(n-1)},V_u^{\varepsilon})\right)\mathrm{d}W_u\\ &+\int_0^t\int_{\mathbb{R}-0}\left(\gamma(S_u^{(n)},y)-\gamma(S_u^{(n-1)},y)\right)\tilde{N}\left(\mathrm{d}u,\mathrm{d}y\right).\end{aligned}$$

Similar computations as for the uniqueness and BDG's inequality, gives

$$\begin{aligned}
\mathbb{E}\left[\sup_{u\le t\wedge\tau_n}|S_u^{(n+1)}-S_u^{(n)}|^2\right] &\le C\mathbb{E}\left[\sup_{u\le t\wedge\tau_n}\Big|\int_0^u\left(f(S_s^{(n)},V_s^{\varepsilon})-f(S_s^{(n-1)},V_s^{\varepsilon})\right)\mathrm{d}s\Big|^2\right]\\
&\quad+C\mathbb{E}\left[\sup_{u\le t\wedge\tau_n}\Big|\int_0^u\left(g(S_s^{(n)},V_s^{\varepsilon})-g(S_s^{(n-1)},V_s^{\varepsilon})\right)\mathrm{d}W_s\Big|^2\right]\\
&\quad+C\mathbb{E}\left[\sup_{u\le t\wedge\tau_n}\Big|\int_0^u\int_{\mathbb{R}-0}\left(\gamma(S_s^{(n)},y)-\gamma(S_s^{(n-1)},y)\right)\tilde{N}(\mathrm{d}s,\mathrm{d}y)\Big|^2\right]\\
&\le C\mathbb{E}\left[\int_0^{t\wedge\tau_n}\big|f(S_u^{(n)},V_u^{\varepsilon})-f(S_u^{(n-1)},V_u^{\varepsilon})\big|^2\mathrm{d}u\right]\\
&\quad+C\mathbb{E}\left[\int_0^{t\wedge\tau_n}\big|g(S_u^{(n)},V_u^{\varepsilon})-g(S_u^{(n-1)},V_u^{\varepsilon})\big|^2\mathrm{d}u\right]\\
&\quad+C\mathbb{E}\left[\int_0^{t\wedge\tau_n}\int_{\mathbb{R}-0}\big|\gamma(S_u^{(n)},y)-\gamma(S_u^{(n-1)},y)\big|^2\nu(\mathrm{d}y)\mathrm{d}u\right]\\
&\le C\mathbb{E}\left[\int_0^{t\wedge\tau_n}\sup_{s\in[0,u]}\big|S_s^{(n)}-S_s^{(n-1)}\big|^2\mathrm{d}u\right],
\end{aligned}\tag{4}$$

for some constant C, and from the *Assumption* 2, we have

$$\begin{aligned}
\mathbb{E}\left[\sup_{u\le t\wedge\tau_n}|S_u^{(1)}|^2\right] &\le C\mathbb{E}\left[\sup_{u\le t\wedge\tau_n}\Big|\int_0^u f\left(S_s^{(0)},V_s^{\varepsilon}\right)\mathrm{d}s\Big|^2\right]\\
&\quad+C\mathbb{E}\left[\sup_{u\le t\wedge\tau_n}\Big|\int_0^u g\left(S_s^{(0)},V_s^{\varepsilon}\right)\mathrm{d}W_s\Big|^2\right]\\
&\quad+C\mathbb{E}\left[\sup_{u\le t\wedge\tau_n}\Big|\int_0^u\int_{\mathbb{R}-0}\gamma\left(S_s^{(n)},y\right)\tilde{N}(\mathrm{d}s,\mathrm{d}y)\Big|^2\right]\\
&\le C\mathbb{E}\left[\int_0^{t\wedge\tau_n}\Big|f\left(S_u^{(0)},V_u^{\varepsilon}\right)\Big|^2\mathrm{d}u\right]\\
&\quad+C\mathbb{E}\left[\int_0^{t\wedge\tau_n}\Big|g\left(S_u^{(0)},V_u^{\varepsilon}\right)\Big|^2\mathrm{d}u\right]\\
&\quad+C\mathbb{E}\left[\int_0^{t\wedge\tau_n}\int_{\mathbb{R}-0}\Big|\gamma\left(S_u^{(0)},y\right)\Big|^2\nu(\mathrm{d}y)\mathrm{d}u\right]\\
&\le C.
\end{aligned}\tag{5}$$

So by induction based on Eqs. (4) and (5), for all $n\in\mathbb{N}$, we obtain the key estimate

$$\mathbb{E}\left[\sup_{u\le t\wedge\tau_n}\big|S_u^{(n+1)}-S_u^{(n)}\big|^2\right]\le\frac{C}{n!}.\tag{6}$$

Our first observation is that $(S_t^{(n)},n\in\mathbb{N})$ is convergent in $L^2(\Omega)$ for each $t\in[0,T]$. Indeed, for each $m,n\in\mathbb{N}$, we have, for each $t\in[0,T]$,

$$\mathbb{E}\left[\big|S_t^{(n)}-S_t^{(m)}\big|^2\right]\le C\sum_{k=m}^{n-1}\mathbb{E}\left[\big|S_t^{(k+1)}-S_t^{(k)}\big|^2\right]\le C\sum_{k=m}^{n-1}\frac{1}{k!}.$$

Since the series on the right converges, each $(S_t^{(n)}, n \in \mathbb{N})$ is Cauchy and hence, convergent to some $S_t \in L^2(\Omega)$ for every $t \in [0,T]$. Now we establish the almost sure convergence of $(S_t^{(n)}, n \in \mathbb{N}, t \in [0,T])$. Apply the Chebyshevs inequality in Eq. (6), we deduce that

$$\mathbb{P}\left[\sup_{u \le t\wedge\tau_n} \left|S_u^{(n+1)} - S_u^{(n)}\right| \ge \frac{1}{2^n}\right] \le \frac{C}{n!},$$

from which we have

$$\mathbb{P}\left[\limsup_{n\to\infty} \sup_{u \le t\wedge\tau_n} \left|S_u^{(n+1)} - S_u^{(n)}\right| \ge \frac{1}{2^n}\right] = 0,$$

by Borel-Cantellis lemma. Therefore, we deduce that $(S_t^{(n)}, n \in \mathbb{N}, t \in [0,T])$ is almost surely uniformly convergent to $(S_t, t \in [0,T])$ and consequently,

$$\int_0^{t\wedge\tau_n} f\left(S_u^{(n)}, V_u^\varepsilon\right) \mathrm{d}u \longrightarrow \int_0^{t\wedge\tau_n} f\left(S_u, V_u^\varepsilon\right) \mathrm{d}u,$$
$$\int_0^{t\wedge\tau_n} g\left(S_u^{(n)}, V_u^\varepsilon\right) \mathrm{d}W_u \longrightarrow \int_0^{t\wedge\tau_n} g\left(S_u, V_u^\varepsilon\right) \mathrm{d}W_u,$$
$$\int_0^{t\wedge\tau_n} \int_{\mathbb{R}-0} \gamma\left(S_u^{(n)}, y\right) \tilde{N}(\mathrm{d}u, \mathrm{d}y) \longrightarrow \int_0^{t\wedge\tau_n} \int_{\mathbb{R}-0} \gamma(S_u, y) \tilde{N}(\mathrm{d}u, \mathrm{d}y),$$

as $n \to \infty$. So the desired result would be produced. □

In what follows, we study the existence and uniqueness of the solution to approximated rough stochastic volatility V_t^ε in system (2).

Theorem 2. *Under locally Lipschitz and linear growth conditions [29], the process V_t^ε admits a unique solution.*

Proof. To prove the uniqueness of the solution, let $\hat{V}_t^\varepsilon$ and $\bar{V}_t^\varepsilon$ be two solutions of process V_t^ε . For every integer $n \ge 1$, define

$$\hat{\tau}_n := T \wedge \inf\{t \in [0,T] \ : |\hat{V}_t^\varepsilon| \ge n\},$$

and also,

$$\bar{\tau}_n := T \wedge \inf\{t \in [0,T] \ : |\bar{V}_t^\varepsilon| \ge n\}.$$

Set $\tau_n := \hat{\tau}_n \wedge \bar{\tau}_n$. Then

$$\begin{aligned}
\mathbb{E}\big[|\hat{V}^\varepsilon_{t\wedge\tau_n} - \bar{V}^\varepsilon_{t\wedge\tau_n}|^2\big] &= \mathbb{E}\big[|\frac{1}{\Gamma(\alpha)}\int_0^{t\wedge\tau_n} (t+\varepsilon-u)^{\alpha-1} \kappa\left(\hat{V}_u^\varepsilon - \bar{V}_u^\varepsilon\right)\mathrm{d}u \\
&\quad + \frac{1}{\Gamma(\alpha)}\int_0^{t\wedge\tau_n} (t+\varepsilon-u)^{\alpha-1} \sigma\left(\sqrt{\hat{V}_u^\varepsilon} - \sqrt{\bar{V}_u^\varepsilon}\right)\mathrm{d}B_u|^2\big] \\
&\le 2\mathbb{E}\big[\frac{1}{\Gamma(\alpha)}|\int_0^{t\wedge\tau_n} (t+\varepsilon-u)^{\alpha-1} \kappa\left(\hat{V}_u^\varepsilon - \bar{V}_u^\varepsilon\right)\mathrm{d}u|^2\big] \\
&\quad + 2\mathbb{E}\big[\frac{1}{\Gamma(\alpha)}|\int_0^{t\wedge\tau_n} (t+\varepsilon-u)^{\alpha-1} \sigma\left(\sqrt{\hat{V}_u^\varepsilon} - \sqrt{\bar{V}_u^\varepsilon}\right)\mathrm{d}B_u|^2\big] \\
&\le \frac{2t\kappa^2}{\Gamma(\alpha)}\mathbb{E}\big[\int_0^{t\wedge\tau_n} (t+\varepsilon-u)^{2(\alpha-1)} |\hat{V}_u^\varepsilon - \bar{V}_u^\varepsilon|^2 \mathrm{d}u\big] \\
&\quad + \frac{2\sigma^2}{\Gamma(\alpha)}\mathbb{E}\big[\int_0^{t\wedge\tau_n} (t+\varepsilon-u)^{2(\alpha-1)} |\sqrt{\hat{V}_u^\varepsilon} - \sqrt{\bar{V}_u^\varepsilon}|^2 \mathrm{d}u\big].
\end{aligned}$$

Given local condition, there exists $a>0$, so that

$$\begin{aligned}\mathbb{E}\big[|\hat{V}^{\varepsilon}_{t\wedge\tau_n}-\bar{V}^{\varepsilon}_{t\wedge\tau_n}|^2\big] \le \frac{2T\kappa^2}{\Gamma(\alpha)}\mathbb{E}\Big[\int_0^{t\wedge\tau_n}(t+\varepsilon-u)^{2(\alpha-1)}|\hat{V}^{\varepsilon}_u-\bar{V}^{\varepsilon}_u|^2\mathrm{d}u\Big]\\ +\frac{2\sigma^2}{a^2\Gamma(\alpha)}\mathbb{E}\Big[\int_0^{t\wedge\tau_n}(t+\varepsilon-u)^{2(\alpha-1)}|\hat{V}^{\varepsilon}_u-\bar{V}^{\varepsilon}_u|^2\mathrm{d}u\Big],\end{aligned}$$

and from the local Lipchits conditin, we have

$$\mathbb{E}\big[|\hat{V}^{\varepsilon}_{t\wedge\tau_n}-\bar{V}^{\varepsilon}_{t\wedge\tau_n}|^2\big] \le \frac{1}{\Gamma(\alpha)}\left(T\kappa^2+\frac{\sigma^2}{a^2}\right)C_n^2\int_0^t(t+\varepsilon-u\wedge\tau_n)^{2(\alpha-1)}\mathbb{E}\big[|\hat{V}^{\varepsilon}_{u\wedge\tau_n}-\bar{V}^{\varepsilon}_{u\wedge\tau_n}|^2\big]\mathrm{d}u.$$

Now, we apply the Gronwall inequality to conclude that $\{\hat{V}^{\varepsilon}_{t\wedge\tau_n}, 0\le t\le T\}$ and $\{\bar{V}^{\varepsilon}_{t\wedge\tau_n}, 0\le t\le T\}$ are modification of each another and thus are indiscernible. Letting $n\to\infty$, we see that the same is true for $\{\hat{V}^{\varepsilon}_t, 0\le t\le T\}$ and $\{\bar{V}^{\varepsilon}_t, 0\le t\le T\}$.

To prove the existence of the solution, we define $V_t^{\varepsilon(0)} := V_0^{\varepsilon} = V_0$ and $V_t^{\varepsilon(n)} := V_t^{\varepsilon(n)}(\omega)$ inductively as follows,

$$V_t^{\varepsilon(n+1)} = V_0^{\varepsilon}+\frac{1}{\Gamma(\alpha)}\int_0^t(t+\varepsilon-u)^{\alpha-1}\kappa\left(\theta-V_u^{\varepsilon(n)}\right)\mathrm{d}u+\frac{1}{\Gamma(\alpha)}\int_0^t(t+\varepsilon-u)^{\alpha-1}\sigma\sqrt{V_u^{\varepsilon(n)}}\mathrm{d}B_u.$$

Then similar computation as the uniqueness, we have

$$\mathbb{E}\big[|V^{\varepsilon(n+1)}_{t\wedge\tau_n}-V^{\varepsilon(n)}_{t\wedge\tau_n}|^2\big] \le \frac{2}{\Gamma(\alpha)}\left(T\kappa^2+\frac{\sigma^2}{a^2}\right)C_n^2\int_0^t(t+\varepsilon-u\wedge\tau_n)^{2(\alpha-1)}\mathbb{E}\big[|V^{\varepsilon(n)}_{u\wedge\tau_n}-V^{\varepsilon(n-1)}_{u\wedge\tau_n}|^2\big]\mathrm{d}u,$$

for $n\ge 1$, $t\le T$ and

$$\begin{aligned}\mathbb{E}\big[|V_t^{\varepsilon(1)}-V_t^{\varepsilon(0)}|^2\big] &\le \frac{2}{\Gamma(\alpha)}\left(T\kappa^2+\frac{\sigma^2}{a^2}\right)C^2\left(1+\mathbb{E}\big[|V_t^{\varepsilon}|^2\big]\right)\\ &\le C.\end{aligned} \tag{7}$$

By induction on n, we have

$$\mathbb{E}\big[|V^{\varepsilon(n+1)}_{t\wedge\tau_n}-V^{\varepsilon(n)}_{t\wedge\tau_n}|^2\big] \le \frac{C}{n!}.$$

The required assertion now follows from the Doob martingale inequality and the Fatou lemma. □

4 Numerical illustration

In this section, we examine the American put option on an underlying asset whose price is approximated by the rough Heston jump-diffusion model (2).

An American option is a type of financial derivative that gives the holder the right, but not the obligation, to buy (in the case of a call option) or sell (in the case of a put option) an underlying asset at a specified strike price at any time before or on the expiration date. This flexibility to exercise the option at any point up to maturity adds a layer of complexity to the pricing model compared to European options, which can only be exercised at expiration.

In the field of option pricing, pricing American–style options is a significant issue. One of the most effective and straightforward solutions for this type of problem is the Least–Square Monte–Carlo (LSM) method, which is based on the Longstaff–Schwarts algorithm [24].

The LSM method is based on the property that the conditional expectation of a random process minimizes the mean squared distance between a simulated sample of this process and an adopted Borel measurable function approximated by a regression in a subspace of basis functions. Here, we provide a detailed explanation of the steps involved:

To accurately price an American option under this complex model, we follow these detailed steps:

1. Simulate Asset Price Paths:
   - Use Monte Carlo methods to generate multiple paths for the underlying asset price S_t and the volatility V_t^{ε}.
   - Incorporate jumps in S_t by generating Poisson-distributed jump times and double exponential jump sizes.
2. Initialize Payoff Matrix:
   - Calculate the intrinsic value (payoff) of the American option at each time step for all simulated paths.
   - For a put option, the payoff is $\max(K - S_t, 0)$, where K is the strike price.
3. Backward Induction Using LSM:
   - Begin with the terminal payoff at maturity and proceed backward to the initial time.
   - At each time step, identify paths where the option is in-the-money (i.e., the intrinsic value is positive).
   - Estimate the continuation value (CV) via polynomial regression on the current asset price.
   - Update the option value by comparing the payoff and the estimated CV, taking the maximum to reflect the possibility of early exercise.
   - Apply the discount factor to account for the time value of money.
4. Calculate the Option Price:
   - The final American option price is obtained by averaging the option values across all simulated paths at the initial time step.

The volatility process is given by

$$V_t^{\varepsilon} = V_0^{\varepsilon} + \frac{1}{\Gamma(\alpha)} \int_0^t (t+\varepsilon-s)^{\alpha-1} \kappa(\theta - V_s)\,\mathrm{d}s + \frac{1}{\Gamma(\alpha)} \int_0^t (t+\varepsilon-s)^{\alpha-1} \sigma \sqrt{V_s}\,\mathrm{d}B_s,$$

where $0 < \varepsilon \ll 1$. The steps for discretizing this process are as follows:

1. Time Discretization:
   - Divide the time interval $[0, T]$ into N steps with a small time increment $\Delta t = \frac{T}{N}$.

- Let $t_j = j\Delta t$ for $j = 0, 1, \ldots, N$.
- Use the Euler-Maruyama method to approximate solutions of the stochastic differential equations.

2. Discretize Mean-Reverting Component:

- Approximate the integral $\int_0^{t_j} (t_j + \varepsilon - s)^{\alpha-1} \kappa(\theta - V_s)\mathrm{d}s$ using a discrete sum over time steps:

$$\int_0^{t_j} (t_j + \varepsilon - s)^{\alpha-1} \kappa(\theta - V_s)\mathrm{d}s \approx \sum_{k=0}^{j-1} (t_j + \varepsilon - t_k)^{\alpha-1} \kappa(\theta - V_{t_k})\Delta t.$$

3. Discretize Stochastic Component:

- Approximate the integral $\int_0^{t_j} (t_j + \varepsilon - s)^{\alpha-1} \sigma\sqrt{V_s}\mathrm{d}B_s$ using a discrete sum and Brownian motion increments:

$$\int_0^{t_j} (t_j + \varepsilon - s)^{\alpha-1} \sigma\sqrt{V_s}\mathrm{d}B_s \approx \sum_{k=0}^{j-1} (t_j + \varepsilon - t_k)^{\alpha-1} \sigma\sqrt{V_{t_k}}\Delta B_{t_k},$$

where ΔB_{t_k} is the Brownian motion increment.

The asset price dynamic is given by

$$\mathrm{d}S_t = rS_t\mathrm{d}t + \sqrt{V_t}S_t\mathrm{d}W_t + \mathrm{d}\left(\sum_{i=1}^{N_t} (X_i - 1)\right).$$

The steps for discretizing this process are as follows:

- Discretize this process using the Euler-Maruyama scheme and include the jump component:

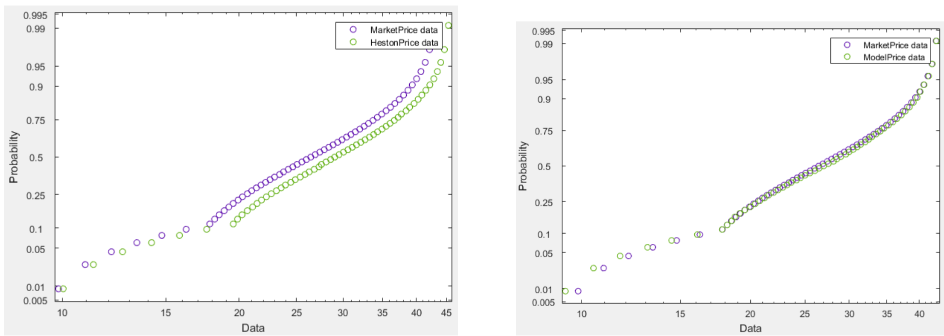
$$S_{t_{j+1}} = S_{t_j} + S_{t_j}(r - q)\Delta t + S_{t_j}\sqrt{V_{t_j}}\Delta W_{t_j} + \text{Jump Component}.$$

- The jump component is given by:

$$\text{Jump Component} = \sum_{i=1}^{N_{\Delta t}} (X_i - 1),$$

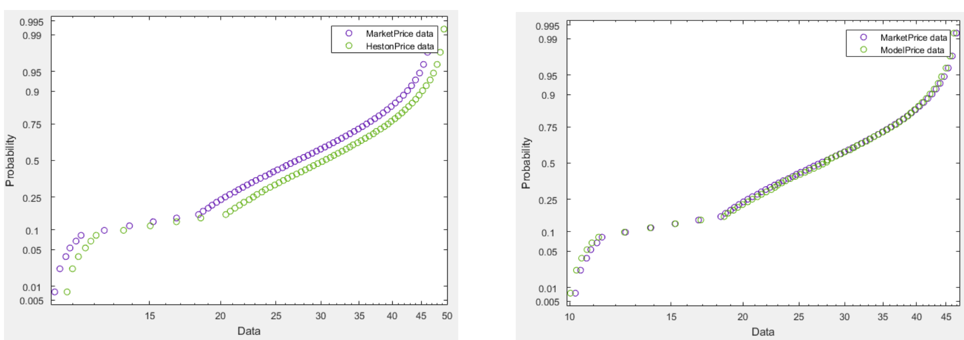
where $N_{\Delta t}$ is the Poisson-distributed number of jumps within the time step Δt and X_i are the i.i.d. jump sizes following a double exponential distribution.

By following these steps, we can effectively price American options under our complex model, capturing the nuances of market dynamics and providing accurate pricing. In this work, we take advantage of this framework and study the behavior of the American put option prices from Aug. 2019 to Sep. 2019, Oct. 2019, Nov. 2019, Dec. 2020 and Jan. 2021, respectively, in order to confirm the accuracy and efficiency of our presented model. The adopted parameters related to our study include $\kappa = 0.1669$, $\theta = 0.2930$, $\sigma = 0.1436$, $\rho = -0.5$, $\lambda = 1$, $p = 0.5$, $\eta_1 = 25$, $\eta_2 = 30$, and $H = 0.3$.



(a) Heston–model and SPY prices **(b)** RHJd–model and SPY prices

Figure 1: Log–Normal probability test for prices generated by Heston model, Rough Heston Jump–diffusion (RHJd) model and SPY market for a maturity time of 499 days.



(a) Heston–model and SPY prices **(b)** RHJd–model and SPY prices

Figure 2: Log–Normal probability test for prices generated by Heston model, Rough Heston Jump–diffusion (RHJd) model and SPY market for a maturity time of 527 days.

Figures 1 and 2 demonstrate log–normal probability test for the prices predicted by the Heston model, the rough Heston jump-diffusion model, and the prices of the SPY market for two maturity times of 499 and 527 days.

In Figures 3 to 8, we provide a comparison of the American put option price estimated by the classical Heston model and the rough Heston jump–diffusion model by considering SPY American put option data with expiration dates 19, 54, 72, 100, 499, and 527 days as a set of benchmark data in which the initial stock price is 287.97. As shown, both models perform well in assessing the market for short–term dates. But, for long–term dates, the accuracy of rough Heston jump–diffusion model in evaluating our sample of data is evident.

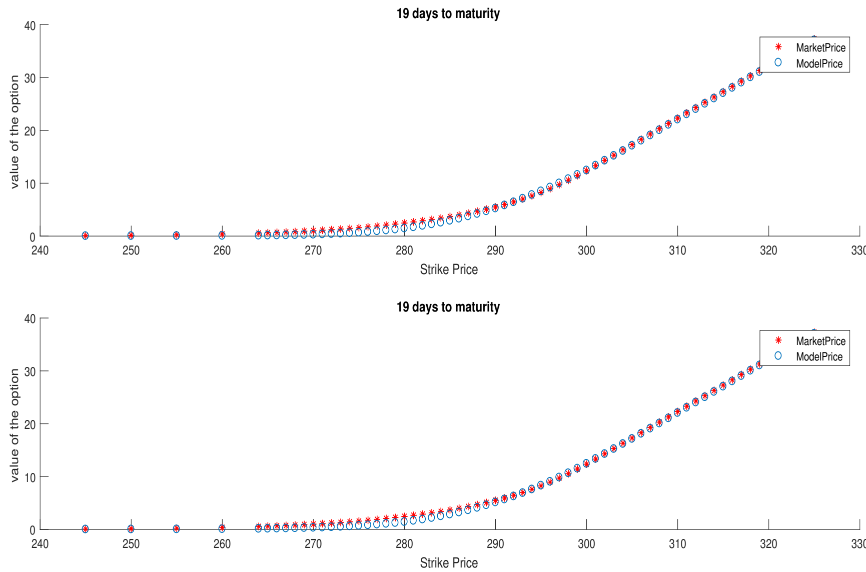


Figure 3: The American put option price estimated by the Heston (up) and rough Heston jump–diffusion (down) models vs. SPY market data from Aug. 2019 for time to maturity T of 19 days.

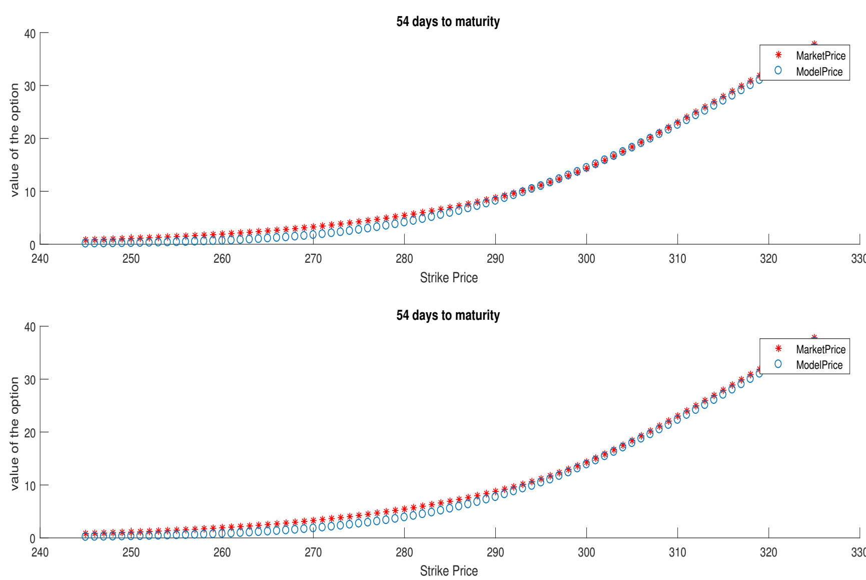


Figure 4: The American put option price estimated by the Heston (up) and rough Heston jump–diffusion (down) models vs. SPY market data from Aug. 2019 for time to maturity T of 54 days.

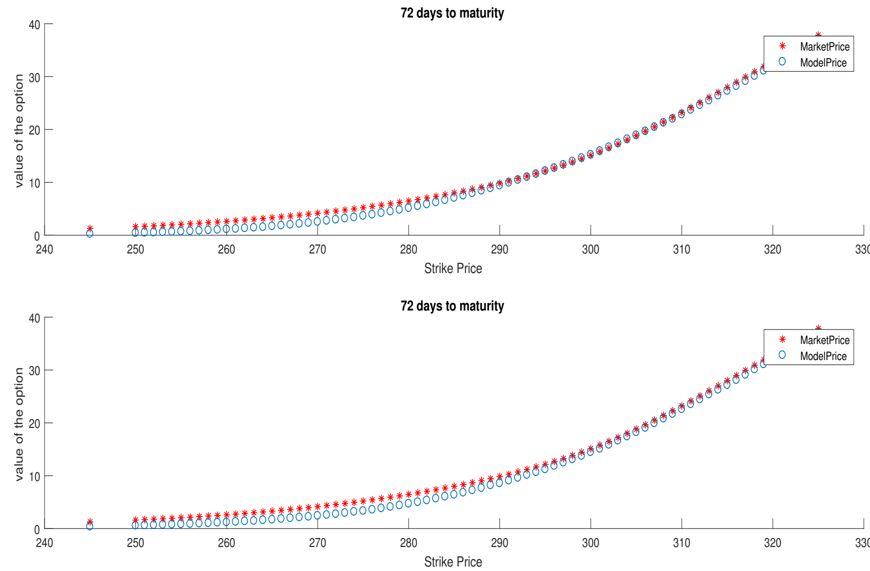


Figure 5: The American put option price estimated by the Heston (up) and rough Heston jump–diffusion (down) models vs. SPY market data from Aug. 2019 for time to maturity T of 72 days.

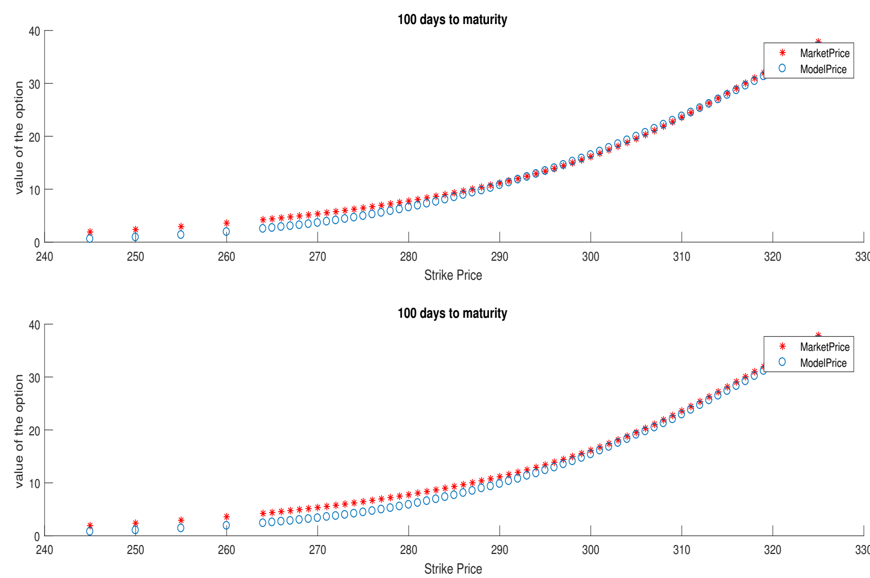


Figure 6: The American put option price estimated by the Heston (up) and rough Heston jump–diffusion (down) models vs. SPY market data from Aug. 2019 for time to maturity T of 100 days.

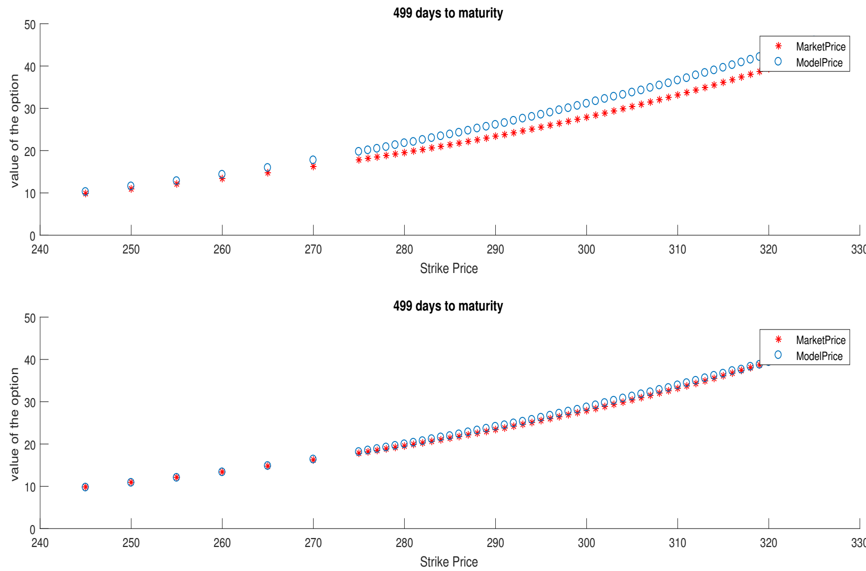


Figure 7: The American put option price estimated by the Heston (up) and rough Heston jump–diffusion (down) models vs. SPY market data from Aug. 2019 for time to maturity T of 499 days.

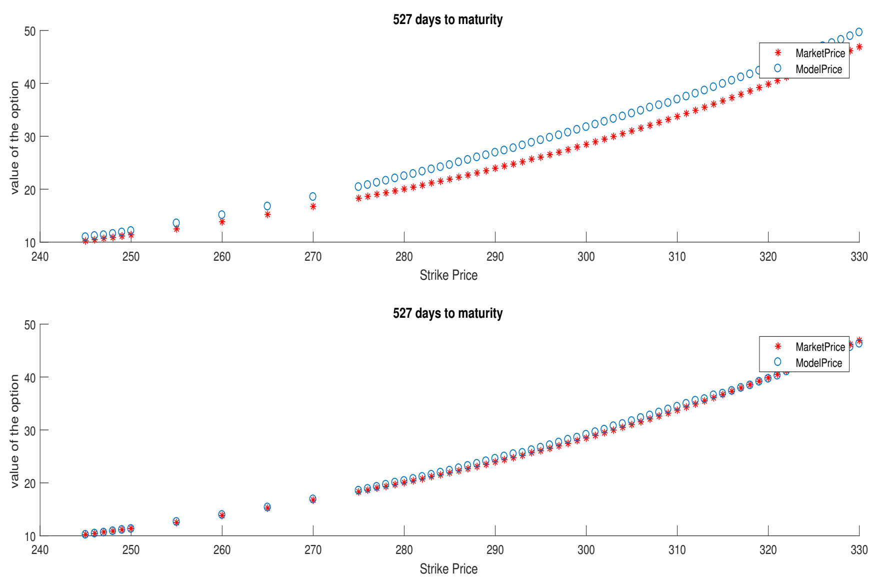


Figure 8: The American put option price estimated by the Heston (up) and rough Heston jump–diffusion (down) models vs. SPY market data from Aug. 2019 for time to maturity T of 527 days.

5 Conclusion

In this study, we introduced and validated a novel stochastic volatility model incorporating a double exponential jump-diffusion process with a memory feature embedded in the volatility component. To demonstrate the practical application of our model, we conducted a case study focusing on the pricing of American options. Using the LSM algorithm, we priced these options and compared the results with those from the classical Heston model and real data from the *S&P* 500 index options. The comparative analysis revealed that our proposed model accurately captures the market dynamics and provides precise and reliable pricing of American options. The inclusion of the memory feature and double exponential jumps not only enhances the model's realism but also its robustness. Our findings underscore the significance of integrating these features for more accurate pricing of financial derivatives, validating the effectiveness and reliability of our approach.

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