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Optical solitary solution of two coupled fractional equations with new modification of exp function method

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ABSTRACT

In past years, various methods were applied to achieve the analytical solution of the fractional partial differential equation (FPDE). However, most have not been applied directly to manipulate the system of equations. In the current paper, the modified exp function method is expanded to attain the answer of a fractional two-dimensional system, without reducing fractional equations. Furthermore, the proposed methods have been used to achieve the analytical solutions of the coupled space Time-Fractional Boussinesq-Burgers System and coupled Time-Fractional Long System. The proposed methods are highly accurate, flexible, effective, and programmable to solve nonlinear evolution equations. Moreover, the plots of obtained solutions have been illustrated for some parameters.

1. Introduction

Finding analytical solutions of FPDE has a vital role in various sciences because many physical phenomena can be explained by analyzing them and that is why many researchers are interested in working on this topic in recent years.

Plenty of definitions had been presented about derivative of fractional order. The Riemann–Liouville and Caputo are two prevalent interpretations of them. In the current paper Riemann-Liouville derivative has been applied.

Furthermore, these equations may be applied to extend the diffusion and wave equation. So, recently the FPDE have become the focus of plenty scientists in the field of physics and mathematics and also many researchers to focus on this topic [1-4]. It can provide many methods

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for attaining their solutions, such as the G'/G - expansion method [5-7], the exp-function method [8-28], the homotopy analysis method [29, 30] and so on [30-44].

In almost all of these methods, to solve a system of equations, first, it will be converted into an equation and then the desired method will be applied. The motivation of the present paper is to develop the modified exp function method, the Kudryashov method to directly solve a system of differential equations. To show the ability and characteristics of the method, two fractional differential equations known called coupled space Time-Fractional Boussinesq-Burgers System and coupled Time-Fractional long System have been solved and their soliton solution have been obtained.

2. Coupled Modified Exp function Method

In the current part, we extended the Coupled modified exp function methods to resolve the coupled fractional differential equation. let's consider the following nonlinear system of FPDE:

$$\begin{cases} F\left(u, D_t^{\gamma} u, D_x^{\mu}\, \mathrm{v}, \ldots\right)=0, \\ G\left(u, D_t^{\gamma} u, D_x^{\mu}\, \mathrm{v}, \ldots\right)=0, \end{cases} \quad 0<\gamma, \mu \leq 1. \tag{1}$$

Where $D_t^{\gamma} u, D_x^{\mu} v$,... are the modified Riemann-Liouville derivatives. By using the nonlinear fractional variable

$$\xi=\frac{k x^{\mu}}{\Gamma(1+\mu)}-\frac{\omega t^{\gamma}}{\Gamma(1+\gamma)}, \tag{2}$$

Where τ and υ are nonzero parameters, ***Eq. (1)*** turns to a system of ODE

$$\begin{cases} f(u, v, u', v', \ldots)=0, \\ g(u, v, u', v', \ldots)=0. \end{cases} \tag{3}$$

In the Coupled modified exp function method (CMEF) the solution of system (3) have been imaged as:

$$\begin{aligned} u(\xi) &= \sum_{i=-M}^{M} a_i\left(exp(\varphi(\xi))\right)^i, \\ v(\xi) &= \sum_{i=-N}^{N} b_i\left(exp(\varphi(\xi))\right)^i, \end{aligned} \tag{4}$$

Where $\varphi(\xi)$ satisfies the nonlinear ODE in the form as fallows

$$\varphi' = \exp(-\varphi)+\alpha \exp(\varphi)+\beta \tag{5}$$

a_i, b_i, α and β are parameters to be handled further. The solution of ***Eq. (5)*** are as follows

$$
\begin{aligned}
&1. If \alpha \neq 0, \beta^2-4\alpha>0, \varphi(\xi)=ln\left(-\frac{\sqrt{\beta^2-4\alpha}}{2\alpha}tanh\left(\frac{\sqrt{\beta^2-4\alpha}}{2}(\xi+c)\right)-\frac{\beta}{2\alpha}\right),\\
&2. If \alpha \neq 0, \beta^2-4\alpha<0, \varphi(\xi)=ln\left(\frac{\sqrt{4\alpha-\beta^2}}{2\alpha}tan\left(\frac{\sqrt{4\alpha-\beta^2}}{2}(\xi+c)\right)-\frac{\beta}{2\alpha}\right),\\
&3. If \alpha \neq 0, \beta^2-4\alpha=0, \varphi(\xi)=ln\left(-\frac{2\beta(\xi+c)+4}{\beta(\xi+c)}\right),\\
&4. If \alpha = 0, \beta \neq 0, \varphi(\xi)=-ln\left(\frac{\beta}{exp(\beta(\xi+c))-1}\right),\\
&5. If \alpha = 0, \beta = 0, \varphi(\xi)=ln(\xi+c),
\end{aligned}
\tag{6}
$$

To obtain the numbers M and N, we strike a balance between the sentences with the topmost derivative and the topmost nonlinear order in ***Eq. (3)***. Placing ***Eq. (5)*** into ***Eq. (3)*** and considering ***Eq. (6)*** precede an algebraic system including powers of $exp(\varphi(\xi))$. By setting the coefficient of these power to zero, unknown parameters are acquire. Finally substituting over determine value in ***Eq. (5)***, the solutions of the ***Eq. (1)*** will be achieved.

3. CMEFM for Coupled Space Time-Fractional Boussinesq-Burgers System

To illustrate the application of the method, let us consider the following Coupled space Time-Fractional Boussinesq-Burgers System

$$
\begin{cases}
D_t^{\gamma}u-\frac{1}{2}v_x+2uu_x=0,\\
D_t^{\gamma}v-\frac{1}{2}u_{xxx}+2(uv)_x=0,
\end{cases}
\tag{7}
$$

At first, we defined

$$
\xi=\frac{kx^{\gamma}}{\Gamma(1+\gamma)}+\frac{\omega t^{\gamma}}{\Gamma(1+\gamma)},
\tag{8}
$$

So, ***Eq. (7)*** convert to,

$$
\begin{cases}
-\omega u'-\frac{1}{2}ku'+2kuu'=0,\\
-\omega v'-\frac{1}{2}k^3u'''+2k(uv)'=0.
\end{cases}
\tag{9}
$$

The solution of (9) will be image as ***Eq. (4)***, Where $\varphi(\xi)$ satisfy in ***Eq. (5)***. Homogeneous balance between linear and nonlinear terms in each equation of (9) leads to $M=1$, and $N=2$. So the solution (4), will be written as follows

$$
\begin{cases}
u(\xi)=a_1e^{\varphi(\xi)}+a_0+a_{-1}e^{-\varphi(\xi)},\\
v(\xi)=b_2e^{2\varphi(\xi)}+b_1e^{\varphi(\xi)}+b_0+b_{-1}e^{-\varphi(\xi)}+b_{-2}e^{-2\varphi(\xi)}.
\end{cases}
\tag{10}
$$

Putting (10) in (9), and putting the coefficient of $e^{\varphi(\xi)}$ equal to zero, yields a system of algebraic equations,

$$\frac{1}{2}\left(\left(-4a_{-1}^2+2b_{-2}\right)\beta-4a_0a_{-1}+b_{-1}\right)k+\omega a_{-1}=0$$

$$6k^3a_{-1}\beta+\frac{1}{2}\left(-12\beta a_{-1}b_{-2}-8a_{-1}b_{-1}-8a_0b_{-2}\right)k+2\omega b_{-2}=0$$

$$\frac{1}{2}\left(\left(-4a_{-1}^2+2b_{-2}\right)\alpha+\beta\left(-4a_{-1}a_0+b_{-1}\right)\right)k+\omega a_{-1}\beta=0$$

$$e2_2 := \frac{7}{2}a_{-1}\left(\beta^2+\frac{8}{7}\alpha\right)k^3+\frac{1}{2}\left(-12a_{-1}b_{-2}\alpha+\left(-8a_{-1}b_{-1}-8a_0b_{-2}\right)\beta-4a_1b_{-2}\right.$$
$$\left.-4a_0b_{-1}-4b_0a_{-1}\right)k+2\left(b_{-2}\beta+\frac{1}{2}b_{-1}\right)\omega=0$$

$$\frac{1}{2}\left(\left(-4a_{-1}a_0+b_{-1}\right)\alpha+4a_0a_1-b_1\right)k+\omega\left(\alpha a_{-1}-a_1\right)=0$$

$$\frac{1}{2}a_{-1}\beta\left(\beta^2+8\alpha\right)k^3+\frac{1}{2}\left(\left(-8a_{-1}b_{-1}-8a_0b_{-2}\right)\alpha-4\beta\left(a_{-1}b_0+a_0b_{-1}+a_1b_{-2}\right)\right)k$$
$$+2\omega b_{-2}\alpha+\omega b_{-1}\beta=0$$

$$\frac{1}{2}\left(\left(4a_0a_1-b_1\right)\beta+4a_1^2-2b_2\right)k-\omega a_1\beta=0$$

$$\frac{1}{2}\left(\beta^2+2\alpha\right)\left(\alpha a_{-1}-a_1\right)k^3+\frac{1}{2}\left(\left(-4a_{-1}b_0-4a_0b_{-1}-4a_1b_{-2}\right)\alpha+4a_0b_1\right.$$
$$\left.+4a_{-1}b_2+4b_0a_1\right)k+\omega\left(\alpha b_{-1}-b_1\right)=0$$

$$\frac{1}{2}\left(\left(4a_0a_1-b_1\right)\alpha-2\beta\left(-2a_1^2+b_2\right)\right)k-\omega a_1\alpha=0$$

$$-\frac{1}{2}\beta a_1\left(\beta^2+8\alpha\right)k^3+\frac{1}{2}\left(\left(4a_{-1}b_2+4a_0b_1+4a_1b_0\right)\beta+8a_1b_1+8a_0b_2\right)k$$
$$-\omega\left(\beta b_1+2b_2\right)=0$$

$$-k\alpha\left(-2a_1^2+b_2\right)=0$$

$$\frac{7}{2}\left(\beta^2+\frac{8}{7}\alpha\right)\alpha a_1k^3+\frac{1}{2}\left(\left(4a_{-1}b_2+4a_0b_1+4a_1b_0\right)\alpha+\left(8a_0b_2+8a_1b_1\right)\beta\right.$$
$$\left.+12a_1b_2\right)k-\omega\left(\alpha b_1+2\beta b_2\right)=0$$

$$-6k^3\beta\alpha^2a_1+\frac{1}{2}\left(\left(8a_0b_2+8a_1b_1\right)\alpha+12a_1b_2\beta\right)k-2\omega b_2\alpha=0$$

$$-3k\alpha a_1\left(\alpha^2k^2-2b_2\right)=0$$

$$-2k\left(a_{-1}^2-\frac{1}{2}b_{-2}\right)=0$$

$$3ka_{-1}\left(k^2-2b_{-2}\right)=0$$

Which solving by maple leads to

Case 1, 2:

$$a_{-1}=0, a_0=\pm\frac{1}{4}\frac{\beta k^2\pm 2\omega}{k}, a_1=\pm\frac{1}{2}k\alpha, b_{-2}=0, b_{-1}=0, b_0=\frac{1}{2}k^2\alpha, b_1=\frac{1}{2}k^2\alpha\beta, b_2=\frac{1}{2}k^2\alpha^2,$$

If $\beta^2-4\alpha>0,$

$$\begin{cases} u(\xi)=\mp\dfrac{k\sqrt{\beta^2-4\alpha}}{4}\tanh\left(\dfrac{\sqrt{\beta^2-4\alpha}}{2}(\xi+c)\right)+\dfrac{\omega}{2k}, \\ v(\xi)=\dfrac{1}{2}k^2(\left(-\dfrac{\sqrt{\beta^2-4\alpha}}{2}\tanh\left(\dfrac{\sqrt{\beta^2-4\alpha}}{2}(\xi+c)\right)-\dfrac{\beta}{2}\right)^2+\beta\left(-\dfrac{\sqrt{\beta^2-4\alpha}}{2}\tanh\left(\dfrac{\sqrt{\beta^2-4\alpha}}{2}(\xi+c)\right)-\dfrac{\beta}{2}\right)+\alpha). \end{cases} \quad (11)$$

$If \beta^2-4\alpha<0,$

$$\begin{cases} u(\xi)=\pm\dfrac{k\sqrt{4\alpha-\beta^2}}{4}tan\left(\dfrac{\sqrt{4\alpha-\beta^2}}{2}(\xi+c)\right)+\dfrac{\omega}{2k}, \\ v(\xi)=\dfrac{1}{2}k^2((\dfrac{\sqrt{4\alpha-\beta^2}}{2}tan\left(\dfrac{\sqrt{4\alpha-\beta^2}}{2}(\xi+c)\right)-\dfrac{\beta}{2})^2+\beta(\dfrac{\sqrt{4\alpha-\beta^2}}{2}tan\left(\dfrac{\sqrt{4\alpha-\beta^2}}{2}(\xi+c)\right)-\dfrac{\beta}{2})+\alpha). \end{cases} \quad (12)$$

$If \beta^2-4\alpha=0,$

$$\begin{cases} u(\xi)=\mp\dfrac{1}{2}k\alpha\dfrac{2\beta(\xi+c)+4}{\beta(\xi+c)}\pm\dfrac{1}{4}\dfrac{\beta k^2\pm 2\omega}{k}, \\ v(\xi)=\dfrac{1}{2}k^2\alpha\left[\alpha\left(\dfrac{2\beta(\xi+c)+4}{\beta(\xi+c)}\right)^2-\dfrac{2\alpha\beta(\xi+c)+4}{(\xi+c)}+1\right]. \end{cases} \quad (13)$$

Where $\xi=\frac{kx^\gamma}{\Gamma(1+\gamma)}+\frac{\omega t^\gamma}{\Gamma(1+\gamma)},$

Case 3, 4:

$$\beta=0, k=\pm\frac{2a_1}{\alpha}, a_{-1}=\frac{a_1}{\alpha}, a_0=\pm\frac{1}{4}\frac{\omega\alpha}{a_1}, b_{-2}=\frac{2{a_1}^2}{\alpha^2}, b_{-1}=0, b_0=0, b_1=0, b_2=2{a_1}^2.$$

$If \alpha<0, \varphi(\xi)=ln\left(-\frac{\sqrt{-\alpha}}{\alpha}tanh\left(\sqrt{-\alpha}(\xi+c)\right)\right),$

$$\begin{cases} u(\xi)=-\dfrac{a_1}{\sqrt{-\alpha}}tanh\left(\sqrt{-\alpha}(\xi+c)\right)\pm\dfrac{1}{4}\dfrac{\omega\alpha}{a_1}-\dfrac{a_1}{\sqrt{-\alpha}\,tanh\left(\sqrt{-\alpha}(\xi+c)\right)}, \\ v(\xi)=-\dfrac{2{a_1}^2}{\alpha}tanh^2\left(\sqrt{-\alpha}(\xi+c)\right)-\dfrac{2{a_1}^2}{\alpha\, tanh^2\left(\sqrt{-\alpha}(\xi+c)\right)}. \end{cases} \quad (14)$$

$If \alpha>0, \varphi(\xi)=ln\left(\frac{1}{\sqrt{\alpha}}tan\left(\sqrt{\alpha}(\xi+c)\right)\right),$

$$\begin{cases} u(\xi)=a_1\dfrac{1}{\sqrt{\alpha}}tan\left(\sqrt{\alpha}(\xi+c)\right)\pm\dfrac{1}{4}\dfrac{\omega\alpha}{a_1}+\dfrac{a_1}{\alpha\dfrac{1}{\sqrt{\alpha}}tan\left(\sqrt{\alpha}(\xi+c)\right)}, \\ v(\xi)=\dfrac{2{a_1}^2}{\alpha}tan^2\left(\sqrt{\alpha}(\xi+c)\right)+\dfrac{2{a_1}^2}{\alpha\, tan^2\left(\sqrt{\alpha}(\xi+c)\right)}. \end{cases} \quad (15)$$

where

$$\xi=\frac{\pm 2a_1x^{\gamma}}{\alpha\Gamma(1+\gamma)}-\frac{\omega t^{\gamma}}{\Gamma(1+\gamma)},$$

Case 5,6

$$\beta=0,k=\pm\frac{2a_1}{\alpha},a_{-1}=\pm\frac{a_1}{\alpha},a_0=\pm\frac{1}{4}\frac{\omega\alpha}{a_1},b_{-2}=\frac{2{a_1}^2}{\alpha^2},b_{-1}=0,b_0=\frac{4{a_1}^2}{\alpha},b_1=0,b_2=2{a_1}^2.$$

$$If\alpha<0,\varphi(\xi)=ln\left(-\frac{1}{\sqrt{-\alpha}}tanh\big(\sqrt{-\alpha}(\xi+c)\big)\right),$$

$$\begin{cases}u(\xi)=-\dfrac{a_1}{\sqrt{-\alpha}}tanh\big(\sqrt{-\alpha}(\xi+c)\big)\pm\dfrac{1}{4}\dfrac{\omega\alpha}{a_1}\pm\dfrac{a_1}{\sqrt{-\alpha}\,tanh\big(\sqrt{-\alpha}(\xi+c)\big)},\\ v(\xi)=-\dfrac{2{a_1}^2}{\alpha}tanh^2\big(\sqrt{-\alpha}(\xi+c)\big)+\dfrac{4{a_1}^2}{\alpha}-\dfrac{2{a_1}^2}{\alpha\,tanh^2\big(\sqrt{-\alpha}(\xi+c)\big)}.\end{cases}\tag{16}$$

$$If\alpha>0,\varphi(\xi)=ln\left(\frac{1}{\sqrt{\alpha}}tan\big(\sqrt{\alpha}(\xi+c)\big)\right),$$

$$\begin{cases}u(\xi)=a_1\dfrac{1}{\sqrt{\alpha}}tan\big(\sqrt{\alpha}(\xi+c)\big)\pm\dfrac{1}{4}\dfrac{\omega\alpha}{a_1}\pm\dfrac{a_1}{\alpha\dfrac{1}{\sqrt{\alpha}}tan\big(\sqrt{\alpha}(\xi+c)\big)},\\ v(\xi)=\dfrac{2{a_1}^2}{\alpha}tan^2\big(\sqrt{\alpha}(\xi+c)\big)+\dfrac{4{a_1}^2}{\alpha}++\dfrac{2{a_1}^2}{\alpha\,tan^2\big(\sqrt{\alpha}(\xi+c)\big)}.\end{cases}\tag{17}$$

Where

$$\xi=\frac{\pm 2a_1x^{\gamma}}{\alpha\Gamma(1+\gamma)}+\frac{\omega t^{\gamma}}{\Gamma(1+\gamma)},$$

Case 7,8

$$a_{-1}=\pm\frac{1}{2}k,\quad a_0=\pm\frac{1}{4}\frac{\beta k^2\pm 2\omega}{k},a_1=0,b_{-2}=\frac{1}{2}k^2,b_{-1}=\frac{1}{2}\beta k^2,b_0=\frac{1}{2}k^2\alpha,b_1=0,b_2=0,$$

$$If\alpha\neq 0,\beta^2-4\alpha>0,$$

$$\begin{cases}u(\xi)=\pm\dfrac{1}{4}\dfrac{\beta k^2\pm 2\omega}{k}\mp\dfrac{\alpha k}{\sqrt{\beta^2-4\alpha}\,tanh\left(\dfrac{\sqrt{\beta^2-4\alpha}}{2}(\dfrac{kx^{\gamma}}{\Gamma(1+\gamma)}+\dfrac{\omega t^{\gamma}}{\Gamma(1+\gamma)}+c)\right)+\beta}\\ v(\xi)=\dfrac{k^2\alpha(\beta^2-4\alpha)\,tanh^2\left(\dfrac{\sqrt{\beta^2-4\alpha}}{2}(\dfrac{kx^{\gamma}}{\Gamma(1+\gamma)}+\dfrac{\omega t^{\gamma}}{\Gamma(1+\gamma)}+c)\right)-\beta\alpha k^2+4\alpha^2k^2}{\left(\sqrt{\beta^2-4\alpha}\,tanh\left(\dfrac{\sqrt{\beta^2-4\alpha}}{2}(\dfrac{kx^{\gamma}}{\Gamma(1+\gamma)}+\dfrac{\omega t^{\gamma}}{\Gamma(1+\gamma)}+c)\right)+\beta\right)^2}.\end{cases}\tag{18}$$

$$If\alpha\neq 0,\beta^2-4\alpha<0,$$

$$\begin{cases}u(\xi)=\pm\dfrac{1}{4}\dfrac{\beta k^2\pm 2\omega}{k}\pm\dfrac{\alpha k}{\sqrt{4\alpha-\beta^2}\,tan\left(\dfrac{\sqrt{4\alpha-\beta^2}}{2}(\dfrac{kx^{\gamma}}{\Gamma(1+\gamma)}+\dfrac{\omega t^{\gamma}}{\Gamma(1+\gamma)}+c)\right)-\beta}\\ v(\xi)=\dfrac{k^2\alpha(4\alpha-\beta^2)\,tan^2\left(\dfrac{\sqrt{4\alpha-\beta^2}}{2}(\dfrac{kx^{\gamma}}{\Gamma(1+\gamma)}+\dfrac{\omega t^{\gamma}}{\Gamma(1+\gamma)}+c)\right)-\beta\alpha k^2+4\alpha^2k^2}{2\left(\sqrt{4\alpha-\beta^2}\,tan\left(\dfrac{\sqrt{4\alpha-\beta^2}}{2}(\dfrac{kx^{\gamma}}{\Gamma(1+\gamma)}+\dfrac{\omega t^{\gamma}}{\Gamma(1+\gamma)}+c)\right)-\beta\right)^2}\end{cases}\tag{19}$$

$If\alpha \neq 0, \beta^2 - 4\alpha = 0,$

$$\begin{cases} u(\xi) = \pm\dfrac{\beta k^2 \pm 2\omega}{4k} \mp \dfrac{k\,\beta\,(\frac{kx^\gamma}{\Gamma(1+\gamma)} + \frac{\omega t^\gamma}{\Gamma(1+\gamma)} + c)}{4\beta(\frac{kx^\gamma}{\Gamma(1+\gamma)} + \frac{\omega t^\gamma}{\Gamma(1+\gamma)} + c) + 8}, \\ v(\xi) = \dfrac{1}{2}\left(k^2\alpha - \dfrac{k^2\beta^2(\frac{kx^\gamma}{\Gamma(1+\gamma)} + \frac{\omega t^\gamma}{\Gamma(1+\gamma)} + c)}{2\beta(\frac{kx^\gamma}{\Gamma(1+\gamma)} + \frac{\omega t^\gamma}{\Gamma(1+\gamma)} + c) + 4} + \dfrac{k^2\beta^2(\frac{kx^\gamma}{\Gamma(1+\gamma)} + \frac{\omega t^\gamma}{\Gamma(1+\gamma)} + c)^2}{(2\beta(\frac{kx^\gamma}{\Gamma(1+\gamma)} + \frac{\omega t^\gamma}{\Gamma(1+\gamma)} + c) + 4)^2}\right). \end{cases} \quad \textbf{(20)}$$

$If\alpha = 0, \beta \neq 0,$

$$\begin{cases} u(\xi) = \pm\dfrac{1}{4}\dfrac{\beta k^2 \pm 2\omega}{k} \pm \dfrac{1}{2}\dfrac{k\,\beta}{exp(\beta(\frac{kx^\gamma}{\Gamma(1+\gamma)} + \frac{\omega t^\gamma}{\Gamma(1+\gamma)} + c)) - 1}, \\ v(\xi) = \dfrac{1}{2}\dfrac{exp(\beta(\frac{kx^\gamma}{\Gamma(1+\gamma)} + \frac{\omega t^\gamma}{\Gamma(1+\gamma)} + c))}{(exp(\beta(\frac{kx^\gamma}{\Gamma(1+\gamma)} + \frac{\omega t^\gamma}{\Gamma(1+\gamma)} + c)) - 1)^2}. \end{cases} \quad \textbf{(21)}$$

$If\alpha = 0, \beta = 0, \varphi(\xi) = ln(\xi + c),$

$$\begin{cases} u(\xi) = \dfrac{\omega}{2k} \pm \dfrac{1}{2}\dfrac{k}{\frac{kx^\gamma}{\Gamma(1+\gamma)} + \frac{\omega t^\gamma}{\Gamma(1+\gamma)} + c} \\ v(\xi) = \dfrac{1}{2}\dfrac{k^2}{(\frac{kx^\gamma}{\Gamma(1+\gamma)} + \frac{\omega t^\gamma}{\Gamma(1+\gamma)} + c)^2}. \end{cases} \quad \textbf{(22)}$$

Graphs of some obtained solutions for specific values of the parameters are drawn in ***Figures 1 to 4***.

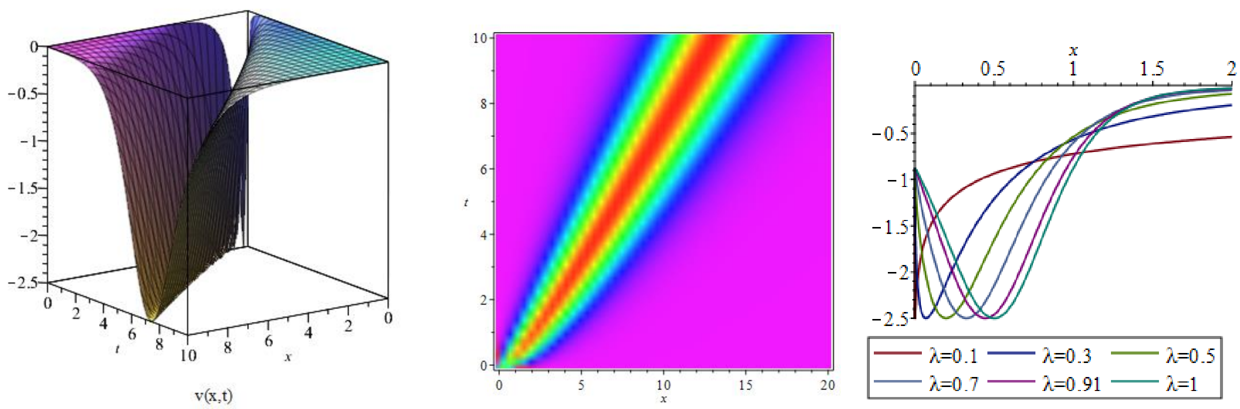

Figure 1. *plots of solution* (11) $for\beta = 3, \alpha = 1, c = 1, \omega = 2, k = -2.$
$(a)\,\lambda = 0.5 (b)\,\lambda = 0.5, (c) t = 0 for different value of \lambda.$

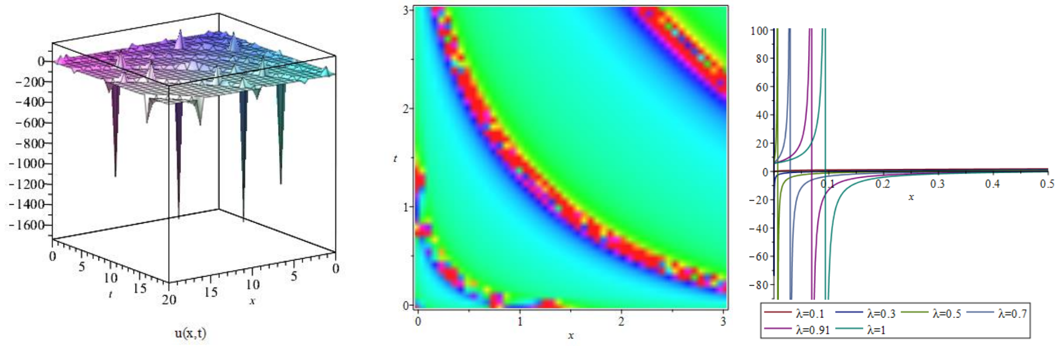

Figure 2. *plots of solution* (12) *for* $\beta = 1, \alpha = 2, c = 1, \omega = 2, k = 2.$
$(a)\, \lambda = 0.5 (b)\, \lambda = 0.5, (c) t = 0$ *for different value of* λ.

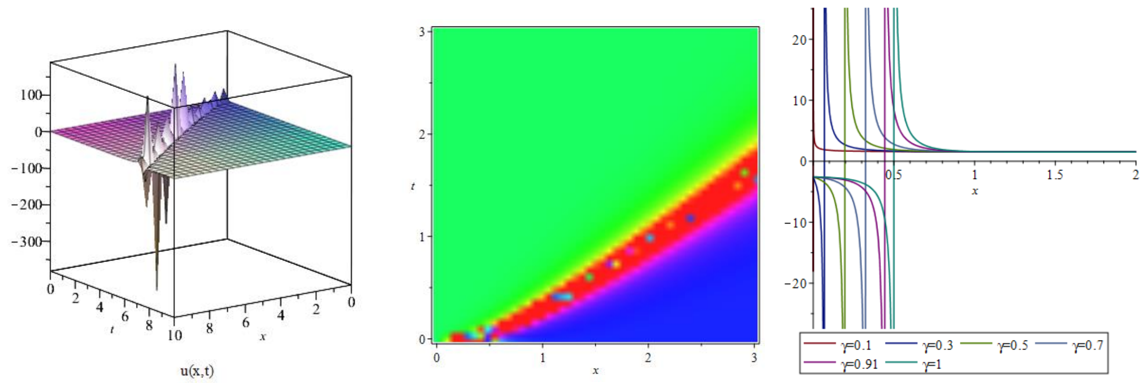

Figure 3. *plots of solution* (16) *for* $\beta = 0, \alpha = 1, c = -1, \omega = -2, k = 1, a_1 = 1.$
$(a)\, \gamma = 0.5 (b)\, \gamma = 0.5, (c) t = 0$ *for different value of* γ.

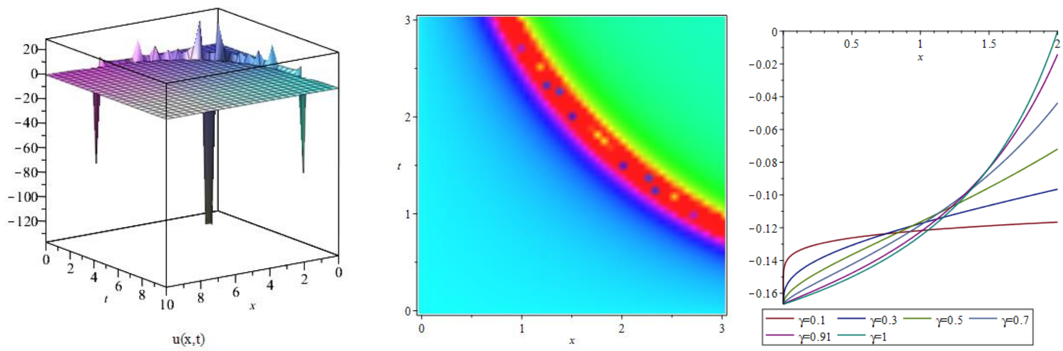

Figure 4. *plots of solution* (16) *for* $\beta = -2, \alpha = 1, c = -2, \omega = 1, k = 1, a_1 = 1.$
$(a)\, \gamma = 0.5 (b)\, \gamma = 0.5, (c) t = 0$ *for different value of* γ.

4. CMEFM for Coupled Time-Fractional Long System

Consider the space time fractional (2+1)-dimensional dispersive long wave equations as follows

$$\begin{cases} \dfrac{\partial^{2\alpha} u}{\partial y^{\alpha} \partial t^{\alpha}} + \dfrac{\partial^{2\alpha} v}{\partial x^{2\alpha}} + \dfrac{\partial^{\alpha}}{\partial y^{\alpha}} \left(u \dfrac{\partial^{\alpha} u}{\partial x^{\alpha}}\right) = 0, \\ \dfrac{\partial^{\alpha} v}{\partial t^{\alpha}} + \dfrac{\partial^{\alpha} u}{\partial x^{\alpha}} + \dfrac{\partial^{\alpha}(uv)}{\partial x^{\alpha}} + \dfrac{\partial^{3\alpha} u}{\partial x^{2\alpha} \partial y^{\alpha}} = 0. \end{cases} \tag{23}$$

At first, we defined

$$\xi = \frac{kx^{\gamma}}{\Gamma(1+\gamma)} - \frac{\omega t^{\gamma}}{\Gamma(1+\gamma)} + \frac{\lambda y^{\gamma}}{\Gamma(1+\gamma)}, \tag{24}$$

So, ***Eq. (23)*** convert to,

$$\begin{cases} -\omega\lambda u'' + k^2 v'' + \lambda k (uu')' = 0, \\ -\omega v' + ku' + k(uv)' + k^2 \lambda u''' = 0. \end{cases} \tag{25}$$

The solution of (25) will be image as ***Eq. (4)***, Where $\varphi(\xi)$ satisfy in ***Eq. (5)***. Homogeneous balance between linear and nonlinear terms in each equation of (25) leads to $M = 1$, and $N = 2$. So the solution (4), will be written as follows

$$\begin{cases} u(\xi) = a_1 e^{\varphi(\xi)} + a_0 + a_{-1} e^{-\varphi(\xi)}, \\ v(\xi) = b_2 e^{2\varphi(\xi)} + b_1 e^{\varphi(\xi)} + b_0 + b_{-1} e^{-\varphi(\xi)} + b_{-2} e^{-2\varphi(\xi)}. \end{cases} \tag{26}$$

Putting (26) in (25), and putting the coefficient of $e^{\varphi(\xi)}$ equal to zero, yields a system of algebraic equations,

$$3k\lambda a_{-1}^2 + 6k^2 b_{-2} = 0$$

$$-6\left(\lambda k + \frac{1}{2} b_{-2}\right) k a_{-1} = 0$$

$$\left(10\beta b_{-2} + 2b_{-1}\right)k^2 + 5\lambda a_{-1}\left(\beta a_{-1} + \frac{2}{5}a_0\right)k - 2\omega\lambda a_{-1} = 0$$

$$-12k^2\lambda a_{-1}\beta + \left(-3\beta a_{-1}b_{-2} - 2a_{-1}b_{-1} - 2a_0 b_{-2}\right)k + 2\omega b_{-2} = 0$$

$$\left(4\beta^2 b_{-2} + 8\alpha b_{-2} + 3\beta b_{-1}\right)k^2 + 2\left(\beta^2 a_{-1} + \frac{3}{2}\beta a_0 + 2a_{-1}\alpha\right)\lambda a_{-1} k - 3\omega\lambda a_{-1}\beta = 0$$

$$-7\left(\beta^2 + \frac{8}{7}\alpha\right)\lambda a_{-1}k^2 + \left(-3b_{-2}a_{-1}\alpha + \left(-2a_{-1}b_{-1} - 2a_0 b_{-2}\right)\beta + \left(-b_0 - 1\right)a_{-1} - b_{-2}a_1 - b_{-1}a_0\right)k + 2\omega\left(\beta b_{-2} + \frac{1}{2}b_{-1}\right) = 0$$

$$\left(6\alpha\beta b_{-2} + \beta^2 b_{-1} + 2\alpha b_{-1}\right)k^2 + \lambda a_{-1}\left(3\alpha\beta a_{-1} + \beta^2 a_0 + 2\alpha a_0\right)k - \omega\lambda a_{-1}\left(\beta^2 + 2\alpha\right) = 0$$

$$-\beta\lambda a_{-1}\left(\beta^2 + 8\alpha\right)k^2 + \left(\left(-2a_{-1}b_{-1} - 2a_0 b_{-2}\right)\alpha - \left(\left(b_0 + 1\right)a_{-1} + b_{-2}a_1 + b_{-1}a_0\right)\beta\right)k + 2\omega\alpha b_{-2} + \omega\beta b_{-1} = 0$$

$$\left(\left(\alpha b_{-1} + b_1\right)\beta + 2\alpha^2 b_{-2} + 2b_2\right)k^2 + \lambda\left(a_0\left(\alpha a_{-1} + a_1\right)\beta + \alpha^2 a_{-1}^2 + a_1^2\right)k - \beta\omega\lambda\left(\alpha a_{-1} + a_1\right) = 0$$

$$\lambda\left(\beta^{2}+2\alpha\right)\left(-\alpha a_{-1}+a_{1}\right)k^{2}+\left(\left(\left(-b_{0}-1\right)a_{-1}-b_{-2}a_{1}-b_{-1}a_{0}\right)\alpha+a_{-1}b_{2}+\left(b_{0}+1\right)a_{1}+a_{0}b_{1}\right)k+\omega\left(\alpha b_{-1}-b_{1}\right)=0$$

$$\left(\beta^{2}b_{1}+2\alpha b_{1}+6\beta b_{2}\right)k^{2}+\lambda a_{1}\left(\beta^{2}a_{0}+2\alpha a_{0}+3\beta a_{1}\right)k-\omega\lambda a_{1}\left(\beta^{2}+2\alpha\right)=0$$

$$a_{1}\beta\lambda\left(\beta^{2}+8\alpha\right)k^{2}+\left(\left(a_{-1}b_{2}+\left(b_{0}+1\right)a_{1}+a_{0}b_{1}\right)\beta+2a_{1}b_{1}+2b_{2}a_{0}\right)k-\omega\left(\beta b_{1}+2b_{2}\right)=0$$

$$\left(3\alpha\beta b_{1}+4\beta^{2}b_{2}+8\alpha b_{2}\right)k^{2}+2\lambda\left(a_{1}\beta^{2}+\frac{3}{2}\beta\alpha a_{0}+2a_{1}\alpha\right)a_{1}k-3\omega\lambda a_{1}\beta\alpha=0$$

$$7\left(\beta^{2}+\frac{8}{7}\alpha\right)\lambda\alpha a_{1}k^{2}+\left(\left(a_{-1}b_{2}+\left(b_{0}+1\right)a_{1}+a_{0}b_{1}\right)\alpha+\left(2a_{0}b_{2}+2a_{1}b_{1}\right)\beta+3a_{1}b_{2}\right)k-\omega\left(\alpha b_{1}+2\beta b_{2}\right)=0$$

$$\alpha\left(\left(\alpha b_{1}+5\beta b_{2}\right)k^{2}+\left(\frac{5}{2}\beta a_{1}^{2}+\alpha a_{0}a_{1}\right)\lambda k-\omega\lambda a_{1}\alpha\right)=0$$

$$12k^{2}\lambda\alpha^{2}a_{1}\beta+\left(\left(2a_{0}b_{2}+2a_{1}b_{1}\right)\alpha+3a_{1}b_{2}\beta\right)k-2\omega b_{2}\alpha=0$$

$$3\alpha^{2}k\lambda a_{1}^{2}+6\alpha^{2}k^{2}b_{2}=0$$

$$6\left(k\alpha^{2}\lambda+\frac{1}{2}b_{2}\right)\alpha k a_{1}=0$$

Which solving by maple leads to

Case 1, 2:

$$\alpha=0,\beta=0,a_{-1}=\pm 2k,\quad a_{0}=\frac{\omega}{k},a_{1}=0,b_{-2}=-2\lambda k,b_{-1}=0,b_{0}=-1,b_{2}=0,$$

$$\begin{cases}u(\xi)=\dfrac{\omega}{k}\pm 2\dfrac{k}{\dfrac{kx^{\gamma}}{\Gamma(1+\gamma)}-\dfrac{\omega t^{\gamma}}{\Gamma(1+\gamma)}+\dfrac{\lambda y^{\gamma}}{\Gamma(1+\gamma)}+c}\\ v(\xi)=b_{1}\dfrac{kx^{\gamma}}{\Gamma(1+\gamma)}-\dfrac{\omega t^{\gamma}}{\Gamma(1+\gamma)}+\dfrac{\lambda y^{\gamma}}{\Gamma(1+\gamma)}+c-1-\dfrac{2\lambda k}{\dfrac{kx^{\gamma}}{\Gamma(1+\gamma)}-\dfrac{\omega t^{\gamma}}{\Gamma(1+\gamma)}+\dfrac{\lambda y^{\gamma}}{\Gamma(1+\gamma)}+c}e^{-2\varphi(\xi)}.\end{cases}\quad\textbf{(27)}$$

Case 3, 4

$$a_{-1}=0,a_{0}=\pm\frac{\beta k^{2}\pm\omega}{k},a_{1}=\pm 2\alpha k,b_{-2}=0,b_{-1}=0,b_{0}=-2\alpha k\lambda-1,b_{1}=-2\alpha\beta k\lambda,b_{2}=-2\alpha^{2}k\lambda,$$

$$If\,\alpha\neq 0,\beta^{2}-4\alpha>0,$$

$$\begin{cases}u(\xi)=\mp k\sqrt{\beta^{2}-4\alpha}\,tanh\left(\dfrac{\sqrt{\beta^{2}-4\alpha}}{2}\left(\dfrac{kx^{\gamma}}{\Gamma(1+\gamma)}-\dfrac{\omega t^{\gamma}}{\Gamma(1+\gamma)}+\dfrac{\lambda x^{\gamma}}{\Gamma(1+\gamma)}+c\right)\right)+\dfrac{\omega}{k},\\ v(\xi)=-\dfrac{k\lambda(\beta^{2}-4\alpha)}{2}\left(tanh^{2}\left(\dfrac{\sqrt{\beta^{2}-4\alpha}}{2}\left(\dfrac{kx^{\gamma}}{\Gamma(1+\gamma)}-\dfrac{\omega t^{\gamma}}{\Gamma(1+\gamma)}+\dfrac{\lambda x^{\gamma}}{\Gamma(1+\gamma)}+c\right)\right)-1\right)-1.\end{cases}\quad\textbf{(28)}$$

$If\alpha \neq 0, \beta^2 - 4\alpha < 0,$

$$\begin{cases} u(\xi) = \pm k\sqrt{4\alpha-\beta^2}\, tan\left(\frac{\sqrt{4\alpha-\beta^2}}{2}(\frac{kx^\gamma}{\Gamma(1+\gamma)} - \frac{\omega t^\gamma}{\Gamma(1+\gamma)} + \frac{\lambda x^\gamma}{\Gamma(1+\gamma)} + c)\right) + \frac{\omega}{k}, \\ v(\xi) = -\frac{k\lambda(4\alpha-\beta^2)}{2}\left(tan^2\left(\frac{\sqrt{4\alpha-\beta^2}}{2}(\frac{kx^\gamma}{\Gamma(1+\gamma)} - \frac{\omega t^\gamma}{\Gamma(1+\gamma)} + \frac{\lambda x^\gamma}{\Gamma(1+\gamma)} + c)\right) + 1\right) - 1. \end{cases} \quad (29)$$

$If\alpha \neq 0, \beta^2 - 4\alpha = 0,$

$$\begin{cases} u(\xi) = \mp 2\alpha k\left(\frac{2\beta(\xi+c)+4}{\beta(\xi+c)}\right) \pm \frac{\beta k^2 \pm \omega}{k}, \\ v(\xi) = -2\alpha^2 k\lambda\left(\frac{2\beta(\xi+c)+4}{\beta(\xi+c)}\right)^2 + 2\alpha\beta k\lambda\left(\frac{2\beta(\xi+c)+4}{\beta(\xi+c)}\right) - 2\alpha k\lambda - 1. \end{cases} \quad (30)$$

Where

$$\xi = \frac{kx^\gamma}{\Gamma(1+\gamma)} - \frac{\omega t^\gamma}{\Gamma(1+\gamma)} + \frac{\lambda y^\gamma}{\Gamma(1+\gamma)}$$

Case 5, 6

$$a_{-1} = \pm 2k, a_0 = \pm\frac{\beta k^2 \pm \omega}{k}, a_1 = 0, b_{-2} = -2\lambda k, b_{-1} = -2\lambda\beta k, b_0 = -2\alpha k\lambda - 1, b_1 = 0, b_2 = 0,$$

$If\alpha \neq 0{,}2\beta^2 - 4\alpha > 0,$

$$\begin{cases} u(\xi) = \pm\frac{\beta k^2 \pm \omega}{k} \pm \frac{2k}{\left(-\frac{\sqrt{\beta^2-4\alpha}}{2\alpha} tanh\left(\frac{\sqrt{\beta^2-4\alpha}}{2}(\xi+c)\right) - \frac{\beta}{2\alpha}\right)}, \\ v(\xi) = -1 - \frac{2k\lambda\alpha(\beta^2-4\alpha)\left(tanh^2\left(\frac{\sqrt{\beta^2-4\alpha}}{2}(\xi+c)\right)+1\right)}{\left(\sqrt{\beta^2-4\alpha}\, tanh\left(\frac{\sqrt{\beta^2-4\alpha}}{2}(\xi+c)\right) + \beta\right)^2}. \end{cases} \quad (31)$$

$If\alpha \neq 0, \beta^2 - 4\alpha < 0,$

$$\begin{cases} u(\xi) = \pm\frac{\beta k^2 \pm \omega}{k} \pm \frac{4\alpha k}{\left(\sqrt{4\alpha-\beta^2}\, tan\left(\frac{\sqrt{4\alpha-\beta^2}}{2}(\xi+c)\right) - \beta\right)}, \\ v(\xi) = -1 + \frac{-k\lambda(4\alpha-\beta^2)\left(tan^2\left(\frac{\sqrt{4\alpha-\beta^2}}{2}(\xi+c)\right) - 1\right)}{\left(\sqrt{4\alpha-\beta^2}\, tan\left(\frac{\sqrt{4\alpha-\beta^2}}{2}(\xi+c)\right) - \beta\right)^2}. \end{cases} \quad (32)$$

$If\alpha \neq 0, \beta^2 - 4\alpha = 0,$ **(33)**

$$\begin{cases} u(\xi) = \pm\dfrac{\beta k^2 \pm \omega}{k} \mp \dfrac{k\beta(\xi+c)}{\beta(\xi+c)+2}, \\ v(\xi) = -2\alpha k\lambda - 1 + \dfrac{\lambda k\beta^2(\xi+c)}{\beta(\xi+c)+2} - \dfrac{\lambda k\beta^2(\xi+c)^2}{2(\beta(\xi+c)+2)^2}. \end{cases}$$

$If\alpha = 0, \beta \neq 0,$

$$\begin{cases} u(\xi) = \pm\dfrac{\beta k^2 \pm \omega}{k} \pm 2k\dfrac{\beta}{exp(\beta(\xi+c))-1}, \\ v(\xi) = -1 - \dfrac{2\lambda k\beta^2\, exp(\beta(\xi+c))}{(exp(\beta(\xi+c))-1)^2}. \end{cases} \quad \textbf{(34)}$$

$If\alpha = 0, \beta = 0,$

$$\begin{cases} u(\xi) = \dfrac{\omega}{k} \pm \dfrac{2k}{\xi+c}, \\ v(\xi) = -1 - \dfrac{2\lambda k}{(\xi+c)^2}. \end{cases} \quad \textbf{(35)}$$

Case 7, 8

$\beta = 0, a_{-1} = \pm 2k, a_0 = \dfrac{\omega}{k}, a_1 = \pm 2\alpha k, b_{-2} = -2\lambda k, b_{-1} = 0, b_0 = -1, b_1 = 0, b_2 = -2\alpha^2\lambda k,$

$If\alpha < 0,$

$$\begin{cases} u(\xi) = \mp 2k\sqrt{-\alpha}\, tanh\left(\sqrt{-\alpha}(\xi+c)\right) + \dfrac{\omega}{k} \mp \dfrac{2\sqrt{-\alpha}k}{tanh\left(\sqrt{-\alpha}(\xi+c)\right)}, \\ v(\xi) = 2\lambda k\alpha\, tanh^2\left(\sqrt{-\alpha}(\xi+c)\right) - 1 + \dfrac{2\alpha\lambda k}{tanh^2\left(\sqrt{-\alpha}(\xi+c)\right)}. \end{cases} \quad \textbf{(36)}$$

$If\alpha > 0,$

$$\begin{cases} u(\xi) = \pm 2k\sqrt{\alpha}\, tan\left(\sqrt{\alpha}(\xi+c)\right) + \dfrac{\omega}{k} \pm \dfrac{2k\sqrt{\alpha}}{tan\left(\sqrt{\alpha}(\xi+c)\right)}, \\ v(\xi) = -2\alpha\lambda k\, tan^2\left(\sqrt{\alpha}(\xi+c)\right) - 1 - \dfrac{2\alpha\lambda k}{tan^2\left(\sqrt{\alpha}(\xi+c)\right)}. \end{cases} \quad \textbf{(37)}$$

$If\alpha = 0,$

$$\begin{cases} u(\xi) = +\dfrac{\omega}{k} \pm \dfrac{2k}{\xi+c}, \\ v(\xi) = -1 - \dfrac{2\lambda k}{(\xi+c)^2}. \end{cases} \quad \textbf{(38)}$$

Where

$$\xi = \frac{kx^\gamma}{\Gamma(1+\gamma)} - \frac{\omega t^\gamma}{\Gamma(1+\gamma)} + \frac{\lambda y^\gamma}{\Gamma(1+\gamma)},$$

Case 9, 10

$$\beta = 0, a_{-1} = \pm 2k, a_0 = \frac{\omega}{k}, a_1 = \pm 2\alpha k, b_{-2} = -2\lambda k, b_{-1} = 0, b_0 = -4\alpha k\lambda - 1, b_1 = 0, b_2 = -2\alpha^2 k\lambda,$$

$If \alpha < 0,$

$$\begin{cases} u(\xi) = \mp 2k\sqrt{-\alpha}\, tanh\left(\sqrt{-\alpha}(\xi + c)\right) + \frac{\omega}{k} \mp \frac{2\sqrt{-\alpha}k}{tanh\left(\sqrt{-\alpha}(\xi + c)\right)}, \\ v(\xi) = 2\lambda k\alpha\, tanh^2\left(\sqrt{-\alpha}(\xi + c)\right) - 4\alpha k\lambda - 1 + \frac{2\alpha\lambda k}{tanh^2\left(\sqrt{-\alpha}(\xi + c)\right)}. \end{cases} \quad \textbf{(39)}$$

$If \alpha > 0,$

$$\begin{cases} u(\xi) = \pm 2k\sqrt{\alpha}\, tan\left(\sqrt{\alpha}(\xi + c)\right) + \frac{\omega}{k} \pm \frac{2k\sqrt{\alpha}}{tan\left(\sqrt{\alpha}(\xi + c)\right)}, \\ v(\xi) = -2\alpha\lambda k\, tan^2\left(\sqrt{\alpha}(\xi + c)\right) - 4\alpha k\lambda - 1 - \frac{2\alpha\lambda k}{tan^2\left(\sqrt{\alpha}(\xi + c)\right)}. \end{cases} \quad \textbf{(40)}$$

$If \alpha = 0,$

$$\begin{cases} u(\xi) = +\frac{\omega}{k} \pm \frac{2k}{\xi + c}, \\ v(\xi) = -4\alpha k\lambda - 1 - \frac{2\lambda k}{(\xi + c)^2}. \end{cases} \quad \textbf{(41)}$$

Where

$$\xi = \frac{kx^\gamma}{\Gamma(1+\gamma)} - \frac{\omega t^\gamma}{\Gamma(1+\gamma)} + \frac{\lambda y^\gamma}{\Gamma(1+\gamma)},$$

Case 11, 12

$$\alpha = \pm\frac{1}{2}\frac{a_1}{k}, a_{-1} = \pm 2k, a_0 = \pm\frac{\beta k^2 \pm \omega}{k}, b_{-2} = -2\lambda k, b_{-1} = -2\lambda\beta k, b_0 = -1, b_1 = \mp\lambda\beta a_1, b_2 = -\frac{1}{2}\frac{\lambda {a_1}^2}{k}$$

$$If a_1 \neq 0, \beta^2 \mp \frac{2a_1}{k} > 0, \varphi(\xi) = ln\left(\mp\frac{\sqrt{\beta^2 k^2 \mp 2ka_1}}{a_1} tanh\left(\frac{\sqrt{\beta^2 \mp \frac{2a_1}{k}}}{2}(\xi + c)\right) \mp \frac{\beta k}{a_1}\right), \quad \textbf{(42)}$$

$$\begin{cases} u(\xi) = a_1 e^{\varphi(\xi)} \pm \frac{\beta k^2 \pm \omega}{k} \pm 2ke^{-\varphi(\xi)}, \\ v(\xi) = -\frac{1}{2}\frac{\lambda {a_1}^2}{k} e^{2\varphi(\xi)} \mp \lambda\beta a_1 e^{\varphi(\xi)} - 1 - 2\lambda\beta k e^{-\varphi(\xi)} - 2\lambda k e^{-2\varphi(\xi)}. \end{cases}$$

$$If a_1 \neq 0, \beta^2 \mp \frac{2a_1}{k} < 0, \varphi(\xi) = ln\left(\pm\frac{\sqrt{-\beta^2 k^2 \pm 2ka_1}}{a_1} tan\left(\frac{\sqrt{-\beta^2 \pm \frac{2a_1}{k}}}{2}(\xi + c)\right) \mp \frac{\beta k}{a_1}\right),$$

$$\begin{cases} u(\xi) = a_1 e^{\varphi(\xi)} \pm \frac{\beta k^2 \pm \omega}{k} \pm 2ke^{-\varphi(\xi)}, \\ v(\xi) = -\frac{1}{2}\frac{\lambda {a_1}^2}{k} e^{2\varphi(\xi)} \mp \lambda\beta a_1 e^{\varphi(\xi)} - 1 - 2\lambda\beta k e^{-\varphi(\xi)} - 2\lambda k e^{-2\varphi(\xi)}. \end{cases} \quad \textbf{(43)}$$

$$If\, a_1 \neq 0, \beta^2 \mp \frac{2a_1}{k} = 0, \varphi(\xi) = ln\left(-\frac{2\beta(\xi+c)+4}{\beta(\xi+c)}\right),$$

$$\begin{cases} u(\xi) = -a_1 \frac{2\beta(\xi+c)+4}{\beta(\xi+c)} \pm \frac{\beta k^2 \pm \omega}{k} \mp \frac{2k\beta(\xi+c)}{2\beta(\xi+c)+4}, \\ v(\xi) = -\frac{1}{2}\frac{\lambda a_1{}^2}{k}\left(\frac{2\beta(\xi+c)+4}{\beta(\xi+c)}\right)^2 \pm \lambda\beta a_1 \frac{2\beta(\xi+c)+4}{\beta(\xi+c)} - 1 + \frac{2\lambda\beta k\beta(\xi+c)}{2\beta(\xi+c)+4} - \frac{2\lambda k\beta^2(\xi+c)^2}{(2\beta(\xi+c)+4)^2}. \end{cases} \quad \textbf{(44)}$$

$$If\, a_1 = 0, \beta \neq 0,$$

$$\begin{cases} u(\xi) = \pm\frac{\beta k^2 \pm \omega}{k} \pm 2k \frac{\beta}{exp(\beta(\xi+c))-1}, \\ v(\xi) = -1 - \frac{2\lambda\beta^2 k\, exp(\beta(\xi+c))}{(exp(\beta(\xi+c))-1)^2}. \end{cases} \quad \textbf{(45)}$$

$$If\, \alpha = 0, \beta = 0,$$

$$\begin{cases} u(\xi) = \frac{\omega}{k} \pm \frac{2k}{\xi+c}, \\ v(\xi) = -1 - \frac{2\lambda k}{(\xi+c)^2}. \end{cases} \quad \textbf{(46)}$$

Where

$$\xi = \frac{kx^\gamma}{\Gamma(1+\gamma)} - \frac{\omega t^\gamma}{\Gamma(1+\gamma)} + \frac{\lambda y^\gamma}{\Gamma(1+\gamma)},$$

Graphs of some obtained solutions for specific values of the parameters are drawn in ***Figures 5 to 8***.

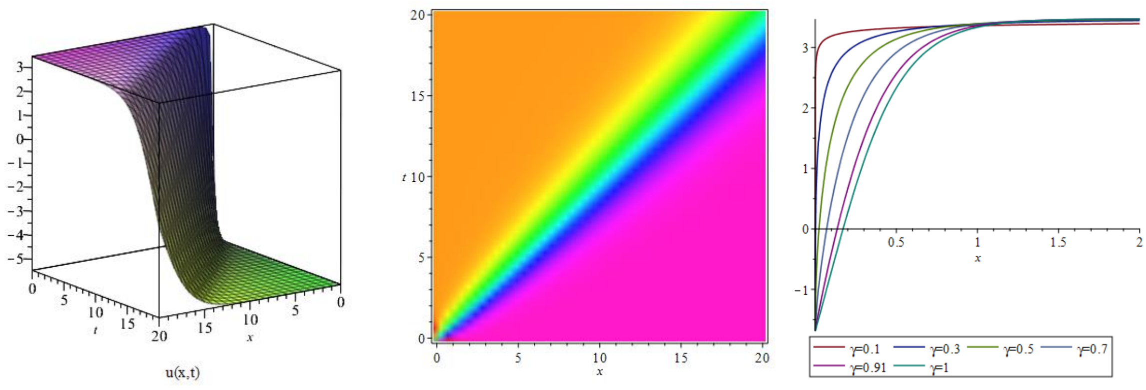

Figure 5. $plots\ of\ s\,olution\ (28)\ for \beta = 3, \alpha = 1, c = -1, \omega = -2, k = 2, \lambda$=1,y=0. $(a)\,\gamma = 0.5 (b)\,\gamma = 0.5, (c) t = 0 for different value of \gamma.$

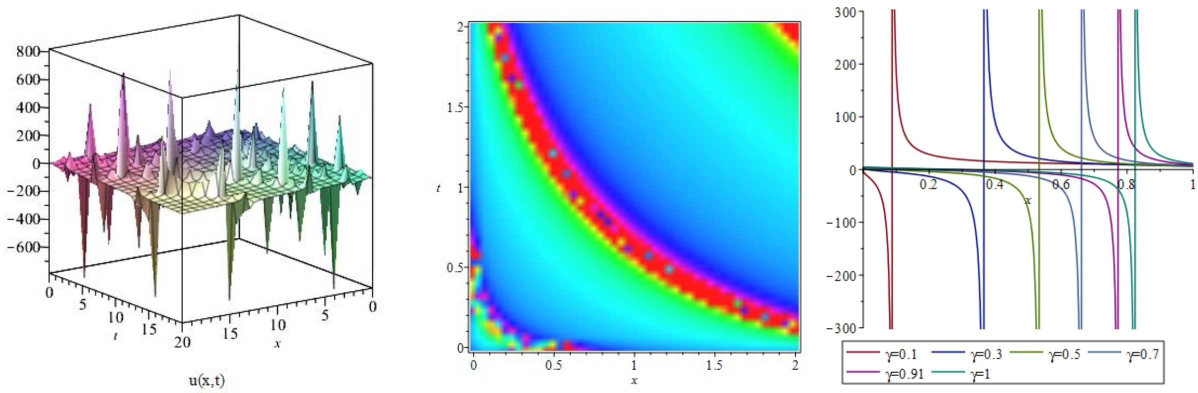

Figure 6. $plots\ of\ s\ olution\ (29)\ for \beta = 1, \alpha = 2, c = 1, \omega = 2, k = 2, \lambda$=1,y=0.
$(a)\,\gamma = 0.5 (b)\,\gamma = 0.5, (c) t = 0 for different value of \gamma.$

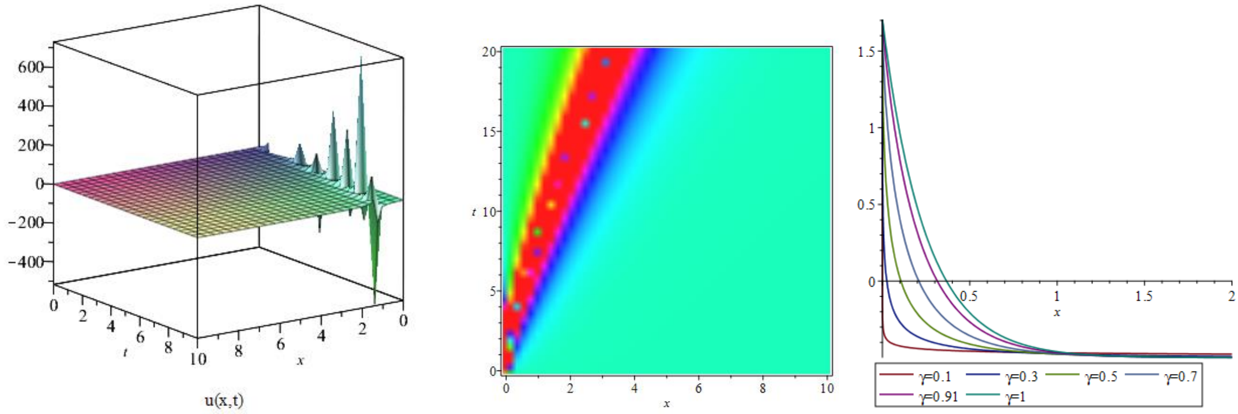

Figure 7. $plots\ of\ s\ olution\ (36)\ for \beta = 0, \alpha = -1, c = -1, \omega = 1, k = -2, \lambda$=1,y=0.
$(a)\,\gamma = 0.5 (b)\,\gamma = 0.5, (c) t = 0 for different value of \gamma.$

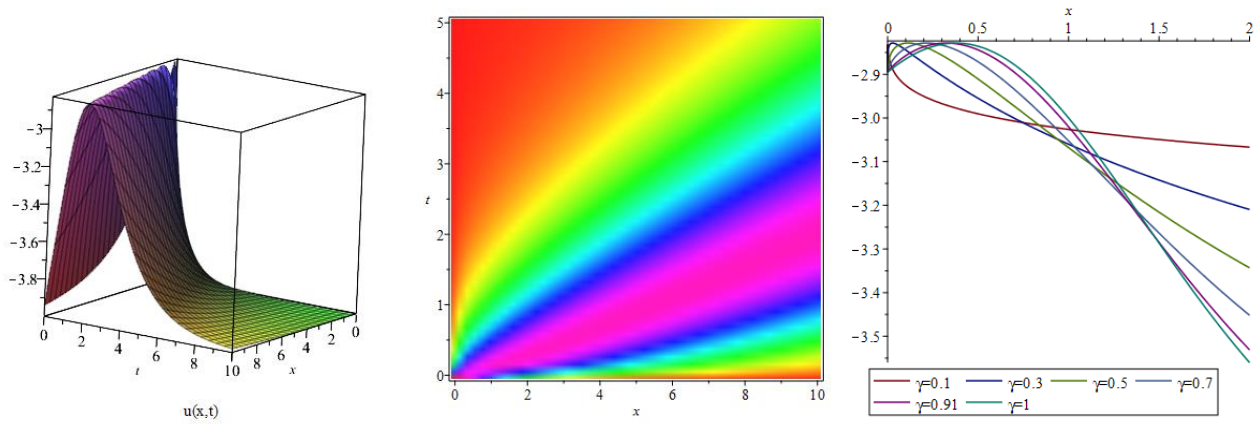

Figure 8. $plots\ of\ s\ olution\ (42)\ for \beta = 2, a_1 = 1, c = -1, \omega = -2, k = 1, \lambda$=1,y=0.
$(a)\,\gamma = 0.5 (b)\,\gamma = 0.5, (c) t = 0 for different value of \gamma.$

5. Conclusions

In the current paper, new extended of a novel method called coupled exp function method has been used to attain the analytical solutions of coupled space Time-Fractional Boussinesq-Burgers System and coupled Time-Fractional long System. As it was stated previously, implementing the method for solving systems of FPDE directly is an innovative attempt. The results indicate that the techniques are useful means to get the analytical solutions of FPDE system.

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