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Some results on 2-absorbing $R_\Gamma-$semimodules over $\Gamma-$semirings

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Abstract. The purpose of this paper is to introduce the notion of 2-absorbing $R_\Gamma-$semimodules over $\Gamma-$semirings, as a generalization of 2-absorbing semimodules over semirings and study various results related to them.

Keywords: k-ideal, $R_\Gamma-$semimodule, Strong $R_\Gamma-$semimodule, 2-absorbing $R_\Gamma-$semimodule, $\Gamma-$ semiring.
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1 Introduction

The notion of 2-absorbing ideals of commutative rings with non-zero identity, as a generalization of prime ideals was introduced by Badawi [1] in 2007. Darani [2] investigated these concepts in commutative semirings and characterized several results in terms of 2-absorbing and weakly 2-absorbing ideals in commutative semirings in 2012. The notion of 2-absorbing semimodules over a commutative semirings with non-zero identity was introduced by Dubey and Sarohe [3] in 2013, which is a generalization of prime semimodules and gave some characterizations related to them.

In 1995, Rao [10] introduced the concept of $\Gamma-$ semirings, as a generalization of semirings, $\Gamma-$ rings and ternary semirings. Dutta and Sardar [4] and Rao [10], studied the concept of ideals, k-ideals and prime ideals in $\Gamma-$ semirings. The notion of 2-absorbing ideals and 2-absorbing primary ideals of commutative $\Gamma-$ semirings were defined by Sangjaer and Pianskool [11] in 2019 and examine the various results of 2-absorbing primary ideals in commutative $\Gamma-$ semirings. The concept of $\Gamma-$ semimodules over $\Gamma-$ semirings was introduced by Sardar and Dasgupta [12] in 2004. Furthermore, Dutta and Dasgupta [5], Galindo and Petalcorin [7,8] studied the properties of $\Gamma-$ semimodules over $\Gamma-$ semirings and prove some results related to them. In this paper, the

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concept of 2-absorbing $R_\Gamma-$ semimodules over $\Gamma-$ semirings are introduced and study various results related to them.

2 Preliminaries

In this section, we recall some basic definitions of $\Gamma-$ semirings and $R_\Gamma-$ semimodules. R represents a $\Gamma-$ semiring throughout this paper.

Definition 1 ([10]). *Let R and Γ be two additive commutative semi-groups. Then R is called a $\Gamma-$ semiring if there exists a mapping $R \times \Gamma \times R \to R$ denoted by $x\alpha y$ for all $x, y \in R$ and $\alpha \in \Gamma$ satisfying the following conditions:*

1. $(x+y)\alpha z = x\alpha z + y\alpha z$.
2. $x(\alpha+\beta)z = x\alpha z + x\beta z$.
3. $x\alpha(y+z) = x\alpha y + x\alpha z$.
4. $(x\alpha y)\beta z = x\alpha(y\beta z)$ *for all* $x, y, z \in R$ *and* $\alpha, \beta \in \Gamma$.

R is a $\Gamma-$ semiring with zero if for all $x \in R$ and $\gamma \in \Gamma$ we have $0\gamma x = 0 = x\gamma 0$ and $x + 0 = x = 0 + x$. A $\Gamma-$ semiring R is said to have an identity element if for all $x \in R$ there exists $\alpha \in \Gamma$ such that $1\alpha x = x = x\alpha 1$ [13]. Let Z_0^+ be a commutative semigroup under addition, which represents the set of non-negative integers. The set $nZ_0^+ = \{nx | x \in Z_0^+\}$ is a commutative semigroup under the usual addition of integers for all $n \in \mathbb{N}$ [11].

Example 1. Let $R = Z_0^+$ be an additive commutative semigroup of positive integers and $\Gamma = 5Z_0^+$ be a commutative semigroup under the usual addition of integers. Then R is a $\Gamma-$ semiring with $(a, \gamma, b) \to (a\gamma b)$, where $a, b \in Z_0^+$ and $\gamma \in 5Z_0^+$.

Example 2. Let $R = (Z_6, +_6)$ be a commutative semigroup of addition modulo 6 and $\Gamma = (Z_3, +_3)$ be a commutative semigroup of addition modulo 3. Then R is a $\Gamma-$ semiring with $(r, \beta, s) \to (r\beta s)$, where $r, s \in R$ and $\beta \in \Gamma$.

Definition 2 ([10]). *A $\Gamma-$ semiring R is said to be commutative if $x\gamma y = y\gamma x$ for all $x, y \in R$ and for all $\gamma \in \Gamma$.*

Definition 3 ([6]). *A non empty subset I of R is said to be a left (right) ideal of R if I is a sub semi-group of $(R, +)$ and $x\alpha y \in I (y\alpha x \in I)$ for all $y \in I, x \in R$ and $\alpha \in \Gamma$. If I is both a left and right ideal of R, then I is known to be an ideal of R.*

Definition 4 ([6]). *An ideal I of a $\Gamma-$ semiring R is said to be k-ideal if for $x, y \in R, x + y \in I$ and $y \in I$ implies that $x \in I$.*

Definition 5 ([11]). *Let R be a commutative $\Gamma-$ semiring. A proper ideal J of R is said to be 2-absorbing ideal if whenever $x, y, z \in R$ and $\alpha, \beta \in \Gamma$ such that $x\alpha y\beta z \in J$ implies that either $x\alpha y \in J$ or $x\beta z \in J$ or $y\beta z \in J$.*

Example 3. By Example 1, R is a $\Gamma-$ semiring, where $R = Z_0^+$ and $\Gamma = 5Z_0^+$. Let $J = 6Z_0^+$ be an ideal of R. Since $1, 2, 3 \in R$ and $5 \in \Gamma$ such that $(1)(5)(2)(5)(3) \in J$, then $(2)(5)(3) \in J$. Hence, J is 2-absorbing ideal.

Definition 6 ([11])**.** *Let R be a $\Gamma-$ semiring and J be an ideal in R. Then $\sqrt{J} = \{x \in R|$ there exists $n \in N$ such that $(x\alpha)^{n-1}x \in J$ for all $\alpha \in \Gamma\}$ is an ideal in R containing J. The ideal $\sqrt{J}$ is called the radical ideal of J and is denoted by Rad(J).*

By [9, Theorem 3.9], $K = \sqrt{J}$ is a 2-absorbing ideal of R with $K\Gamma K \subseteq J \subseteq K$.

Definition 7 ([12])**.** *Let R be a $\Gamma-$ semiring. An additive commutative monoid M is said to be a left $R_\Gamma-$ semimodule if there exists a mapping $R \times \Gamma \times M \to M$ (images to be denoted by $r \in R, \alpha \in \Gamma, m \in M$) satisfying the following conditions:*

1. $r\alpha(m + n) = r\alpha m + r\alpha n$
2. $(r + s)\alpha m = r\alpha m + s\alpha m$
3. $r(\alpha + \beta)m = r\alpha m + r\beta m$
4. $r\alpha(s\beta m) = (r\alpha s)\beta m$
5. $0_R\alpha m = 0_M = r\alpha 0_M$ *for all* $r, s \in R, \alpha, \beta \in \Gamma$ *and* $m, n \in M$.

A right $R_\Gamma-$ semimodule is defined analogously.

Example 4. Every $\Gamma-$ semiring is an $R_\Gamma-$ semimodule. Let R be a $\Gamma-$ semiring and $M = R$. Define a mapping $R \times \Gamma \times R \to R$ with $(x, \alpha, y) \to x\alpha y$. Then R is an $R_\Gamma-$ semimodule.

Example 5. By Example 1, R is a $\Gamma-$ semiring, where $R = Z_0^+$ and $\Gamma = 5Z_0^+$. Let $M = 3Z_0^+$ be an additive commutative monoid. Then M is an $R_\Gamma-$ semimodule with $(r, \alpha, m) \to r\alpha m$, where $r \in R, \alpha \in \Gamma$ and $m \in M$.

Example 6. Let $R = Z_0^+$ and $\Gamma = 3Z_0^+$ be an additive commutative semi-group of positive integers. Then R is a $\Gamma-$ semiring. Let $M = Z_0^+ \oplus 2Z_0^+ = \{(r, z)|r \in Z_0^+, z \in 2Z_0^+\}$ be an additive commutative monoid. Define a mapping $R \times \Gamma \times M \to M$ with $(r, z) \oplus (r_1, z_1) = (r +_{Z_0^+} r_1, z +_{2Z_0^+} z_1)$ and $(r_2, \alpha, (r, z)) \to (r_2\alpha r, z)$, where $r_2 \in R, \alpha \in \Gamma$ and $(r, z), (r_1, z_1) \in M$. Then M is an $R_\Gamma-$ semimodule.

Definition 8 ([12])**.** *A left $R_\Gamma-$ semimodule M is called unity or unitary if there exists $1 \in R$ such that $1\alpha m = m$ for all $m \in M$ and $\alpha \in \Gamma$.*

Definition 9 ([12])**.** *A non empty subset N of a left $R_\Gamma-$ semimodule M is a left $R_\Gamma-$ subsemimodule M if and only if*

1. $x + y \in N$.
2. $r\alpha x \in N$ *for all* $x, y \in N, r \in R$ *and* $\alpha \in \Gamma$.

It is obvious that $0_M \in N$.

Definition 10 ([5]). *A proper $R_\Gamma-$ subsemimodule N of an $R_\Gamma-$ semimodule M is called k $-R_\Gamma-$ subsemimodule if $x, x+y \in N$ and $y \in M$ implies that $y \in N$.*

Definition 11 ([5]). *A proper $R_\Gamma-$ subsemimodule N of an $R_\Gamma-$ semimodule M is called strong $R_\Gamma-$ subsemimodule if for some $x, y \in M$ such that $x+y \in N$ implies that $x \in N$ and $y \in N$.*

Example 7. By Example 1, R is a $\Gamma-$ semiring, where $R = Z_0^+$ and $\Gamma = 5Z_0^+$. Let $M = Z_0^+ \times Z_0^+$ be an additive commutative monoid. Then M is an $R_\Gamma-$ semimodule. Consider $N = 2Z_0^+ \times 2Z_0^+$ be an $R_\Gamma-$ subsemimodule of an $R_\Gamma-$ semimodule M. Then N is a k-$R_\Gamma-$ subsemimodule of M, while N is not a strong $R_\Gamma-$ subsemimodule, since $(3,5)+(5,7) \in N$ but neither $(3,5) \in N$ nor $(5,7) \in N$.

All through here, R will signify with 0 and 1 as zero element and identity element except if in any case expressed.

3 2-absorbing $R_\Gamma-$ semimodules

In this section, we introduce and study the notion of 2-absorbing $R_\Gamma-$ semimodules and investigate the properties in commutative $\Gamma-$ semirings.

Definition 12 ([7]). *Let R be a $\Gamma-$ semiring and M be an $R_\Gamma-$ semimodule and N be a proper $R_\Gamma-$ subsemimodule of M. Then $(N : M) = \{r \in R : r\Gamma M \subseteq N\}$ is called associated ideal of N.*

Theorem 1. *Let M be an $R_\Gamma-$ semimodule and N be a proper k-$R_\Gamma-$ subsemimodule of M. Then $(N : M)$ is a k- ideal of R.*

Proof. Since the intersection of an arbitrary family of k- ideals of R is again k-ideals. Now, we show that $(N : M)$ is a k-ideal of R. Let $r \in R, m_1 \in (N : M)$ and $\alpha \in \Gamma$ with $r+m_1 \in (N : M)$. Then for some $m \in M$, we have $r\alpha m + m_1\alpha m \in N$ and $m_1\alpha m \in N$. Therefore, $r\alpha m \in N$, as N is k-$R_\Gamma-$ subsemimodule of M. So $r \in (N : M)$. Hence, $(N : M)$ is a k- ideal of R. □

Definition 13. *Let M be an $R_\Gamma-$ semimodule. A proper $R_\Gamma-$ subsemimodule N of M is called prime if $r \in R$, $x \in M$ and $\alpha \in \Gamma$ such that $r\alpha x \in N$, then either $r \in (N : M)$ or $x \in N$.*

Definition 14. *Let M be an $R_\Gamma-$ semimodule and N be a proper $R_\Gamma-$ subsemimodule of M. Then N is said to be 2-absorbing $R_\Gamma-$ subsemimodule of M, if whenever $r, s \in R$, $m \in M$ and $\alpha, \beta \in \Gamma$ such that $r\alpha s\beta m \in N$ implies that $r\alpha s \in (N : M)$ or $r\beta m \in N$ or $s\beta m \in N$.*

It is obvious that each prime $R_\Gamma-$ subsemimodule is a 2-absorbing $R_\Gamma-$ subsemimodule of M. The following example shows that the converse need not be true.

Example 8. (i). Let $R = Z_0^+$ and $\Gamma = 2Z_0^+$ be an additive commutative semi-group of positive integers and $M = R \times R$ be an additive commutative monoid. Then M is an $R_\Gamma-$ semimodule. Let $N = \{0\} \times 4Z_0^+$ be an $R_\Gamma-$ subsemimodule of M, then $(N : M) = \{0\}$. Since $1, 2 \in R$, $(0,1) \in M$ and $2 \in \Gamma$ such that $1.2.2.2.(0,1) \in N$ which gives $2.2.(0,1) \in N$. Hence, N is a 2-absorbing $R_\Gamma-$ subsemimodule of M. While, N is not a prime $R_\Gamma-$ subsemimodule of M, since $2.2.(0,1) \in N$ but neither $2 \in (N : M)$ nor $(0,1) \in N$.

(ii). By Example (i), M is an $R_\Gamma-$ semimodule. Let $N = \{0\} \times 6Z_0^+$ be an $R_\Gamma-$ subsemimodule of M, then the associated ideal of N is $\{0\}$. Consider $1, 3 \in R, (0,1) \in M$ and $2 \in \Gamma$ such that $1.2.3.2.(0,1) \in N$ which gives $3.2.(0,1) \in N$. Hence, N is a 2-absorbing $R_\Gamma-$ subsemimodule of M. But, N is not a prime $R_\Gamma-$ subsemimodule of M, since $3.2.(0,1) \in N$ but neither $3 \in (N : M)$ nor $(0,1) \in N$.

Theorem 2. *Let M be an $R_\Gamma-$ semimodule and N be a 2-absorbing k-$R_\Gamma-$ subsemimodule of M. Then $(N : M)$ is a 2-absorbing ideal of R.*

Proof. By Theorem 1, $(N : M)$ is a k-ideal of R. We now show that $(N : M)$ is a 2-absorbing ideal of R. Let $r\alpha s\beta t \in (N : M)$ for some $r, s, t \in R$ and $\alpha, \beta \in \Gamma$. Assume that $r\beta t \notin (N : M)$ or $s\beta t \notin (N : M)$, then $r\beta t\gamma m \notin N$ or $s\beta t\gamma n \notin N$ for some $m, n \in M \setminus N$ and $\beta, \gamma \in \Gamma$. Since $r\alpha s\beta t \in (N : M)$ then $r\alpha s\beta t\Gamma M \subseteq N$ implies that $r\alpha s\beta(t\gamma(m + n)) \in N$, which gives $r\alpha s \in (N : M)$ or $r\beta t\gamma(m + n) \in N$ or $s\beta t\gamma(m + n) \in N$. If $r\beta t\gamma(m + n) \in N$ and $r\beta t\gamma m \notin N$ then we have $r\beta t\gamma n \notin N$, as N is a k-$R_\Gamma-$ subsemimodule of M. Now as $r\alpha s\beta(t\gamma n) \in N$ and N is a 2-absorbing $R_\Gamma-$ subsemimodule of M, so either $r\alpha s \in (N : M)$ or $s\beta t\gamma n \in N$ or $r\beta t\gamma n \in N$. Consequently, $r\alpha s \in (N : M)$. If $s\beta t\gamma(m + n) \in N$ and $s\beta t\gamma n \notin N$ then we have $s\beta t\gamma m \notin N$. Since N is a 2-absorbing $R_\Gamma-$ subsemimodule of M and $r\alpha s\beta(t\gamma m) \in N$, either $r\alpha s \in (N : M)$ or $s\beta t\gamma m \in N$ or $r\beta t\gamma m \in N$. Thus, $r\alpha s \in (N : M)$. Hence, $(N : M)$ is a 2-absorbing ideal of R. □

Corollary 1. *Let R be a $\Gamma-$ semiring, M be an $R_\Gamma-$ semimodule and N be a 2-absorbing k-$R_\Gamma-$ subsemimodule of M. Then $K = \sqrt{N : M} = \{r \in R : r\alpha r \in (N : M), \alpha \in \Gamma\}$ is a 2-absorbing ideal of R with $K\Gamma K \subseteq (N : M) \subseteq K$.*

Proof. Let N be a 2-absorbing $R_\Gamma-$ subsemimodule of an $R_\Gamma-$ semimodule M. Then by Theorem 2, $(N : M)$ is a 2-absorbing ideal of R. By [9, Theorem 3.9], $\sqrt{(N : M)}$ is a 2-absorbing ideal of R. □

In general, the converse of Theorem 2 is not true.

Example 9. By Example 8 (i), M is an $R_\Gamma-$ semimodule, where $R = Z_0^+$, $\Gamma = 2Z_0^+$ and $M = R \times R$. Let $N = \{0\} \times 8Z_0^+$ be an $R_\Gamma-$ subsemimodule of M, then $(N : M) = \{0\}$, which is a 2-absorbing ideal of R. While N is not a 2-absorbing $R_\Gamma-$ subsemimodule of M, since $1, 2 \in R, (0,1) \in M$ and $2 \in \Gamma$ such that $(2)(2)(1)(2)(0,1) \in N$, neither $(2)(2)(1) \in (N : M)$ nor $(2)(2)(0,1) \in N$ nor $(1)(2)(0,1) \in N$.

The converse of the Theorem 2 is true in the case of cyclic $R_\Gamma-$ semimodules.

Definition 15 ([7])**.** *An $R_\Gamma-$ semimodule M is called cyclic $R_\Gamma-$ semimodule if M can be generated by a single element, that is, $M = (m) = R\Gamma m = \{r\alpha m |\ r \in R, \alpha \in \Gamma\}$ for some $m \in M$.*

Theorem 3. *Let N be a 2-absorbing $R_\Gamma-$ subsemimodule of a cyclic $R_\Gamma-$ semimodule M. Then $(N : M)$ is a 2-absorbing ideal of R if and only if N is a 2-absorbing $R_\Gamma-$ subsemimodule of M.*

Proof. Let $(N:M)$ be a 2-absorbing ideal of R and $M = R\Gamma m$, for some $m \in M$. Suppose that $r\alpha s\beta m_1 \in N$ for $r, s \in R, m_1 \in M$ and $\alpha, \beta \in \Gamma$. Then there exists $t \in R$ and $\gamma \in \Gamma$ such that $m_1 = t\gamma m$, and we have $r\alpha s\beta t\gamma m \in N$. Thus, $r\alpha s\beta t \in (N:m) = (N:M)$, gives either $r\alpha s \in (N:M)$ or $r\beta t \in (N:M)$ or $s\beta t \in (N:M)$, as $(N:M)$ be a 2-absorbing ideal of R. Therefore, $r\alpha s \in (N:M)$ or $r\beta x \in N$ or $s\beta x \in N$. Hence, N is a 2-absorbing $R_\Gamma-$ subsemimodule of M. The converse is true by Theorem 2. □

Theorem 4. *Let M be an $R_\Gamma-$ semimodule and N be a 2-absorbing k-$R_\Gamma-$ subsemimodule of M such that $\sqrt{(N:M)} = J$. If $(N:M) \neq J$, then for all $r \in J \setminus (N:M)$ by $N_r = \{m \in M : r\alpha m \in N, \alpha \in \Gamma\}$ is a prime $R_\Gamma-$ subsemimodule of M containing N with $K \subsetneq (N_r : M)$.*

Proof. Let $s \in R \setminus (N_r : M)$, $m \in M$ and $\beta \in \Gamma$ such that $s\beta m \in N_r$. Then $r\alpha s\beta m \in N$, so either $r\alpha s \in (N:M)$ or $r\beta m \in N$ or $s\beta m \in N$, as N is a 2-absorbing $R_\Gamma-$ subsemimodule of M. If $r\alpha s \in (N:M)$, then $s \in (N_r : M)$, which is a contradiction. By definition of N_r, if $r\beta m \in N$ then $m \in N_r$, then there is nothing to prove. If $s\beta m \in N$ and $r\gamma r \in K\Gamma K \subseteq (N:M), \gamma \in \Gamma$, then for some $m \in M$ and $\beta \in \Gamma$ such that $r\beta m \in N_r$. Now, $(r+s)\beta m \in N_r$ implies that $r\gamma(r+s)\beta m \in N$, thus we have either $r\beta m \in N$ or $(r+s)\beta m \in N$ or $r\gamma(r+s) \in (N:M)$, as N is a 2-absorbing $R_\Gamma-$ subsemimodule of M. Moreover, if $(r+s)\beta m \in N$ and $s\beta m \in N$, then $r\beta m \in N$ implies that $m \in N_r$, as N is a k-ideal. If $r\gamma(r+s) \in (N:M)$ and $r\gamma r \in K\Gamma K \subseteq (N:M)$, which gives $r\gamma s \in (N:M)$ implies that $s \in (N_r : M)$, a contradiction. Hence, N_r is a prime $R_\Gamma-$ subsemimodule of M. □

Theorem 5. *Let M be an $R_\Gamma-$ semimodule and N be a k-$R_\Gamma-$ subsemimodule of M, then the following statements hold:*

1. *N_r is a k-$R_\Gamma-$ subsemimodule of M.*
2. *$(N_r : M)$ is a k-ideal of R.*

Proof.

1. Let $m, (m+n) \in N_r$ and $n \in M$. Then $r\alpha m, (r\alpha m + r\alpha n) \in N$, $\alpha \in \Gamma$. So $r\alpha n \in N$ implies that $n \in N_r$, as N is a k-$R_\Gamma-$ subsemimodule of M. Thus, N_r is a k-$R_\Gamma-$ subsemimodule of M.
2. By (1), N_r is a k-$R_\Gamma-$ subsemimodule of M, then by Theorem 1, $(N_r : M)$ is a k-ideal of R.

□

Theorem 6. *Let M be an $R_\Gamma-$ semimodule and N be a 2-absorbing $R_\Gamma-$ subsemimodule of M. If N_1 is an $R_\Gamma-$ subsemimodule of M, then $N_1 \cap N$ is a 2-absorbing $R_\Gamma-$ subsemimodule of N_1.*

Proof. Since N is a proper $R_\Gamma-$ subsemimodule of M, then $N_1 \cap N$ is a proper $R_\Gamma-$ subsemimodule of N_1. Assume that $r\alpha s\beta n_1 \in N_1 \cap N$ for some $r, s \in R, n_1 \in N_1$ and $\alpha, \beta \in \Gamma$. Since N is a 2-absorbing $R_\Gamma-$ subsemimodule of M and $r\alpha s\beta n_1 \in N$, either $r\alpha s \in (N:M)$ or $r\beta n_1 \in N$ or $s\beta n_1 \in N$. If $r\beta n_1 \in N$ or $s\beta n_1 \in N$, then $r\beta n_1 \in N_1 \cap N$ or $s\beta n_1 \in N_1 \cap N$. In case $r\alpha s \in (N:M)$, then $r\alpha s\Gamma M \subseteq N$. Especially, $r\alpha s\Gamma N_1 \subseteq N$ implies that $r\alpha s\Gamma N_1 \subseteq N_1 \cap N$. Therefore, $r\alpha s \in (N_1 \cap N : N_1)$. Hence, $N_1 \cap N$ is a 2-absorbing $R_\Gamma-$ subsemimodule of N_1. □

Theorem 7. *Let M be an $R_\Gamma-$ semimodule and N be intersection of two prime $R_\Gamma-$ subsemimodules of M. Then N is a 2-absorbing $R_\Gamma-$ subsemimodule.*

Proof. Assume that N_1 and N_2 be two prime $R_\Gamma-$ subsemimodules of M. Now we have to show that $N_1 \cap N_2$ is a 2-absorbing $R_\Gamma-$ subsemimodule of M. Let $r, s \in R,\ m \in M$ and $\alpha, \beta \in \Gamma$ such that $r\alpha s\beta m \in N_1 \cap N_2$. Then $r\alpha s\beta m \in N_1$ and $r\alpha s\beta m \in N_2$. Since N_1 is a prime $R_\Gamma-$ subsemimodules of M and $r\alpha s\beta m \in N_1$ implies $r \in (N_1 : M)$ or $s \in (N_1 : M)$ or $m \in N_1$. Also, $r\alpha s\beta m \in N_2$ implies $r \in (N_2 : M)$ or $s \in (N_2 : M)$ or $m \in N_2$. If $r \in (N_1 : M)$ then $r\Gamma M \subseteq N_1$. In a similar way, if $r \in (N_2 : M)$, then $r\Gamma M \subseteq N_2$. Thus $r\Gamma M \subseteq N_1 \cap N_2$ which infers that $r \in (N_1 \cap N_2 : M)$. Similarly, $s \in (N_1 \cap N_2 : M)$. Hence $r\alpha s \in (N_1 \cap N_2 : M)$. Further, if $r \in (N_1 : M)$ and $m \in N_2$, then $r\beta m \in N_1 \cap N_2$. In a similar way, we can prove other cases. Hence, $N_1 \cap N_2$ is a 2-absorbing $R_\Gamma-$ subsemimodule of M. □

While the intersection of two distinct nonzero 2-absorbing $R_\Gamma-$ subsemimodules need not be a 2-absorbing $R_\Gamma-$ subsemimodule of M.

Example 10. By Example 8, $N_1 = \{0\} \times 4Z_0^+$ and $N_2 = \{0\} \times 6Z_0^+$ are 2-absorbing $R_\Gamma-$ subsemimodules of M. Then $N = N_1 \cap N_2 = (\{0\} \times 4Z_0^+) \cap (\{0\} \times 6Z_0^+) = \{0\} \times 12Z_0^+$. Then $(N : M) = \{0\}$. Let $1, 3 \in R,\ (0, 1) \in M$ and $2 \in \Gamma$ such that $1.2.3.2.(0, 1) \in N$ which gives neither $1.2.3 \in (N : M)$ nor $3.2.(0, 1) \in N$ nor $1.2.(0, 1) \in N$. Hence, N is not a 2-absorbing $R_\Gamma-$ subsemimodule of M.

Definition 16. *An $R_\Gamma-$ semimodule M is said to be a multiplication $R_\Gamma-$ semimodule if for each $R_\Gamma-$ subsemimodules N of M is of the form $N = J\Gamma M$, where J an ideal of R.*

Definition 17 ([13])**.** *An element x of a $\Gamma-$ semiring R is said to be a multiplicative $\Gamma-$ idempotent if there exists $\gamma \in \Gamma$ such that $x = x\gamma x$. If every element of R is a multiplicative $\Gamma-$ idempotent, then R is called a multiplicative $\Gamma-$ idempotent $\Gamma-$ semiring.*

Example 11. Let R be a multiplicatively $\Gamma-$ idempotent $\Gamma-$ semiring. Then every ideal of R is a multiplication $R_\Gamma-$ semimodule. Assume that K is an ideal of R and $J \subseteq K$. If $a \in J$ and $\alpha \in \Gamma$, then we have $a = a\alpha a \in J\Gamma K$. Thus, $J = J\Gamma K$. Hence, K is a multiplication $R_\Gamma-$ semimodule.

Theorem 8. *Every cyclic $R_\Gamma-$ semimodule is a multiplication $R_\Gamma-$ semimodule.*

Proof. Let $M(= R\Gamma m)$ be a cyclic $R_\Gamma-$ semimodule and N be an $R_\Gamma-$ subsemimodule of M, then $N = R\Gamma n$ where $n = r\alpha m$ for some $r \in R$ and $\alpha \in \Gamma$. Since $N = R\Gamma n = R\Gamma(r\alpha m) = (R\Gamma r)\alpha m = (R\Gamma r)\Gamma(R\Gamma m) = J\Gamma M$ for $J = R\Gamma r$. Hence, M is a multiplication $R_\Gamma-$ semimodule. □

Theorem 9. *Let N be an $R_\Gamma-$ subsemimodule of a cyclic $R_\Gamma-$ semimodule M. Then N is a 2-absorbing $R_\Gamma-$ subsemimodule of M if and only if $X\Gamma Y\Gamma Z \subseteq N$ implies $X\Gamma Y \subseteq N$ or $Y\Gamma Z \subseteq N$ or $X\Gamma Z \subseteq N$ for some $R_\Gamma-$ subsemimodules X, Y and Z of M.*

Proof. Let N be a 2-absorbing $R_\Gamma-$ subsemimodule of M. Assume that for some $R_\Gamma-$ subsemimodules X, Y and Z of M such that $X\Gamma Y\Gamma Z \subseteq N$. Since M is cyclic, then by Theorem 8, M is a multiplication $R_\Gamma-$ semimodule. Thus, $X = J\Gamma M$, $Y = K\Gamma M$ and $Z = L\Gamma M$ for some ideals J, K and L of R. Then, $X\Gamma Y\Gamma Z = (J\Gamma K\Gamma L)\Gamma M \subseteq N$ implies that $J\Gamma K\Gamma L \subseteq (N : M)$. As N is a 2-absorbing $R_\Gamma -$ subsemimodule of M, then by Theorem 3, $(N : M)$ is a 2-absorbing ideal of R. So either $J\Gamma K \subseteq (N : M)$ or $K\Gamma L \subseteq (N : M)$ or $J\Gamma L \subseteq (N : M)$. Thus, $J\Gamma K\Gamma M \subseteq N$ or $K\Gamma L\Gamma M \subseteq N$ or $J\Gamma L\Gamma M \subseteq N$ which implies that $(J\Gamma M)\Gamma(K\Gamma M) \subseteq N$ or $(K\Gamma M)\Gamma(L\Gamma M) \subseteq N$ or $(J\Gamma M)\Gamma(L\Gamma M) \subseteq N$, Hence, either $X\Gamma Y \subseteq N$ or $Y\Gamma Z \subseteq N$ or $X\Gamma Z \subseteq N$. Conversely, assume that for some ideals J, K and L of R such that $J\Gamma K\Gamma L \subseteq (N : M)$. Then $J\Gamma K\Gamma L\Gamma M \subseteq N$. As M is a cyclic, then by Theorem 8, M is a multiplication $R_\Gamma -$ semimodule. So $(J\Gamma K\Gamma L)\Gamma M \subseteq N$ implies $(J\Gamma M)\Gamma(K\Gamma M)\Gamma(L\Gamma M) \subseteq N$. Thus, either $(J\Gamma M)\Gamma(K\Gamma M) \subseteq N$ or $(K\Gamma M)\Gamma(L\Gamma M) \subseteq N$ or $(J\Gamma M)\Gamma(L\Gamma M) \subseteq N$. Consequently, $J\Gamma K \subseteq (N : M)$ or $K\Gamma L \subseteq (N : M)$ or $J\Gamma L \subseteq (N : M)$. Thus, $(N : M)$ is a 2-absorbing ideal of R. Therefore, by Theorem 3, N is a 2-absorbing $R_\Gamma -$ subsemimodule of M. □

Theorem 10. *Let N be an $R_\Gamma -$ subsemimodule of a cyclic $R_\Gamma -$ semimodule M. Then the following statements are equivalent:*

1. *N is a 2-absorbing $R_\Gamma -$ subsemimodule.*
2. *$(N : M)$ is a 2-absorbing ideal of R.*
3. *$N = P\Gamma M$, where P is a 2-absorbing ideal of R. If $J\Gamma M \subseteq N$ with $J \subseteq P$, then P is maximal.*

Proof. (1) ⇒ (2). By Theorem 3, $(N : M)$ is a 2-absorbing ideal of R.
(2) ⇒ (3). By Theorem 8, M is a multiplicative $R_\Gamma -$ semimodule. Then $N = P\Gamma M$ for some ideal P of R. Thus $P = (N : M)$, which is a 2-absorbing ideal of R, by (ii). Assume that $J\Gamma M \subseteq N$ for some ideal J of R such that $J \subseteq (N : M) = P$. Thus, P is maximal, if $J\Gamma M \subseteq N$, then $J \subseteq P$.
(3) ⇒ (1). Assume that for some ideals J, K and L of R such that $J\Gamma K\Gamma L \subseteq (N : M)$. Then $J\Gamma K\Gamma L\Gamma M \subseteq N$. As M is a cyclic, then by Theorem 8, M is a multiplicative $R_\Gamma -$ semimodule. Since P is a 2-absorbing ideal of R, so $(J\Gamma K\Gamma L)\Gamma M \subseteq N = P\Gamma M$. Then $J\Gamma K\Gamma L \subseteq P$, as P is maximal ideal. Hence, $J \subseteq P$. As P is a 2-absorbing ideal of R so, $J\Gamma K \subseteq P$ or $K\Gamma L \subseteq P$ or $J\Gamma L \subseteq P$ implies that $J\Gamma K\Gamma M \subseteq P\Gamma M = N$ or $K\Gamma L\Gamma M \subseteq P\Gamma M = N$ or $J\Gamma L\Gamma M \subseteq P\Gamma M = N$. Hence, $J\Gamma K \subseteq (N : M)$ or $K\Gamma L \subseteq (N : M)$ or $J\Gamma L \subseteq (N : M)$. Thus, $(N : M)$ is a 2-absorbing ideal of R. By Theorem 3, N is a 2-absorbing $R_\Gamma -$ semimodule of M. □

Definition 18. *An $R_\Gamma -$ subsemimodule N of an $R_\Gamma -$ semimodule M is called pure if $r\Gamma N = N \cap r\Gamma M$ for all $r \in R$.*

Definition 19. *An $R_\Gamma -$ semimodule M is called $M_\Gamma -$ cancellative $R_\Gamma -$ semimodule if $m, n \in M, r \in R$ and $\alpha \in \Gamma$ such that $r\alpha m = r\alpha n$, then $m = n$.*

Theorem 11. *Let N be a proper $R_\Gamma -$ subsemimodule of $M_\Gamma -$ cancellative $R_\Gamma -$ semimodule M. Then N is a 2-absorbing $R_\Gamma -$ subsemimodule of M with $(N : M) = \{0\}$ if and only if N is a pure $R_\Gamma -$ subsemimodule of M.*

Proof. Let N be a 2-absorbing $R_\Gamma-$ subsemimodule of M. Assume that for some $r, s \in R, m_1 \in M$ and $\alpha, \beta \in \Gamma$ such that $r\alpha s\beta m_1 \in r\alpha s\Gamma M \cap N$ and $r\alpha s \neq 0$. As N is a 2-absorbing $R_\Gamma-$ subsemimodule of M so either $r\beta m_1 \in N$ or $s\beta m_1 \in N$. If $s\beta m_1 \in N$, then $r\alpha s\beta m_1 \in r\alpha s\Gamma N$ for some $r \in R$. Thus $r\alpha s\Gamma M \cap N \subseteq r\alpha s\Gamma N$. In a similar way, we may prove the other cases $r\beta m_1 \in N$. The converse is clear. So $r\alpha s\Gamma M \cap N = r\alpha s\Gamma N$. Hence, N is a pure $R_\Gamma-$ subsemimodule of M.

Conversely, let N be a pure $R_\Gamma-$ subsemimodule of M. Assume that for some $r, s \in R, m \in M$ and $\alpha, \beta \in \Gamma$ with $r\alpha s\beta m \in N$ such that $r\alpha s \notin (N : M)$. Then $r\alpha s\beta m \in r\alpha s\Gamma M \cap N = r\alpha s\Gamma N$. Therefore, $r\alpha s\beta m = r\alpha s\beta n$ for some $n \in N$. Since M is a $M_\Gamma-$ cancellative $R_\Gamma-$ semimodule, then $s\beta m = s\beta n \in N$. Consequently, N is a 2-absorbing $R_\Gamma-$ subsemimodule of M. Moreover, assume that $r \in (N : M)$ with $r \neq 0$. Since $N \neq M$, then $r\alpha u \in r\Gamma M \cap M = r\Gamma N$ for some $u \in M \setminus N$, so $r\alpha u = r\alpha v$ for some $v \in N$. Therefore $u = v$, a contradiction. Hence, $(N : M) = \{0\}$. □

Definition 20. *Let M and M_1 be two $R_\Gamma-$ semimodules. A mapping $f : M \to M_1$ is an $R_\Gamma-$ homomorphism if $m, n \in M, r \in R$ and $\alpha \in \Gamma$ then*

1. $f(m + n) = f(m) + f(n)$
2. $f(r\alpha m) = r\alpha f(m)$.

An $R_\Gamma-$ homomorphism f is an $R_\Gamma-$ epimorphism if f is surjective.

Theorem 12. *Let $f : M \to M_1$ be an $R_\Gamma-$ epimorphism of $R_\Gamma-$ semimodules M and M_1. If N is a 2-absorbing $R_\Gamma-$ subsemimodule of M_1, then $f^{-1}(N)$ is a 2-absorbing $R_\Gamma-$ subsemimodule of M.*

Proof. Let $r\alpha s\beta m \in f^{-1}(N)$ for some $r, s \in R, m \in M$ and $\alpha, \beta \in \Gamma$. Then $f(r\alpha s\beta m) = r\alpha s\beta f(m) \in N$. So $r\alpha s \in (N : M_1)$ or $r\beta f(m) \in N$ or $s\beta f(m) \in N$, as N is a 2-absorbing $R_\Gamma-$ subsemimodule of M_1. If $r\alpha s \in (N : M_1)$ then $r\alpha s\Gamma M_1 \subseteq N$ implies that $r\alpha s\Gamma M = r\alpha s\Gamma f^{-1}(M_1) = f^{-1}(r\alpha s\Gamma M_1) \subseteq f^{-1}(N)$. Thus, $r\alpha s \in (f^{-1}(N) : M)$. If $r\beta f(m) \in N$ then $f(r\beta m) \in N$ implies that $r\beta m \in f^{-1}(N)$. Likewise, if $s\beta f(m) \in N$ then $f(s\beta m) \in N$ implies that $s\beta m \in f^{-1}(N)$. Consequently, $r\alpha s \in (f^{-1}(N) : M)$ or $r\beta m \in f^{-1}(N)$ or $s\beta m \in f^{-1}(N)$. Hence, $f^{-1}(N)$ is a 2-absorbing $R_\Gamma-$ subsemimodule of M. □

If f is not an $R_\Gamma-$ epimorphism, then $f^{-1}(N)$ is not a proper $R_\Gamma-$ subsemimodule of M.

Example 12. By Example 2, R is a $\Gamma-$ semiring, where $R = (Z_6, +_6)$ and $\Gamma = (Z_3, +_3)$. Let $M = (Z_6, +_6)$ be additive commutative monoid of addition modulo 6. Then M is an $R_\Gamma-$ semimodule over R. Now, $f : (Z_6, +_6) \to (Z_6, +_6)$ with $f(m) = 2m$ for all $m \in M$ is an $R_\Gamma-$ homomorphism, while it is not an $R_\Gamma-$ epimorphism. Let $N = \{0, 2, 4\}$ then $f^{-1}(N) = M$.

Theorem 13. *Let $f : M \to M_1$ be an $R_\Gamma-$ epimorphism of $R_\Gamma-$ semimodules M and M_1 with $f(0) = 0$ and N be a strong k-$R_\Gamma-$ subsemimodule of M. If N is a 2-absorbing $R_\Gamma-$ subsemimodule of M with $ker(f) \subseteq N$, then f(N) is a 2-absorbing $R_\Gamma-$ subsemimodule of M_1.*

Proof. Let $r, s \in R, m_1 \in M_1$ and $\alpha, \beta \in \Gamma$ such that $r\alpha s\beta m_1 \in f(N)$, then $r\alpha s\beta m_1 = f(n)$ for some $n \in N$. As f is an $R_\Gamma -$ epimorphism and $m_1 \in M_1$, then $f(m) = m_1$ for some $m \in M$. Also, N is a strong $R_\Gamma -$ subsemimodule of M and $n \in N$, then $n + n_1 = 0$ for some $n_1 \in N$. Thus $f(n + n_1) = 0$. Therefore, $r\alpha s\beta m_1 + f(n_1) = f(r\alpha s\beta m) + f(n_1) = f(r\alpha s\beta m + n_1) = 0$ which gives $r\alpha s\beta m + n_1 \in ker(f) \subseteq N$. So $r\alpha s\beta m \in N$, as N is a k- $R_\Gamma -$ subsemimodule of M. Since N is a 2-absorbing $R_\Gamma -$ subsemimodule, we have $r\alpha s \in (N : M)$ or $r\beta m \in N$ or $s\beta m \in N$. If $r\alpha s \in (N : M)$, then $r\alpha s\Gamma M \subseteq N$ implies that $f(r\alpha s\Gamma M) = r\alpha s\Gamma f(m) = r\alpha s\Gamma M_1 \subseteq f(N)$. Therefore $r\alpha s \in (f(N) : M_1)$. If $r\beta m \in N$, then $f(r\beta m) = r\beta f(m) = r\beta m_1 \in f(N)$. If $s\beta m \in N$, then $s\beta m_1 \in f(N)$. Thus $r\alpha s \in (f(N) : M_1)$ or $r\beta m_1 \in f(N)$ or $s\beta m_1 \in f(N)$. Hence, $f(N)$ is a 2-absorbing $R_\Gamma -$ subsemimodule of M_1. □

Theorem 14. *Let M be an $R_\Gamma -$ semimodule, N and N_1 be $R_\Gamma -$ subsemimodules of M. If N is a 2-absorbing k-$R_\Gamma -$ subsemimodule of an $R_\Gamma -$ semimodule M and $r\alpha s\Gamma N_1 \subseteq N$ for some $r, s \in R$ and $\alpha, \beta \in \Gamma$ then $r\alpha s \in (N : M)$ or $r\Gamma N_1 \subseteq N$ or $s\Gamma N_1 \subseteq N$.*

Proof. Assume that $r\alpha s \notin (N : M)$ or $r\Gamma N_1 \nsubseteq N$ or $s\Gamma N_1 \nsubseteq N$ for some $\alpha \in \Gamma$. Then $r\beta n' \notin N$ and $s\beta n'' \notin N$ for some $n', n'' \in N_1$ and $\beta \in \Gamma$. Since N is a 2-absorbing $R_\Gamma -$ subsemimodule of M and $r\alpha s\beta n' \in N$, $r\alpha s\beta n'' \in N$. We have $s\beta n' \in N$ and $r\beta n'' \in N$. Now as $r\alpha s\beta(n' + n'') \in N$ and $r\alpha s \notin (N : M)$, then either $r\beta(n' + n'') \in N$ or $s\beta(n' + n'') \in N$, since N is a 2-absorbing $R_\Gamma -$ subsemimodule of M. If $r\beta(n' + n'') \in N$ and $r\beta n'' \in N$. Then $r\beta n' \in N$, as N is a k-ideal, which is a contradiction. Similarly, if $s\beta(n' + n'') \in N$, as N is a k- ideal and $s\beta n' \in N$, we get $s\beta n'' \in N$, which is a contradiction. Hence, $r\alpha s \in (N : M)$ or $r\Gamma N_1 \subseteq N$ or $s\Gamma N_1 \subseteq N$. □

Theorem 15. *Let M be an $R_\Gamma -$ semimodule, N be a k-$R_\Gamma -$ subsemimodule of M and $(N : M)$ be a k-ideal of R. If N is a 2-absorbing $R_\Gamma -$ subsemimodule of M, then $J\Gamma K\Gamma L \subseteq N$ for some ideals J,K of R and an $R_\Gamma -$ subsemimodule L of M, such that $J\Gamma K \subseteq (N : M)$ or $J\Gamma L \subseteq N$ or $K\Gamma L \subseteq N$.*

Proof. Let N be a 2-absorbing $R_\Gamma -$ subsemimodule of M. Assume that for some ideals J, K of R and an $R_\Gamma -$ subsemimodule L of M such that $J\Gamma K\Gamma L \subseteq N$ and $J\Gamma K \nsubseteq (N : M)$. Suppose that $J\Gamma L \nsubseteq N$ or $K\Gamma L \nsubseteq N$. Then $j\Gamma L \nsubseteq N$ and $k\Gamma L \nsubseteq N$ for some $j \in J, k \in K$ and $\alpha \in \Gamma$. Since $j\alpha k\Gamma L \subseteq N$ and $j\Gamma L \nsubseteq N$ or $k\Gamma L \nsubseteq N$, then by Theorem 14, we have $j\alpha k \in (N : M)$. While $J\Gamma K \nsubseteq (N : M)$, then $j_1\alpha k_1 \notin (N : M)$ for some $j_1 \in J, k_1 \in K$ and $\alpha \in \Gamma$. Also, $j_1\alpha k_1\Gamma L \subseteq N$ and $j_1\alpha k_1 \notin (N : M)$, then by Theorem 14, we have $j_1\Gamma L \subseteq N$ or $k_1\Gamma L \subseteq N$. Here three cases arises:

Case 1: Suppose that $j_1\Gamma L \subseteq N$ and $k_1\Gamma L \nsubseteq N$. Since $j\alpha k_1\Gamma L \subseteq N$, while $k_1\Gamma L \nsubseteq N$ and $j\Gamma L \nsubseteq N$, then by Theorem 14, we have $j\alpha k_1 \in (N : M)$. Since N is k-ideal, $j_1\Gamma L \subseteq N$ and $j\Gamma L \nsubseteq N$ then $(j + j_1)\Gamma L \nsubseteq N$. However, $(j + j_1)\alpha k_1\Gamma L \subseteq N$ and $k_1\Gamma L \subseteq N$ and $(j+j_1)\Gamma L \nsubseteq N$, then by Theorem 14, we get $(j+j_1)\alpha k_1 \in (N : M)$. Since $(j+j_1)\alpha k_1 \in (N : M)$ and $j\alpha k_1 \in (N : M)$, then $j_1\alpha k_1 \in (N : M)$, which is a contradiction.

Case 2: Assume that $j_1\Gamma L \nsubseteq N$ and $k_1\Gamma L \subseteq N$. Since $j_1\alpha k\Gamma L \subseteq N$, while $j_1\Gamma L \nsubseteq N$ and $k\Gamma L \nsubseteq N$, then by Theorem 14, we have $j_1\alpha k \in (N : M)$. Since N is a k- ideal, $k_1\Gamma L \subseteq N$ and $k\Gamma L \nsubseteq N$ then $(k+k_1)\Gamma L \nsubseteq N$. Now as, $j_1\alpha(k+k_1)\Gamma L \subseteq N$ and $j_1\Gamma L \nsubseteq N$ and $(k+k_1)\Gamma L \nsubseteq N$, then by Theorem 14, we have $j_1\alpha(k + k_1) \in (N : M)$. Since $j_1\alpha(k + k_1) \in (N : M)$ and $j_1\alpha k \in (N : M)$, we have $j_1\alpha k_1 \in (N : M)$, as (N : M) is k-ideal. Which is a contradiction.

Case 3: Suppose that $j_1\Gamma L \subseteq N$ and $k_1\Gamma L \subseteq N$. Since $k_1\Gamma L \subseteq N$ and $k\Gamma L \nsubseteq N$, and N is k-$R_\Gamma-$ subsemimodule of M, then we have $(k+k_1)\Gamma L \nsubseteq N$. Since $j\alpha(k+k_1)\Gamma L \subseteq N$ while $j\Gamma L \nsubseteq N$ and $(k+k_1)\Gamma L \nsubseteq N$, we have $j\alpha(k+k_1) \in (N:M)$. As $j\alpha k \in (N:M)$ and $j\alpha(k+k_1) \in (N:M)$, we have $j\alpha k_1 \in (N:M)$, as $(N:M)$ is k-ideal. Moreover, $j_1\Gamma L \subseteq N$ and $j\Gamma L \nsubseteq N$, so $(j+j_1)\Gamma L \nsubseteq N$. Also, $(j+j_1)\alpha k\Gamma L \subseteq N$ and neither $(j+j_1)\Gamma L \subseteq N$ nor $k\Gamma L \subseteq N$, then we can deduce that $(j+j_1)\alpha k \in (N:M)$ and $j\alpha k \in (N:M)$ implies that $j_1\alpha k \in (N:M)$. Since $(j+j_1)\alpha(k+k_1)\Gamma L \subseteq N$ while neither $(j+j_1)\Gamma L \subseteq N$ nor $(k+k_1)\Gamma L \subseteq N$, then by Theorem 14, we have $(j+j_1)\alpha(k+k_1) \in (N:M)$. Since $j_1\alpha k, j\alpha k_1, j\alpha k \in (N:M)$, then we have $j_1\alpha k_1 \in (N:M)$, as $(N:M)$ is k-ideal. Which is a contradiction. Hence, $J\Gamma L \subseteq N$ or $K\Gamma L \subseteq N$. Therefore, $J\Gamma K \subseteq (N:M)$ or $J\Gamma L \subseteq N$ or $K\Gamma L \subseteq N$. □

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