Journal of Algebra and Related Topics
Vol. 13, No. 02, 2025, pp. 137-148. Research Article

Extensions of soft fractional ideals using soft semistar operations approach on integral domains

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Abstract. A comprehensive mathematical technique for handling uncertainty is the Molodtsov introduced idea of soft sets. In this paper, the operations leading to themselves from the set of undeniable soft fractional ideals are instigated. We provide some extensions of soft fractional ideals using the notion of overrings. We bring out the notion of soft semistar operations in relation to undeniable soft fractional ideals and connect it to the current notions of star and semistar operations. We also demonstrate the formation of a complete soft lattice from the collection of all soft semistar operations on integral domains.

Keywords: Soft R-submodule, Star operation, Semistar operation, Soft fractional ideal, Overring.
AMS Subject Classification 2010: 13A15, 13G05, 16D25, 03E75.

1 Introduction

As a crucial supplement to several existing mathematical techniques for addressing uncertainty. Molodtsov [12] introduced the concept of soft set theory through a set-valued mapping. Maji et al. [11] instigated a conceptual research of the soft set theory in depth, which includes superset and subset of a soft set, operations of union and intersection, null soft set etc, and discussed their properties. Jun and Park [9] and Jun [8] instigated various paths in connection with soft sets applications in the ideal theory of soft bck/bci algebras. Many authors explored a few procedures on the soft set theory as well. Aktas and Çağman [3] compare the soft set concepts and soft sets' properties to the associated rough set and fuzzy set concepts, then transferred this concept to the groups and defined soft groups along with some of their properties. Acar et al. [1] instigated and explained the basic ideas behind soft rings. Sun et al. [16] instigated the idea of soft modules and pointed out several properties of the same by utilising the concept of modules and soft set theory. Türkmen and Pancar [19] developed some soft submodule properties over a

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Received: 06 March 2024/ Revised: 03 June 2024/ Accepted: 08 June 2024
DOI: 10.22124/JART.2024.26958.1644

module and illustrated these concepts with examples. Atagün and Sezgin [5] carried out the soft substructures of modules, fields, and rings algebrically. They also proposed the concepts of soft subfield, soft submodule of R-module, soft subrings, and soft ideals of a ring and exemplified these concepts. Taouti and Khan [17] proposed soft fields, soft integral domains, and also introduced fractions of soft rings. Taouti et al. [18] investigated the idealistic soft rings, and defined the soft fractional ideal of soft rings. The authors have also studied fractional ideals with a few fundamental soft operations. This fractional ideal concept has its unique significance while studying domains, especially that of Dedekind domains.

Anderson and Anderson [4] introduced the concept of set of star operations and demonstrated that the star operations set on an integral domain is a complete lattice. Ryûki and Takasi [15] introduced the idea of semistar operations on an integral domain and studied several properties of the same. Akira and Ryûki [2] introduce and study semistar operations on the collection of all nonzero fractional ideals of integral domain R (nonzero R-submodules of M, where M is a fraction field of R). Also, the semistar operations set on R and how these operations relate to rings that are intermediate between R and M are examined by the authors. Picozza [13, 14] extended the understanding of the relation between (semi)star operations (i.e., the semistar operations that are restricted to the set of fractional ideals, are star operations) on the overring of R and the semistar operations on an integral domain R. Karaaslan et al. [10] demonstrated the notions of soft lattice, complete soft lattice, distributive soft lattice etc. with their related properties. Gilmer and Ohm [7] studied integral domains with quotient overrings and exemplified these concepts. For an overview of the definitions and results, we refer [6].

2 Preliminary

In this section, we recall certain definitions and results needed for our purpose. We consider R be an integral domain with fraction field M, and U be the given universal set throughout this paper.

Definition 1 ([12]). *A soft set (β, C) over U is a mapping from a set of parameters C into $P(U)$. i.e., $\beta : C \longrightarrow P(U)$, where $P(U)$ denotes the power set of U.*

In other words, A function $f_A : C \longrightarrow P(U)$ such that $f_A(x) = \phi$ if $x \notin A$, is called a soft set over U, where C is a set of parameters and $A \subseteq C$.

The set of all soft sets over U is denoted by $\mathcal{S}(U)$ and $\mathcal{L}$ ($\mathcal{L} \subseteq \mathcal{S}(U)$) stands for a soft lattice with greatest lower bound and a least upper bound which satisfies the distributave law.

Definition 2 ([11]). *For two soft sets (δ, C) and (ω, D) over a common universe U, we say that (δ, C) is a soft subset of (ω, D) denoted by $(\delta, C) \mathrel{\widetilde{\subset}} (\omega, D)$, if it fulfils:*

(i) $C \subset D$,

(ii) $\forall\ l \in C$, $\delta(l)$ and $\omega(l)$ are identical approximations.

Definition 3. *For a soft set δ over a field M and $\delta_\alpha = \{l \in M : \delta(l) \supseteq \alpha\}$ is called the level set, for every $\alpha \in \mathcal{L}$.*

Assuming L be the subset of $R \subseteq M$ whose characteristic function is symbolized by χ_L. Now for every $\alpha \in \mathcal{L} \setminus R$, the soft subset of M symbolized by χ_L^α which is outlined by

$$\chi_L^\alpha(l) = \begin{cases} U & ; \text{ if } l \in R \\ \alpha & ; \text{ if } l \in M - R \end{cases}$$

Definition 4 ([1])**.** *Let (δ, A) be a soft set. The set $supp(\delta, A) = \{l \in A \mid \delta(l) \neq \phi\}$ is called the support of the soft set (δ, A). A soft set is said to be non-null if its support is not equal to the empty set.*

Definition 5 ([1])**.** *Let (δ, A) be a non-null soft set over R. Then (δ, A) is called a soft ring over R if $\delta(l)$ is a subring of R, for all $l \in A$.*

Definition 6 ([1])**.** *Let (G, B) be a soft ring over R. A non-null soft set (β, J) over R is called soft ideal of (G, B) and is symbolized by $(\beta, J) \widetilde{\lhd} (G, B)$ if it satisfies:*

(i) $J \subset B$,

(ii) $\beta(l)$ *is an ideal of* $G(l)$*, for all* $l \in supp(\beta, J)$.

Definition 7 ([18])**.** *A soft subset δ is said to be a soft ideal of a ring R if $\delta(p-q) \supseteq \delta(p) \cap \delta(q)$ and $\delta(pq) \supseteq \delta(p) \cup \delta(q)$.*

A soft subset δ of R is said to be an ideal of R if and only if $\delta(0) \supseteq \delta(p)$ for every $p \in R$ and δ_α is an ideal of R for every $\alpha \in \mathcal{L}$.

Definition 8 ([18])**.** *Let R be a ring contained in a field M and (δ, M) be a soft subset over the field M. Then, δ is said to be a soft R-submodule of M if it fulfils :*

(i) $\delta(e - f) \supseteq \delta(e) \cap \delta(f), \forall\ e, f \in M$,

(ii) $\delta(re) \supseteq \delta(e), \forall\ e \in M,\ r \in R$,

(iii) $\delta(0) = R$.

A soft subset δ of M is a soft R-submodule of M if and only if $\delta(0) = R$ and δ_α is an R-submodule of M for every $\alpha \in \mathcal{L}$. We denote the set by $\delta_* = \{e \in M : \delta(e) = \delta(0)\}$. If soft sets δ and ω of M exist, then the definitions of soft sets $\delta \circ \omega$ and $\delta\omega$ of M are outlined by

$$(\delta \circ \omega)(l) = \begin{cases} \cup\{\delta(p) \cap \omega(q) : l = pq \ ; p, q \in M\}, \forall\ l \in M \\ \qquad 0 \ ; \ \text{otherwise} \end{cases}$$

and

$$(\delta\omega)(l) = \cup\{\bigcap_{i=1}^{n}(\delta(p_i) \cap \omega(q_i)) : p_i, q_i \in M,\ n \geq i \geq 1,\ n \in \mathbb{N},\ l = \sum_{i=1}^{n} p_i q_i\}, \forall\, l \in M.$$

Definition 9 ([18])**.** *For $d \in M$ and $\delta \in \mathcal{L}$, let d_δ denotes the soft subset of M and is defined by for every $l \in M$, $d_\delta(l) = \delta$ if $l = d$ and $d_\delta(l) = 0$, otherwise. We call $d_\delta(l)$ a soft singleton.*

Definition 10 ([18]). *Let M be the field of fractions of an integral domain R. A soft R-submodule δ of M is called a soft fractional ideal of R, if there exists $d \in R$; $d \neq 0$ such that $d_R \circ \delta \subseteq \chi_R^\omega$, for some $\omega \in \mathcal{L} - R$.*

Definition 11 ([10]). *Let $\mathcal{L} \subseteq \mathcal{S}(U)$, $\curlyvee$ and $\curlywedge$ be two binary operations on $\mathcal{L}$. If the set $\mathcal{L}$ is equipped with two commutative and assocative binary operations $\curlyvee$ and $\curlywedge$ which are connected by the absorption law, then the algebraic structure $(\mathcal{L}, \curlyvee, \curlywedge)$ is called a soft lattice.*

Definition 12 ([10]). *Let $(\mathcal{L}, \curlyvee, \curlywedge, \preceq)$ be a soft lattice. If every subsets of $\mathcal{L}$ have both a greatest lower bound and a least upper bound, then it is called complete soft lattice.*

Definition 13 ([7]). *Let R be an integral domain with quotient field M. By an overring of R we shall mean any ring D such that $R \subset D \subset M$.*

3 Soft R-submodule operations

In this section, we shall study soft R-submodule operations, with their properties including practical uses. Also, we shall discuss why it is so important in the field of soft set theory.

Definition 14. *Let $\{\delta_i \mid i = 1, \ldots, n\}$ be a collection of soft sets of M. Then the soft set $\sum\limits_{i=1}^{n} \delta_i$ of M is outlined by*

$$(\sum_{i=1}^{n} \delta_i)(l) = \cup\{\cap(\delta_i(l_i) : i = 1, \ldots, n) \mid l = \sum_{i=1}^{n} l_i, l_i \in M\}, \quad \forall l \in M.$$

Also, a soft set $\widetilde{\bigcap}_{i\in I}\delta_i$ of M is outlined by $(\widetilde{\bigcap}_{i\in I}\delta_i)(l) = \cap\{\delta_i(l) \mid i \in I\}$, $\forall\ l \in M$, where I is a nonempty index set.

Definition 15. *Let δ_i be a soft R-submodule of M for every $i = 1, \ldots, n$, and the intersection $\langle \delta_1, \delta_2, \ldots, \delta_n \rangle$ of all soft R-submodules of M that contain all the δ_i's. Then $\langle \delta_1, \delta_2, \ldots, \delta_n \rangle$ is said to be soft R-submodule of M generated by $\delta_1, \delta_2, \ldots, \delta_n$.*

Theorem 1. *Let δ and ω be two soft R-submodules of M. Then $\delta\omega$ is a soft R-submodule of M.*

Proof. Case I. Let $w = (\delta\omega)(k) \cap (\delta\omega)(l)$ for all $k, l \in M$ and let $0 \subseteq \zeta \subset w$ be arbitrary. Then

$$\zeta \subset (\delta\omega)(k) = \widetilde{\bigcup}\{\widetilde{\bigcap}_{i=1}^{n}(\delta(k_i) \cap \omega(k_i^{'})) : k_i, k_i^{'} \in M, n \in \mathbb{N}, k = \sum_{i=1}^{n} k_i k_i^{'}\},$$
$$\text{and } \zeta \subset (\delta\omega)(l) = \widetilde{\bigcup}\{\widetilde{\bigcap}_{i=1}^{n}(\delta(l_i) \cap \omega(l_i^{'})) : l_i, l_i^{'} \in M, n \in \mathbb{N}, l = \sum_{i=1}^{n} l_i l_i^{'}\}.$$

Thus, there are representations $k = \sum\limits_{i=1}^{p} k_i k_i^{'}$ and $l = \sum\limits_{i=1}^{q} l_i l_i^{'}$, where $k_i, k_i^{'}, l_i, l_i^{'} \in M$ such that $\zeta \subset \widetilde{\bigcap}_{i=1}^{p}(\delta(k_i) \cap \omega(k_i^{'}))$ and $\zeta \subset \widetilde{\bigcap}_{i=1}^{q}(\delta(l_i) \cap \omega(l_i^{'}))$, i.e., $\zeta \subset \delta(k_i) \cap \omega(k_i^{'})$ and $\zeta \subset \delta(l_i) \cap \omega(l_i^{'})$ for each i. Therefore, $\zeta \subset \delta(k_i), \zeta \subset \delta(l_i), \zeta \subset \omega(k_i^{'}), \zeta \subset \omega(l_i^{'})$ with $k = \sum\limits_{i=1}^{p} k_i k_i^{'}$ and $l = \sum\limits_{i=1}^{q} l_i l_i^{'}$.

So, $\zeta \subset \delta(k_i) \cap \delta(l_i) \subseteq \delta(k_i + l_i)$ for each $i = 0, \ldots, p$, and $\zeta \subset \omega(k_i^{'}) \cap \omega(l_i^{'}) \subseteq \omega(k_i^{'} + l_i^{'})$ for each $i = 0, \ldots, q$. Thus, a representation of $k + l = \sum_{i=1}^{n}(k_i k_i^{'} + l_i l_i^{'})$ exists such that $\zeta \subset \delta(k_i + l_i) \cap \omega(k_i^{'} + l_i^{'})$ for each $i = 0, \ldots, n$. Therefore, $\zeta \subset \widetilde{\bigcup}\{\widetilde{\bigcap}_{i=1}^{n}(\delta(m_i) \cap \omega(m_i^{'})) : m_i, m_i^{'} \in M, n \in \mathbb{N}, k + l = \sum_{i=1}^{n} m_i m_i^{'}\}$. It follows then that $\zeta \subset (\delta\omega)(k + l)$ for any arbitrary $0 \subseteq \zeta \subset w$. Hence, $(\delta\omega)(k) \cap (\delta\omega)(l) = w \subseteq (\delta\omega)(k + l)$.

Case II. Let $w = \delta\omega(k)$ and $0 \subseteq \zeta \subset w$ be arbitrary. Then $\zeta \subset (\delta\omega)(k) = \widetilde{\bigcup}\{\widetilde{\bigcap}_{i=1}^{n}(\delta(k_i) \cap \omega(k_i^{'})) : k_i, k_i^{'} \in M, n \in \mathbb{N}, k = \sum_{i=1}^{n} k_i k_i^{'}\}$. Thus, for some representation $k = \sum_{i=1}^{n} k_i k_i^{'}$ with $k_i, k_i^{'} \in M$, we have $\zeta \subset \widetilde{\bigcap}_{i=1}^{n}(\delta(k_i) \cap \omega(k_i^{'}))$. So, $\zeta \subset \delta(k_i) \cap \omega(k_i^{'}) \subseteq \delta(rk_i) \cap \omega(k_i^{'})$ for each i, for every $r \in R$ and the representation $rk = \sum_{i=1}^{n}(rk_i)k_i^{'}$. It follows then that $\zeta \subset \widetilde{\bigcup}\{\widetilde{\bigcap}_{i=1}^{n}(\delta(k_i) \cap \omega(k_i^{'})) : k_i, k_i^{'} \in M, n \in \mathbb{N}, rk = \sum_{i=1}^{n} k_i k_i^{'}\} = \delta\omega(rk)$ for any arbitrary $0 \subseteq \zeta \subset w$. Hence, $\delta\omega(k) = w \subseteq \delta\omega(rk)$ for all $r \in R$ and $k \in M$. □

Proposition 1. *For $0 \neq d \in M$, let δ, ω and μ be arbitrary soft subsets of M. Then*

(i) $(d_R \circ \delta)\omega = d_R \circ (\delta\omega)$.

(ii) If $\delta \,\widetilde{\subseteq}\, \omega$, then $\delta\mu \,\widetilde{\subseteq}\, \omega\mu$.

(iii) Suppose δ, ω and μ are soft R-submodules of M. Then $\delta \circ \omega \,\widetilde{\subseteq}\, \mu$ if and only if $\delta\omega \,\widetilde{\subseteq}\, \mu$.

Proof. (i) We have to show that $(d_R \circ \delta)\omega(k) = \delta\omega(k/d)$, let $k \in M$, and $0 \neq d \in M$. Then

$$(d_R \circ \delta)\omega(k) = \widetilde{\bigcup}\{\widetilde{\bigcap}_{i=1}^{n}(\delta(k_i/d) \cap \omega(k_i^{'})) : k_i, k_i^{'} \in M, n \in \mathbb{N}, k = \sum_{i=1}^{n} k_i k_i^{'}\}.$$

But a representation of $k = \sum_{i=1}^{n}(dk_i)k_i^{'}$ can be obtained from any representation of $k/d = \sum_{i=1}^{n} k_i k_i^{'}$, so that $(d_R \circ \delta(dk_i), \omega(k_i)) = (\delta(k_i), \omega(k_i^{'}))$. Note that

$$\delta\omega(k/d) = \widetilde{\bigcup}\{\widetilde{\bigcap}_{i=1}^{n}(\delta(k_i), \omega(k_i^{'})) : k_i, k_i^{'} \in M, n \in \mathbb{N}, k/d = \sum_{i=1}^{n} k_i k_i^{'}\}.$$

Thus, $(d_R \circ \delta)\omega(k) \supseteq \delta\omega(k/d) = d_R \circ (\delta\omega)$. On the other side, we utilize the fact that any representation of $k = \sum_{i=1}^{n} k_i k_i^{'}$ yields a representation of $k/d = \sum_{i=1}^{n}(k_i/d)k_i^{'}$ along with the ordered pair $(\delta(k_i/d), \omega(k_i^{'}))$. Therefore, $(d_R \circ \delta)\omega(k) \subseteq \delta\omega(k/d)$. Hence, $(d_R \circ \delta)\omega(k) = \delta\omega(k/d)$.

(ii) We have to show that $\delta\mu(k) \,\widetilde{\subseteq}\, \omega\mu(k)$ for all $k \in M$ under the supposition that $\delta(k) \,\widetilde{\subseteq}\, \omega(k)$. For all i, we have $\delta(k_i) \cap \mu(k_i^{'}) \,\widetilde{\subseteq}\, \omega(k_i) \cap \mu(k_i^{'})$ for any representation of $k = \sum_{i=1}^{n} k_i k_i^{'}; k_i, k_i^{'} \in M, n \in \mathbb{N}$. It follows that $\delta\mu(k) \,\widetilde{\subseteq}\, \omega\mu(k)$.

(iii) Suppose that $\delta \circ \omega \,\widetilde{\subseteq}\, \mu$. Let $w = \delta\omega(k)$ for all $k \in M$ and $0 \subseteq \zeta \subset w$ be arbitrary. Then

$$\zeta \subset (\delta\omega)(k) = \widetilde{\bigcup}\{\widetilde{\bigcap}_{i=1}^{n}(\delta(k_i) \cap \omega(k_i^{'})) : k_i, k_i^{'} \in M, n \in \mathbb{N}, k = \sum_{i=1}^{n} k_i k_i^{'}\}.$$

Thus, $\zeta \subset \widetilde{\bigcap}_{i=1}^{n}(\delta(k_i) \cap \omega(k_i^{'}))$ for some representation $k = \sum_{i=1}^{n} k_i k_i^{'}$ with $k_i, k_i^{'} \in M$. Since $\delta \circ \omega \mathrel{\widetilde{\subseteq}} \mu$, $(\delta(k_i) \cap \omega(k_i^{'})) \mathrel{\widetilde{\subseteq}} \mu(k_i k_i^{'})$ for all i. Since μ is a soft R-submodule of M, then it implies that $\zeta \subset \widetilde{\bigcap}_{i=1}^{n} \mu(k_i k_i^{'}) \mathrel{\widetilde{\subseteq}} \mu(\sum_{i=1}^{n} k_i k_i^{'} = k)$. Hence, $\delta\omega(k) = w \mathrel{\widetilde{\subseteq}} \mu(k)$. Conversely, since we have δ, ω and μ are soft R-submodules of M. If $\delta\omega \mathrel{\widetilde{\subseteq}} \mu$, then the definition makes it obvious that $\delta \circ \omega \mathrel{\widetilde{\subseteq}} \delta\omega \mathrel{\widetilde{\subseteq}} \mu$. □

Proposition 2. *Let δ and ω be two arbitrary soft R-submodules of M. Then $(\delta\omega)_\alpha = \delta_\alpha \omega_\alpha$ for every $\alpha \in \mathcal{L} \subseteq \mathcal{S}(U)$.*

Proof. Let $k \in (\delta\omega)_\alpha$, and $0 \subseteq \zeta \subset \alpha$. Then

$$\zeta \subset (\delta\omega)(k) = \widetilde{\bigcup}\{\widetilde{\bigcap}_{i=1}^{n}(\delta(k_i) \cap \omega(k_i^{'})) : k_i, k_i^{'} \in M, n \in \mathbb{N}, k = \sum_{i=1}^{n} k_i k_i^{'}\}.$$

Thus, there is a representation $k = \sum_{i=1}^{p} k_i k_i^{'}$ where $k_i, k_i^{'} \in M$ such that $\zeta \subset \delta(k_i) \cap \omega(k_i^{'})$ for all i, which implies that $\zeta \subset \delta(k_i)$ and $\zeta \subset \omega(k_i^{'})$ for all $0 \subseteq \zeta \subset \alpha$, so $k_i \in \delta_\alpha$, and $k_i^{'} \in \omega_\alpha$ for all i. Thus $k \in \delta_\alpha \omega_\alpha$. Hence, $(\delta\omega)_\alpha \subseteq \delta_\alpha \omega_\alpha$. Conversely, let $k \in \delta_\alpha \omega_\alpha$ with $k = \sum_{i=1}^{p} k_i k_i^{'}$ where $k_i \in \delta_\alpha$ and $k_i^{'} \in \omega_\alpha$. By definition, we have $(\delta\omega)(k) \supseteq \delta(k_i) \cap \omega(k_i^{'})$ for all i. Since $\delta(k_i) \supseteq \alpha$ and $\omega(k_i^{'}) \supseteq \alpha$ implies $(\delta\omega)(k) \supseteq \alpha$. Thus $k \in (\delta\omega)_\alpha$. Therefore, $\delta_\alpha \omega_\alpha \subseteq (\delta\omega)_\alpha$. Hence, $(\delta\omega)_\alpha = \delta_\alpha \omega_\alpha$. □

4 Extensions of soft fractional ideals over overrings

In the study of algebraic theory, integral domains are the fundamental mathematical structures at the core of algebra, and we frequently investigate their inner workings. In this section, we demonstrate certain extensions of soft fractional ideals by utilizing the notion of quotient overrings. By doing so, we make soft fractional ideals more accessible and highlight their practical significance within the broader framework of soft set theory.

Proposition 3. *Consider an overring D of R. Then any soft D-submodule of M is a soft R-submodule of M.*

Proof. The proof is routine and can be easily obtained from the definitions of overring and soft submodule over a ring, thus omitted. □

Proposition 4. *Let D be an overring of R. If ω is a soft fractionary ideal of R, then $\omega\chi_D^{(v)}$ is a soft fractionary ideal of D. Additionally, if $\omega_* \neq 0$, then $(\omega\chi_D^{(v)})_* \neq 0$.*

Proof. Assume ω is a soft fractionary ideal of R. Then there is $0 \neq d \in R$ such that $d_R \circ \omega \mathrel{\widetilde{\subseteq}} \chi_R^{(\alpha)}$ for some $\alpha \in \mathcal{L} \setminus R$. By Theorem 1, if $\chi_D^{(v)}$ is a soft R-submodule, then $\omega\chi_D^{(v)}$ is a soft R-submodule. Since we do have $d_R \circ \omega\chi_D^{(v)} = (d_R \circ \omega)\chi_D^{(v)} \mathrel{\widetilde{\subseteq}} \chi_R^{(\alpha)}\chi_D^{(v)}$. By Proposition 1, we claim that $\chi_R^{(\alpha)}\chi_D^{(v)} \mathrel{\widetilde{\subseteq}} \chi_D^{(\alpha)}$. i.e., $\chi_R^{(\alpha)} \circ \chi_D^{(v)} \mathrel{\widetilde{\subseteq}} \chi_D^{(\alpha)}$. For this, let $k \in D$, then $k = Rk$ is a representation of k and it follows that $(\chi_R^{(\alpha)} \circ \chi_D^{(v)})(k) = \alpha = \chi_D^{(\alpha)}(k)$. Now suppose $k \in M \setminus D$. Then for any representation $k = lm$ of k, either l or m is not in D, in this case $(\chi_R^{(\alpha)} \circ \chi_D^{(v)})(k) = 0 = \chi_D^{(\alpha)}(k)$. Thus the claim is proved and $d_R \circ \omega\chi_D^{(v)} \mathrel{\widetilde{\subseteq}} \chi_D^{(\alpha)}$. i.e., $\chi_R^{(\alpha)} \circ \chi_D^{(v)} \mathrel{\widetilde{\subseteq}} \chi_D^{(\alpha)}$. Hence, $\omega\chi_D^{(v)}$ is a soft fractionary ideal of D. Also, assume that $\omega_* \neq 0$, i.e., there is $0 \neq b \in M$ such that $\omega(b) = R$. Recall that $\omega \mathrel{\widetilde{\subseteq}} \omega \circ \chi_D^{(v)} \mathrel{\widetilde{\subseteq}} \omega\chi_D^{(v)}$. Using the representation $k = Rk$, we can actually state that $\omega(k) \subseteq \omega \circ \chi_D^{(v)}(k)$. Therefore, $R = \omega(b) \subseteq \omega \circ \chi_D^{(v)}(b)$ and $\omega \circ \chi_D^{(v)}(b) = R$. Hence, $(\omega\chi_D^{(v)})_* \neq 0$. □

Lemma 1. *Consider J be a finitely generated ideal of an overring D of R. Then $\exists$ a finitely generated R-submodule J_v of M such that $J = J_vD$ and $\chi_J^{(v)} = \chi_{J_v}^{(v)}\chi_D^{(v)}$.*

Proof. We write a finitely generated ideal $J = b_1D + b_2D + \cdots + b_nD$ of D and consider $\langle (b_1)_R, (b_2)_R, \ldots, (b_n)_R \rangle$ be the soft fractionary ideal of D generated by $(b_1)_R, (b_2)_R, \ldots, (b_n)_R$. Then

$$\langle (b_1)_R, (b_2)_R, \ldots, (b_n)_R \rangle = \langle \chi_{\{b_1, b_2, \ldots, b_n\}}^{(v)} \rangle = \chi_J^{(v)}$$

Let the soft fractionary ideal $\langle (b_1)_R, (b_2)_R, \ldots, (b_n)_R \rangle$ of R generated by $(b_1)_R, (b_2)_R, \ldots, (b_n)_R$. Then

$$\langle (b_1)_R, (b_2)_R, \ldots, (b_n)_R \rangle = \langle \chi_{\{b_1, b_2, \ldots, b_n\}}^{(v)} \rangle = \chi_{b_1R + b_2R + \cdots + b_nR}^{(v)}$$

A finitely generated set $J_v = b_1R + b_2R + \cdots + b_nR$ is an R-submodule of M and obviously $J = J_vD$. Now we claim that $\chi_J^{(v)} = \chi_{J_v}^{(v)}\chi_D^{(v)}$. It is still necessary to demonstrate that $\chi_J^{(v)} \mathrel{\widetilde{\subseteq}} \chi_{J_v}^{(v)}\chi_D^{(v)}$ and $\chi_{J_v}^{(v)}\chi_D^{(v)} \mathrel{\widetilde{\subseteq}} \chi_J^{(v)}$, i.e., $\chi_{J_v}^{(v)} \circ \chi_D^{(v)} \mathrel{\widetilde{\subseteq}} \chi_J^{(v)}$. For this, let $k \in J$, then $k = \sum_{i=1}^{n} b_ik_i$ where $b_i \in M$ and $k_i \in D$. But $\chi_{J_v}^{(v)}(b_i) = R$ and $\chi_D^{(v)}(k_i) = R$ for each i. Thus, $\chi_{J_v}^{(v)}\chi_D^{(v)}(k) = R = \chi_J^{(v)}(k)$ for every $k \in J$. Therefore, $\chi_J^{(v)} \mathrel{\widetilde{\subseteq}} \chi_{J_v}^{(v)}\chi_D^{(v)}$. Also, let $k \in M \setminus J$ and $k = lm$ be any arbitrary representation of k. If $m \notin D$, then $\chi_D^{(v)}(m) = 0$. If $m \in D$, then $l \notin J$ as J is an ideal of D. Since $l \notin J \mathrel{\widetilde{\supseteq}} J_v$, it follows that $\chi_{J_v}^{(v)}(l) = 0$. Thus $\chi_{J_v}^{(v)} \circ \chi_D^{(v)}(k) = 0 = \chi_J^{(v)}(k)$ for $k \in M \setminus J$. Therefore $\chi_{J_v}^{(v)} \circ \chi_D^{(v)} \mathrel{\widetilde{\subseteq}} \chi_J^{(v)}$. Hence, $\chi_J^{(v)} = \chi_{J_v}^{(v)}\chi_D^{(v)}$. □

Remark 1. *It is clear by Lemma 1 that $\chi_J^{(v)}$ is a finitely generated soft fractionary ideal of D and $\chi_{J_v}^{(v)}$ is a finitely generated soft fractionary ideal of R.*

5 Soft semistar operations

In this section, the notion of soft semistar operation on an integral domain R is defined along with several illustrations within the framework of soft fractional ideals. Symbolizing the set of all soft fractional ideals of R as $S_F(R)$ and the set of all soft R-submodules of M as $S_{\bar{S}}(R)$, we delve into their relationships and significance. Our goal is to untangle the intricate relationships between these algebraic structures and reveal how they connect to the principles of soft set theory, making these concepts more accessible and understandable.

Definition 16. *A mapping $\omega \to \omega^*$ on $S_F(R)$ that fulfils the below listed three properties is called a soft star-operation on R.*

(i) $(\chi_R^{(\alpha)})^* = \chi_R^{(\alpha)}$ *and* $(d_R \circ \omega)^* = d_R \circ \omega^*$,

(ii) $\omega \,\widetilde{\subseteq}\, \omega^*$ *; if* $\delta \,\widetilde{\subseteq}\, \omega$, *then* $\delta^* \,\widetilde{\subseteq}\, \omega^*$,

(iii) $(\omega^*)^* = \omega^*$, *for all* $0 \neq d \in M$, *for some* $\alpha \in \mathcal{L} \setminus R$ *and* $\delta, \omega \in S_F(R)$.

Definition 17. *A mapping $\star : S_{\bar{S}}(R) \to S_{\bar{S}}(R), \omega \to \omega^\star$ that fulfils the below listed three properties is called a soft semistar operation on R.*

($\star_1$) $(d_R \circ \omega)^\star = d_R \circ \omega^\star$,

($\star_2$) $\delta \,\widetilde{\subseteq}\, \omega \Longrightarrow \delta^\star \,\widetilde{\subseteq}\, \omega^\star$,

($\star_3$) $\omega \,\widetilde{\subseteq}\, \omega^\star$, *and* $\omega^{\star\star} := (\omega^\star)^\star = \omega^\star$, *for all* $0 \neq d \in M$ *and* $\delta, \omega \in S_{\bar{S}}(R)$.

In addition, the equality $(\chi_R^{(\alpha)})^\star = \chi_R^{(\alpha)}$ for some $\alpha \in \mathcal{L} \setminus R$ is not required, and we symbolize the set $\{\omega^\star : \omega \in S_{\bar{S}}(R)\}$ by $S_{\bar{S}}^\star(R)$.

Proposition 5. *Consider the collection of soft R-submodules $\{\delta_i \mid i \in I\}$ of M and $\star$ be a soft semistar operation on R. Then*

(i) $(\sum_i \delta_i)^\star = (\sum_i \delta_i^\star)^\star$, *if* $(\sum_i \delta_i)^\star \in S_{\bar{S}}(R)$,

(ii) $\widetilde{\bigcap}_i \delta_i^\star = (\widetilde{\bigcap}_i \delta_i^\star)^\star$, *where I is a nonempty index set.*

Proof. (i) For each $i, \delta_i \,\widetilde{\subseteq}\, \sum_i \delta_i$; we have $\delta_i^\star \,\widetilde{\subseteq}\, (\sum_i \delta_i)^\star$. Therefore it is obvious $\sum_i \delta_i^\star \,\widetilde{\subseteq}\, (\sum_i \delta_i)^\star$. Since $(\sum_i \delta_i)^\star \in S_{\bar{S}}(R)$, there exists $0 \neq d \in R$ such that $d_R \circ (\sum_i \delta_i)^\star \subseteq \chi_R^\alpha$, for some $\alpha \in \mathcal{L}$. So $d_R \circ \sum_i \delta_i^\star \,\widetilde{\subseteq}\, d_R \circ (\sum_i \delta_i)^\star \,\widetilde{\subseteq}\, \chi_R^\alpha$. Therefore, $\sum_i \delta_i^\star \in S_{\bar{S}}(R)$ and $(\sum_i \delta_i^\star)^\star \,\widetilde{\subseteq}\, (\sum_i \delta_i)^{\star\star} = (\sum_i \delta_i)^\star$. It is obvious to prove the reverse since $\sum_i \delta_i \,\widetilde{\subseteq}\, \sum_i \delta_i^\star$. Hence, $(\sum_i \delta_i)^\star = (\sum_i \delta_i^\star)^\star$.
(ii) We need only to show that $(\widetilde{\bigcap}_i \delta_i^\star)^\star \,\widetilde{\subseteq}\, \widetilde{\bigcap}_i \delta_i^\star$. But this follows since the containment $\widetilde{\bigcap}_i \delta_i^\star \,\widetilde{\subseteq}\, \delta_j^\star$ implies $(\widetilde{\bigcap}_i \delta_i^\star)^\star \,\widetilde{\subseteq}\, \delta_j^{\star\star} = \delta_j^\star$, for each j. □

Proposition 6. *Let $\star$ be a soft semistar operation on R. If ω is a soft fractionary ideal of R, then $\exists\ 0 \neq d \in R$ such that $d_R \circ \omega^\star \,\widetilde{\subseteq}\, (\chi_R^{(\alpha)})^\star$ for some $\alpha \in \mathcal{L} \setminus R$.*

Proof. Let $\omega \in S_F(R)$. Then $\exists\ 0 \neq d \in R$ such that $d_R \circ \omega \,\widetilde{\subseteq}\, \chi_R^{(\alpha)}$ for some $\alpha \in \mathcal{L} \setminus R$. Hence, $d_R \circ \omega^\star = (d_R \circ \omega)^\star \,\widetilde{\subseteq}\, (\chi_R^{(\alpha)})^\star$. □

Definition 18. *A soft semistar operation $\star$ on R is said to be a soft (semi)star operation on R if $(\chi_R^{(\alpha)})^\star = \chi_R^{(\alpha)}$.*

Remark 2. *From Proposition 6, we observe that every soft (semi)star operation whose restriction $\star|_{S_F(R)}$ to the set $S_F(R)$ of R is a soft star operation.*

Example 1. The identity soft semistar operation symbolized by d and outlined by $\omega \to \omega^d := \omega$, for every $\omega \in S_{\bar{S}}(R)$ is the trivial illustration of soft semistar operation. Also, the e-semistar operation outlined by $\omega \to \omega^e := \chi_R^{(\alpha)}$ for some $\alpha \in \mathcal{L} \setminus R$ is another trivial illustration of soft semistar operation on R. It becomes evident that $e \supseteq \star \supseteq d$ for any soft semistar operation $\star$ on R.

Consider $\star$ be a soft star operation on R and M is the fraction field of R. We outline $\star_e$ on R by

$$\omega^{\star_e} := \begin{cases} \omega^\star & ; \text{ if } \omega \in S_F(R) \\ \chi_M & ; \text{ otherwise} \end{cases}$$

Proposition 7. *Consider $\star$ be a soft star operation on R. Then the trivial extension $\star_e$ of $\star$ is a soft semistar operation on R.*

Proof. To prove $(\star_1)$, for all $\omega \in S_{\bar{S}}(R) \setminus S_F(R)$ and $0 \neq d \in M$, it suffices to verify that $(d_R \circ \omega)^{\star_e} = d_R \circ \omega^{\star_e}$. Clearly, we have $(d_R \circ \omega)^{\star_e} = \chi_M = d_R \circ \chi_M = d_R \circ \omega^{\star_e}$. Now to prove $(\star_2)$, let $\delta \,\widetilde{\subseteq}\, \omega$. Then obviously we have $\delta^{\star_e} \,\widetilde{\subseteq}\, \chi_M = \omega^{\star_e}$ if $\omega \in S_{\bar{S}}(R) \setminus S_F(R)$. However, it follows that $\delta \in S_F(R)$ if $\omega \in S_F(R)$ and it is obvious that $\delta^{\star_e} \,\widetilde{\subseteq}\, \omega^{\star_e}$ by restriction to $\star$. To prove $(\star_3)$, if $\omega \in S_F(R)$, then first we observe that $\omega \,\widetilde{\subseteq}\, \omega^\star = \omega^{\star_e}$ and if $\omega \in S_{\bar{S}}(R) \setminus S_F(R)$, then $\omega \,\widetilde{\subseteq}\, \chi_M = \omega^{\star_e}$, and it is clearly visible that $\omega^{\star_e \star_e} = \omega^{\star_e}$. □

Remark 3. *In general, a soft semistar operation $\star$ on R with fraction field M is associated to a mapping $\star_e$ outlined by*

$$\omega^{\star_e} = \begin{cases} \omega^\star & ; \text{ if } \omega \in S_F(R) \\ \chi_M & ; \text{ if } \omega \in S_{\bar{S}}(R) \setminus S_F(R) \end{cases}$$

Theorem 2. *Consider $\star$ be a soft semistar operation on R with fraction field M. Then for some $0 \neq d \in M$ and $\alpha \in \mathcal{L} \setminus R$, $d_R \circ (\chi_R^{(\alpha)})^\star \,\widetilde{\subseteq}\, \chi_R^{(\alpha)}$ iff $\star_e$ is a soft semistar operation.*

Proof. For some $0 \neq d \in M$, assume that $d_R \circ (\chi_R^{(\alpha)})^\star \,\widetilde{\subseteq}\, \chi_R^{(\alpha)}$. Then the definition of $\star_e$ satisfies $(\star_1)$ and $(\star_2)$. To prove $(\star_3)$, if $\omega \in S_F(R)$, then by Proposition 6, $\exists\ 0 \neq d' \in M$ such that $d'_R \circ \omega^\star \,\widetilde{\subseteq}\, (\chi_R^{(\alpha)})^\star$. Thus, $(d'd)_R \circ \omega^\star = d_R \circ d'_R \circ \omega^\star \,\widetilde{\subseteq}\, d_R \circ (\chi_R^{(\alpha)})^\star \,\widetilde{\subseteq}\, \chi_R^{(\alpha)}$ for some $\alpha \in \mathcal{L} \setminus R$. So, $\omega^\star \in S_F(R)$. Thus $\omega^{\star_e \star_e} = (\omega^\star)^{\star_e} = \omega^{\star\star} = \omega^\star$. The case $\omega \in S_{\bar{S}}(R) \setminus S_F(R)$ is obvious from the definition. Hence, $\star_e$ is a soft semistar operation. Conversely, suppose that $d_R \circ (\chi_R^{(\alpha)})^\star \,\widetilde{\nsubseteq}\, \chi_R^{(\alpha)}$ for some $0 \neq d \in M$. Then $(\chi_R^{(\alpha)})^\star \notin S_F(R)$ and $(\chi_R^{(\alpha)})^{\star_e \star_e} = (\chi_R^{(\alpha)\star_e})^{\star_e} = \chi_M$. Therefore $(\star_3)$ doesnot hold. Hence, $\star_e$ is not a soft semistar operation. □

Remark 4. *If two soft semistar operations $\star_1$ and $\star_2$ on R, then $(\star_1)|_{S_F(R)} = (\star_2)|_{S_F(R)}$ if and only if $(\star_1)_e = (\star_2)_e$, where $\star|_{S_F(R)} = \star_e|_{S_F(R)}$. Therefore $\star_e$ coincides with the soft star opeation's trivial extension $\star|_{S_F(R)}$ if $\star$ is a soft (semi)star operation. For example, the identity soft star operation's trivial extension coincides with the soft semistar operation d_e.*

Example 2. Consider ω be a soft R-submodule of M. Examine the mapping outlined by $\omega \to \omega^\star$, where $\omega^\star := \widetilde{\bigcap}\{k_R \circ \chi_R^{(\alpha)} : \omega \,\widetilde{\subseteq}\, k_R \circ \chi_R^{(\alpha)}, 0 \neq k \in M, \alpha \in \mathcal{L} \setminus R\}$. It is to be noted that

$$\omega^\star = \begin{cases} \omega_v & ; \text{ if } \omega \in S_F(R) \\ \chi_M & ; \text{ if } \omega \in S_{\bar{S}}(R) \setminus S_F(R) \end{cases}$$

As none of the $k_R \circ \chi_R^{(\alpha)}, \alpha \in \mathcal{L} \setminus R$ would contain ω by definition. As a result, the soft semistar operation $\star$ is the soft v-star operation's trivial extension known as soft v-semistar operation. The soft v-semistar operation is actually a soft (semi)star operation on R. Thus we have $(\chi_R^{(\alpha)})_v = \chi_R^{(\alpha)}$ by definition of v.

Proposition 8. *Consider $\star$ be a soft (semi)star operation. Then $\star \subseteq v$.*

Proof. Since $\star|_{S_F(R)}$ is a soft star operation, it is clearly visible that $\omega^\star \mathrel{\widetilde{\subseteq}} \omega_v\ \forall\ \omega \in S_F(R)$. If $\omega \in S_{\bar{S}}(R) \setminus S_F(R)$, then $\omega^\star \mathrel{\widetilde{\subseteq}} \chi_M = \omega_v$. Hence, $\omega^\star \mathrel{\widetilde{\subseteq}} \omega_v$ for every $\omega \in S_{\bar{S}}(R)$ and $\star \subseteq v$. □

Proposition 9. *The mapping $\star_D$, called the extension to the overring D is a soft semistar operation.*

Proof. We know that D is an overring of R. By Proposition 3, it is clearly visible that $\chi_D^{(v)}$ is a soft R-submodule of M. Therefore by Theorem 1, $\omega\chi_D^{(v)} \in S_{\bar{S}}(R)$ and $\star_D$ is well defined. Obviously, we have $d_R \circ \omega\chi_D^{(v)} = (d_R \circ \omega)\chi_D^{(v)}$ from Proposition 1 and $\star_1$ is verified. Now to verify $(\star_2)$, suppose that $\delta \mathrel{\widetilde{\subseteq}} \omega$. Then by Proposition 1, $\delta\chi_D^{(v)} \mathrel{\widetilde{\subseteq}} \omega\chi_D^{(v)}$ and thus verified. Now to prove $\star_3$, suppose $\omega \in S_{\bar{S}}(R)$. We have to demonstrate that $\omega \mathrel{\widetilde{\subseteq}} \omega\chi_D^{(v)}$, then it suffices to demonstrate that $\omega \mathrel{\widetilde{\subseteq}} \omega \circ \chi_D^{(v)}$. By using the representation $k = Rk$ of k, then we have $\omega(k) \cap \chi_D^{(v)}(R) = \omega(k)$. Thus, by definition $\omega(k) \subseteq \omega \circ \chi_D^{(v)}(k)$. We must demonstrate that $\omega\chi_D^{(v)} = \omega\chi_D^{(v)}\chi_D^{(v)}$, i.e., $\omega\chi_D^{(v)}\chi_D^{(v)} \mathrel{\widetilde{\subseteq}} \omega\chi_D^{(v)}$. It suffices to demonstrate that $\chi_D^{(v)}\chi_D^{(v)} \mathrel{\widetilde{\subseteq}} \chi_D^{(v)}$, i.e., $\chi_D^{(v)} \circ \chi_D^{(v)} \mathrel{\widetilde{\subseteq}} \chi_D^{(v)}$ by Proposition 1. For this, let $w \supset \zeta \supseteq 0$ and $w = \chi_D^{(v)} \circ \chi_D^{(v)}(k)$. Then $\zeta \subset \chi_D^{(v)}(l) \cap \chi_D^{(v)}(m)$ for some representation $k = lm$ with $l, m \in M$. We may assume that $k \notin D$ as if $k \in D$ instantly have $\chi_D^{(v)} \circ \chi_D^{(v)}(k) \mathrel{\widetilde{\subseteq}} \chi_D^{(v)}(k)$. So then either l or m is not in D. Therefore in any case $\chi_D^{(v)}(l) \cap \chi_D^{(v)}(m) = 0$, i.e., $\zeta = 0 = \chi_D^{(v)}(k)$. Hence $w = 0 = \chi_D^{(v)}(k)$ as desired. □

Corollary 1. *Consider the collection $\{\star_i\}$ of non empty soft semistar operations (respectively (semi)star operations) on R. Define $\delta^\star = \widetilde{\bigcap}_i \delta^{\star_i}$ for every $\delta \in S_{\bar{S}}(R)$. Then the collection $\{\star_i\}$ with meet $\star$ and $\star$ is a soft semistar operation (respectively (semi)star operation) on R.*

Remark 5. *On the set of soft semistar operations, we define a partial order by $\omega^{\star_1} \mathrel{\widetilde{\subseteq}} \omega^{\star_2}$ if and only if $\star_1 \subseteq \star_2$ for every $\omega \in S_{\bar{S}}(R)$. It is routine to verify that the conditions given below are satisfied:*

(i) $\star_1 \subseteq \star_2$,

(ii) $(\omega^{\star_2})^{\star_1} = \omega^{\star_2}$ *for every* $\omega \in S_{\bar{S}}(R)$,

(iii) $(\omega^{\star_1})^{\star_2} = \omega^{\star_2}$ *for every* $\omega \in S_{\bar{S}}(R)$,

(iv) $S_{\bar{S}}^{\star_1}(R) \mathrel{\widetilde{\subseteq}} S_{\bar{S}}^{\star_2}(R)$.

Moreover, the collection of all soft semistar operations on R is symbolized by $SS_S(R)$ and the collection of all soft (semi)star operations on R is symbolized by $S(S)_S(R)$. Since the partially ordered set $SS_S(R)$ (respectively $S(S)_S(R)$) with greatest element the e-semistar operation (respectively the v-semistar operation) is closed under arbitrary meets. This implies that $SS_S(R)$ (respectively $S(S)_S(R)$) is a complete soft lattice with join outlined by $\widetilde{\bigcup}\star_i = \widetilde{\bigcap}\{\star \in SS_S(R) \mid \star \supseteq \star_i\}$ (respectively $\widetilde{\bigcup}\star_i = \widetilde{\bigcap}\{\star \in S(S)_S(R) \mid \star \supseteq \star_i\}$) for each i.

6 Conclusion

Motivated while studying and analysing the notions with semistar operations on integral domains proposed by Ryûki and Takasi [15] and Akira and Ryûki [2]. The concept of soft semistar operations plays a dominant role in the study of soft fractional ideals of integral domains. Using the notion of quotient overring, we demonstrate some extensions of soft fractional ideals. We bring out the new concept, how the complete soft lattice is formed by the soft semistar operations set in relation to soft fractional ideals on an integral domain. In this paper, using these concepts and freshly defined concepts, we were able to provide some illustrations, including soft d-semistar operation, soft e-semistar operation and soft v-semistar operation. It is a natural task to continue characterizing overrings and other integral domains like Prüfer domains, Krull domains and Mori domains etc, by utilizing the soft semistar operations technique of undeniable soft fractional ideals. The class of soft semistar operations has a wider range of applications as compared to the semistar operations set. It becomes clear that a more accurate classification will be obtained by applying soft semistar procedure techniques. The results of this study will generate novel concepts and serve as a strong foundation for further investigation in the future research work mentioned above.

Acknowledgments

The authors express their sincere gratitude to the referee for meticulous review of the manuscript and valuable comments, which led to improvements in the quality of this paper.

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