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Robust exponential concurrent learning adaptive control for systems preceded by dead-zone input nonlinearity

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Abstract. A concurrent learning (CL) adaptive control is proposed for a class of nonlinear systems in the presence of dead-zone input nonlinearity to guarantee the exponential convergence of the tracking and the parameter estimation errors. The proposed method enriches and encompasses the conventional filtering-based CL by proposing robust and optimal terms. The optimal term is designed by solving a suitable quadratic programming optimization problem based on control Lyapunov function theory which also meets the need for prescribed control bounds. A suitable robust term is proposed to tackle the presence of the dead-zone input nonlinearity. Recent methods of adaptive CL tune the control parameters using trial and error, which is a tedious task. In this paper, by some analysis and proposing two nonlinear optimization problems, the values of the control parameters are derived. The nonlinear optimization problems are solved using the time-varying iteration particle swarm optimization algorithm. The simulation results indicate the effectiveness of the proposed method.

Keywords: Concurrent learning, robust adaptive control, dead-zone nonlinearity, quadratic programming, control Lyapunov function, particle swarm optimization.
AMS Subject Classification 2010: 93A30, 93C40, 93D05.

1 Introduction

Lyapunov method has been used effectively for the analysis and synthesis of affine nonlinear control systems. In the beginning, it has been used for nonlinear stability analysis through defining a suitable Lyapunov function. Later by proposing control Lyapunov function (CLF), control signal is derived in a way to make the time derivative of a candidate Lyapunov function negative pointwise [10]. Since for nonlinear affine systems the time derivative of a CLF is also affine in control signal, the control design can be formulated as a quadratic programming (QP) [1]. The QP formulation is beneficial since it can

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design the control signal in an optimal fashion at the same time it is able to add other control constraints such as prescribed control bounds [27].

The problem with QP-CLF method is that either there should be no uncertainty in the model or all the uncertainties should be bounded with known bounds. However, uncertainties are unavoidable in practical applications. Furthermore, assuming that all uncertainties are bounded is also has no practical interest. Even if we consider this assumption plausible, it does not guarantee the convergence of CLF to zero and may result in a high control effort [3].

One of the methods for handling uncertainties is adaptive control. In adaptive control uncertainties are modeled as unknown parameters with known basis functions. The key issue with the conventional adaptive methods such as model reference adaptive control (MRAC) [13], tracking-error-besed (TEB) [24], and composite adaptive method [25] is that despite ensuring that the tracking error approaches zero in the long run, the convergence of the estimated parameters to their actual values cannot be guaranteed. Ensuring the convergence of estimated parameters to their true values has some advantages such as the exponential closed-loop stability and the availability of a learned model which can be used for monitoring and planning. However, to guarantee this convergence a condition called persistent excitation (PE) of the states is required, which is energy consuming and causes excessive stress on the system [6,8,25]. The root of this issue comes from the fact that conventional adaptive control methods use only the instantaneous data in their adaptive laws. To cope with this problem, Chowdhary et al. in [8] proposed a new method called concurrent learning (CL) adaptive control method. The CL method records highly informative previous data and brings them in the adaptive laws. It is mathematically proved that if the quality of the recorded data is high enough, the simultaneous exponential convergence of the tracking and estimation errors to zero will be guaranteed.

To tackle uncertainties based on adaptive control and at the same time solve a QP-CLF to derive adaptive control signal, recently some valuable methods have been proposed. For instance, in [27] using QP-CLF and adding the safety constraints, an adaptive mechanism is considered to estimate uncertain term in the model. However, uncertainty should be bounded and modeled as an additive term. In [4] with the combination of CL and QP-CLF an adaptive control method is designed for a second-order canonical system with known basis functions. In the same direction in [5], considering a single layer hidden neural network as universal approximator with the combination of CL and QP-CLF with adding safety constraints and control bounds an adaptive control method is proposed in the presence of unstructured uncertainties. Both methods proposed in [4] and [5] only prove that the tracking and estimation errors exponentially converge to compact sets and remain there after, i.e., all the closed-loop signals are uniformly ultimately bounded (UUB). Furthermore, the above methods require the derivative of the second state for recording data which is handled via fixed-point smoothing method that may lose accuracy and cause numerical instability. To resolve the above issues, Azimi et al. in [3] proposed filtering-based CL and QP-CLF adaptive control method for the nth order canonical system. The suggested method in [3] can tackle system parameters and control coefficient at the same time and proves the exponential convergence of the tracking and estimation errors to zero. Furthermore, safety constraints and control bounds have also been met while the need for nth order derivative of the state has been relaxed.

The above methods are just applied to affine nonlinear systems. However, in practice the input nonlinearities are inevitable and they can be seen in many mechanical and electrical devices [20]. Dealing with input nonlinearities are of utmost importance, since they seriously threaten the performance of the system and ultimately jeopardize the closed-loop stability [9]. The input nonlinearities can come from two main sources; it may happen due to the nonlinear behavior of the actuators, such as saturation, dead-zone and hysteresis or the model may inherently be nonlinear in control input [11, 12]. From the above two sources of input nonlinearities the former is more common. The input nonlinearity in this paper is due to the dead-zone actuator. The dead-zone actuator which is a non-smooth input nonlinearity can be seen in servomotors, pneumatic and hydraulic valves, and electrical circuits [19, 23]. It can also severely damage the performance of the closed-loop system and even may lead the system to become unstable.

In this paper, an nth-order controllable canonical system preceded by dead-zone actuator is considered. The unknown parameters of the system and the control coefficient are estimated using the modified filtering-based CL method. In the proposed controller two terms are considered; an optimal term and a robust term. The optimal term is derived by defining a suitable QP-CLF problem which also considers the control bounds. The robust term is defined to deal with nonlinearities caused by the dead-zone actuator. In the recent methods such as [3–5] choosing the values of the control parameters has been done by trial and error. However, since there exist many control parameters and the values of parameters play an important role in closed-loop performance, this method is tedious and non-effective. In this paper, through some analysis and proposing two optimization problems, the values of the control parameters are derived. The proposed optimization problems are highly nonlinear and cannot be solved utilizing conventional methods. Therefore, a metaheuristic method based on time-varying iteration particle swarm optimization (TVIPSO) [17, 21] which is a recent variant of PSO is used. The result is the suboptimal values of the control parameters leading to reach to an effective performance with suitable control effort. Eventually, the proposed method is applied to a jerk chaotic system. Simulation results validate the efficiency and applicability of the proposed method. The contributions of this manuscript can be summarized as follows:

1. The recent methods on the combination of CL and QP-CLF such as [3–5] have focused on affine nonlinear systems. Here, the non-affine nonlinear systems are considered. The input nonlinearity is due to dead-zone which is a commonplace nonlinearity in actuators [12, 19]. The proposed method utilizes a robust structure to cope with the nonlinearity imposed by the dead-zone.

2. Since the number of control parameters is large and choosing their values is not a straightforward task, in this manuscript, a novel method is proposed to choose their values. By theoretical analysis and proposing two optimization problems and solving them using the TVIPSO [17, 21], the values of the control parameters are derived to achieve a desirable performance.

The rest of this manuscript is organized as follows: Section 2 describes the mathematical formulation of the problem. The proposed method in the presence of the dead-zone actuator and stability analysis are described in Section 3. Section 4 proposes an optimization method to derive the control parameter values. Finally, to study the capability and applicability, in Section 5 the proposed method is applied to a jerk chaotic system.

2 Problem formulation

The following non-affine nonlinear system is considered

$$x^{(n)} = a^T \varphi(X) + b\rho(X)\psi(u), \tag{1}$$

where $a = [a_1, \dots, a_m]^T \in \mathbb{R}^m$ is an unknown vector and b with $0 \neq |b| \leq b_{\max}$ is an unknown real number with known sign and bound. The state vector is defined as $X = [x, \dot{x}, \dots, x^{(n-1)}]^T \in \mathbb{R}^n$, the vector function

$\varphi(X) = [\varphi_1(X), \ldots, \varphi_m(X)]^T \in \mathbb{R}^m$ and the function $\rho(X) \neq 0$ are known. $\psi(u)$ represents the dead-zone input nonlinearity as

$$\psi(u) = \begin{cases} \kappa(u - \omega_r), & u \geq \omega_r, \\ 0, & -\omega_l < u < \omega_r, \\ \kappa(u + \omega_l), & u \leq -\omega_l. \end{cases} \tag{2}$$

where u is the control signal and κ, ω_r and ω_l are known positive constants.

The control objective is to design an adaptive control law to guarantee the exponential convergence of the tracking error $e = x - x_d$ to zero and the unknown parameters to their true values in the presence of dead-zone input nonlinearity, where x_d is the desired signal which is known, smooth and bounded.

3 Proposed adaptive control and stability analysis

3.1 Proposed adaptive control law in the presence of dead-zone

System (1) can be expressed as

$$\eta^T \xi(x^{(n)}, \varphi(X)) = \rho(X)\psi(u), \tag{3}$$

where $\eta = \frac{1}{b}[1, a^T]^T$ and $\xi(x^{(n)}, \varphi(X)) = [x^{(n)}, -\varphi^T(X)]^T$.

From (2), we have $\psi(u) = \kappa u + \mathscr{F}(u)$, where

$$\mathscr{F}(u) = \begin{cases} -\kappa\omega_r, & u \geq \omega_r, \\ -\kappa u, & -\omega_l < u < \omega_r, \\ \kappa\omega_l, & u \leq -\omega_l. \end{cases} \tag{4}$$

It can be immediately obtained that $\mathscr{F}(u)$ is bounded. Thus, system (3) can be written as

$$\frac{1}{b}e^{(n)} - \frac{1}{b}z_{opt} + \eta^T \xi(x_d^{(n)} + z_{opt}, \varphi(X)) = \kappa\rho(X)u + \frac{1}{b}d_{\mathscr{F}}(u,X), \tag{5}$$

where $d_{\mathscr{F}}(u,X) = b\rho(X)\mathscr{F}(u)$ and $|d_{\mathscr{F}}(u,X)| \leq \Delta$, with $\Delta = b_{\max}|\rho(X)|\max(\kappa\omega_r, \kappa\omega_l)$. $z_{opt} = \alpha_{opt} + \alpha_r$, where α_{opt} is an optimal signal derived via solving the quadratic programming in (16) and α_r is the robust structure defined as in (9). Defining the control law as

$$u = \frac{1}{\kappa\rho(X)} \hat{\eta}^T \xi(x_d^{(n)} + z_{opt}, \varphi(X)), \tag{6}$$

where $\hat{\eta}$ is an estimate of η. The nonlinear system (5) can be expressed as

$$e^{(n)} = \alpha_{opt} + \alpha_r + b\tilde{\eta}^T \xi(x_d^{(n)} + z_{opt}, \varphi(X)) + d_{\mathscr{F}}(u,X), \tag{7}$$

where $\tilde{\eta} = \hat{\eta} - \eta$. Also, by defining $\mathscr{E} = [e, \dot{e}, \ldots, e^{(n-1)}]^T$, (7) yields

$$\dot{\mathscr{E}} = \mathscr{A}\mathscr{E} + \mathscr{B}(\alpha_{opt} + \alpha_r + d_{\mathscr{F}}(u,X) + \mathscr{H}(\tilde{\eta})), \tag{8}$$

where $\mathscr{A} = \begin{bmatrix} \mathbf{0} & I \\ 0 & \mathbf{0} \end{bmatrix} \in \mathbb{R}^{n\times n}$, $\mathscr{B} = \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix} \in \mathbb{R}^n$, $\mathscr{H}(\tilde{\eta}) = b\tilde{\eta}^T \xi(x_d^{(n)} + z_{opt}, \varphi(X))$. The robust structure

$$\alpha_r = -(\Delta + \upsilon)\text{sgn}(\mathscr{E}^T P \mathscr{B}), \tag{9}$$

is considered, where υ is a small positive constant and $\text{sgn}(\cdot)$ is the sign function.

In the beginning, let us consider $\mathscr{H}(\tilde{\eta}) = 0$. Thus, system (8) becomes

$$\dot{\mathscr{E}} = \mathscr{A}\mathscr{E} + \mathscr{B}(\alpha_{opt} - (\Delta + \upsilon)\text{sgn}(\mathscr{E}^T P \mathscr{B}) + d_{\mathscr{F}}(u, X)). \tag{10}$$

Definition 1. *For the dynamics* (10) *a continuously differentiable function* $V(\mathscr{E}) : \mathbb{R}^n \to \mathbb{R}$ *is an exponential control Lyapunov function (ECLF) if there exist positive constants* β_1, β_2 *and* β_3 *and a set of controls* $\mathscr{U}$ *such that [3]*

$$\beta_1 \|\mathscr{E}\|^2 \le V(\mathscr{E}) \le \beta_2 \|\mathscr{E}\|^2,$$
$$\inf_{\alpha_{opt} \in \mathscr{U}} \dot{V}(\mathscr{E}, \alpha_{opt}) \le -\beta_3 V(\mathscr{E}), \tag{11}$$

where $\dot{V}(\mathscr{E}, \alpha_{opt})$ *is the time derivative of* $V(\mathscr{E})$ *considering* (10).

The following Lyapunov function candidate is considered for the purpose of formulating an ECLF-based controller when $\mathscr{H}(\tilde{\eta}) = 0$

$$V(\mathscr{E}) = \mathscr{E}^T P \mathscr{E}, \tag{12}$$

where $P = P^T \succ 0$ is the solution of the following algebraic Riccati equation

$$\mathscr{A}^T P + P\mathscr{A} - P\mathscr{B}\mathscr{B}^T P + Q = 0, \tag{13}$$

with $Q = Q^T \succ 0$.

Using (10) the time derivative of (12) becomes

$$\begin{aligned}\dot{V}(\mathscr{E}, \alpha_{opt}) &= \mathscr{E}^T(\mathscr{A}^T P + P\mathscr{A})\mathscr{E} + 2\mathscr{E}^T P\mathscr{B}\alpha_{opt} - 2(\Delta + \upsilon)\left|\mathscr{E}^T P\mathscr{B}\right| + 2\mathscr{E}^T P\mathscr{B} d_{\mathscr{F}}(u, X) \\ &\le \mathscr{E}^T(\mathscr{A}^T P + P\mathscr{A})\mathscr{E} + 2\mathscr{E}^T P\mathscr{B}\alpha_{opt} - 2(\Delta + \upsilon)\left|\mathscr{E}^T P\mathscr{B}\right| + 2\left|\mathscr{E}^T P\mathscr{B}\right|\Delta \\ &\le \mathscr{E}^T(\mathscr{A}^T P + P\mathscr{A})\mathscr{E} + 2\mathscr{E}^T P\mathscr{B}\alpha_{opt}.\end{aligned} \tag{14}$$

Based on Definition 1, in order to guarantee $V(\mathscr{E})$ to be an ECLF, a family of controllers α_{opt} should be searched to satisfy the following constraint:

$$\Upsilon_0(\mathscr{E}) + \Upsilon_1(\mathscr{E})\alpha_{opt} \le 0, \tag{15}$$

where $\Upsilon_0(\mathscr{E}) = \mathscr{E}^T(\mathscr{A}^T P + P\mathscr{A})\mathscr{E} + \mu V(\mathscr{E})$ and $\Upsilon_1(\mathscr{E}) = 2\mathscr{E}^T P\mathscr{B}$ with μ is a chosen positive constant. It should be noted that constraint (15) yields $\dot{V}(\mathscr{E}, \alpha_{opt}) \le -\mu V(\mathscr{E})$ and in turn guarantees $V(\mathscr{E})$ to be exponentially convergent. In order to find the α_{opt}, the following QP can be considered:

$$\begin{aligned}\alpha_{opt} = &\underset{\alpha \in \mathbb{R}}{\text{argmin}} \frac{1}{2}\alpha^2, \\ \text{s.t.} \quad &\Upsilon_0(\mathscr{E}) + \Upsilon_1(\mathscr{E})\alpha \le 0.\end{aligned} \tag{16}$$

Based on the discussions above and [3], $V(\mathscr{E})$ is a valid ECLF in the absence of $\mathscr{H}(\widetilde{\eta})$. Obviously, in the presence of $\mathscr{H}(\widetilde{\eta})$ we have

$$\begin{aligned}\dot{V}(\mathscr{E},\alpha_{opt}) &\leq -\mu V(\mathscr{E}) + 2\mathscr{E}^T P\mathscr{B}b\widetilde{\eta}^T\xi(x_d^{(n)} + z_{opt}, \varphi(X)) \\ &\leq -\mu V(\mathscr{E}) + \nu(\|\widetilde{\eta}\|),\end{aligned} \tag{17}$$

where $\nu(\cdot) \in \mathscr{K}_\infty$. Evidently if $\widetilde{\eta}$ does not converge to zero, the exponential convergence of $V(\mathscr{E})$ is no longer guaranteed. Therefore, a suitable adaptive law should be used to guarantee the exponential convergence of $\widetilde{\eta}$ to zero. Based on the same discussion as in [3], in order to handle the unavailability of $x^{(n)}$, by filtering both sides of (3) with stable filter $H(s) = \frac{\varsigma}{s+\varsigma}$, we have

$$\begin{aligned}&\int_0^t h(t-\tau)\eta^T\xi(x^{(n)}, \varphi(X))d\tau = \int_0^t h(t-\tau)\rho(X)\psi(u)d\tau, \\ \Rightarrow &\int_0^t h(t-\tau)[\frac{1}{b}x^{(n)} - \frac{a^T}{b}\varphi(X)]d\tau = \int_0^t h(t-\tau)\rho(X)\psi(u)d\tau,\end{aligned} \tag{18}$$

where $h(t) = \varsigma e^{-\varsigma t}$ with $\varsigma > 0$. Using partial integration, we have

$$\frac{1}{b}\int_0^t h(t-\tau)x^{(n)}d\tau = \frac{1}{b}\left[h(0)x^{(n-1)}(t) - h(t)x^{(n-1)}(0) - \int_0^t \frac{d}{d\tau}(h(t-\tau))x^{(n-1)}(\tau)d\tau\right]. \tag{19}$$

Thus,

$$\int_0^t h(t-\tau)\eta^T\xi(x^{(n)}, \varphi(X))d\tau = \eta^T\xi_f(X), \tag{20}$$

where

$$\xi_f(X) = \begin{bmatrix} h(0)x^{(n-1)}(t) - h(t)x^{(n-1)}(0) - \int_0^t \frac{d}{d\tau}(h(t-\tau))x^{(n-1)}(\tau)d\tau \\ -\int_0^t h(t-\tau)\varphi(X(\tau))d\tau \end{bmatrix}. \tag{21}$$

Therefore, using (18) we have

$$\eta^T\xi_f(X) = \int_0^t h(t-\tau)\rho(X)\psi(u)d\tau. \tag{22}$$

The adaptive law is proposed as follows

$$\dot{\widehat{\eta}} = -\Gamma\left(\xi(x_d^{(n)} + z_{opt}, \varphi(X))\mathscr{E}^T P\mathscr{B}\mathrm{sgn}(b) + \sum_{i=1}^r \xi_f(X_i)\varepsilon_i^T\right), \tag{23}$$

where $\Gamma \succ 0 \in \mathbb{R}^{(m+1)\times(m+1)}$ is the adaption gain matrix, X_i is the ith recorded state vector and $\varepsilon = \widehat{\eta}^T\xi_f(X) - \eta^T\xi_f(X) = \widetilde{\eta}^T\xi_f(X)$, where ε_i is calculated for the ith recorded state vector. It should be noted that to realize ε, $\eta^T\xi_f(X)$ is not needed and it is obtained through the integral in the right hand side of (22). Furthermore, the stored data should be chosen to make the matrix $\Theta = [\xi_f(X_1), \ldots, \xi_f(X_r)] \in \mathbb{R}^{(m+1)\times r}$ highly informative and $\Theta\Theta^T \succ 0$. For implementation, this can be done by choosing the vector which is sufficiently far away from the previous recorded vector, i.e., mathematically if

$$\frac{\|\xi_f(X) - \xi_{fp}\|^2}{\|\xi_f(X)\|} \geq \delta,$$

then $\xi_f(X)$ is recorded, where ξ_{fp} is the previously recorded vector and δ is a design positive constant. It is noteworthy that r, which is the number of recorded data, should satisfy $r \geq m+1$. Using the adaptive law in (23), in Subsection 3.2 it will be proved that $\widetilde{\eta}$ converges to zero exponentially.

Remark 1. *After Θ is completed, i.e., the number of columns becomes r_{max}, where r_{max} is the maximum value of r chosen by the designer, the new data $\xi_f(X)$ will be chosen if it increases the $\sigma_{min}(\Theta)$, i.e, the minimum singular value of Θ. To this end, Algorithm 1 in [7, 8] is utilized. The algorithm successively replaces each column of Θ with the current data point $\xi_f(X)$ and stores each obtained minimum singular value in a variable. Then the maximum of these values is found, and if the resulting configuration increases the current minimum singular value, the algorithm will replace the corresponding data point with the new data point $\xi_f(X)$.*

Remark 2. *In practice usually the control signal u should be bounded by prescribed known lower and upper bounds, i.e., for given positive constants u_{max} and u_{min} we should have $-u_{min} \leq u \leq u_{max}$. Therefore, to avoid having aggressive control signal and at the same time to have a numerically robust problem, the QP in (16) is modified as*

$$\begin{aligned}
\alpha_{opt} = & \underset{(\alpha,\theta)\in\mathbb{R}^2}{argmin} \frac{1}{2}\alpha^2 + \vartheta\theta^2, \\
s.t. \quad & \Upsilon_0(\mathscr{E}) + \Upsilon_1(\mathscr{E})\alpha \leq \theta, \\
& \frac{1}{\hat{b}\kappa\rho(X)}\alpha \leq u_{max} - \frac{1}{\kappa\rho(X)}\widehat{\eta}^T\xi(x_d^{(n)} + \alpha_r, \varphi(X)), \\
& -\frac{1}{\hat{b}\kappa\rho(X)}\alpha \leq u_{min} + \frac{1}{\kappa\rho(X)}\widehat{\eta}^T\xi(x_d^{(n)} + \alpha_r, \varphi(X)),
\end{aligned} \tag{24}$$

where θ is the slack variable to prioritize bounded control and smooth control signal over tracking. It should be noted that based on the same discussion in [26], if the QP is feasible, the weight ϑ can be chosen large enough to make the slack variable $\theta_{opt} \approx 0$.

3.2 Stability analysis

To prove the exponential stability of the closed-loop system the following theorem is expressed.

Theorem 1. *Consider system (1) preceded by the dead-zone input nonlinearity in (2). Suppose the control law in (6), the robust term in (9), the QP in (16) and the adaptive law in (23). If x_d is able to derive informative data i.e., for $r \geq m+1$, Θ is full rank and thus $\Theta\Theta^T$ is positive-definite, then $\mathscr{E}$ and $\widetilde{\eta}$ will exponentially tend to zero for any $\mathscr{E}(0)$ and for unknown parameters $\eta \in \mathbb{R}^{m+1}$.*

Proof. Consider the following candidate for the Lyapunov function

$$V_T(\mathscr{E},\widetilde{\eta}) = V(\mathscr{E}) + \widetilde{\eta}^T\Gamma^{-1}|b|\widetilde{\eta}. \tag{25}$$

The time derivative of $V_T(\mathscr{E},\widetilde{\eta})$ using (12)-(17) yields

$$\dot{V}_T(\mathscr{E},\widetilde{\eta}) \leq -\mu\mathscr{E}^T P\mathscr{E} + 2\widetilde{\eta}^T|b|\Gamma^{-1}\left(\Gamma\mathscr{E}^T P\mathscr{B}\xi(x_d^{(n)} + z_{opt}, \varphi(X))\text{sgn}(b) + \dot{\widehat{\eta}}\right).$$

Using the adaptive law (23) we have

$$\dot{V}_T(\mathscr{E},\widetilde{\eta}) \leq -\mu\mathscr{E}^T P\mathscr{E} - 2|b|\widetilde{\eta}^T\Theta\Theta^T\widetilde{\eta},$$

where $\Theta\Theta^T \succ 0$. Thus,

$$\dot{V}_T(\mathscr{E},\widetilde{\eta}) \leq -\mu\lambda_{\min}(P)\,\|\mathscr{E}\|^2 - 2\,|b|\,\lambda_{\min}(\Theta\Theta^T)\,\|\widetilde{\eta}\|^2 \\ \leq -\varpi V_T(\mathscr{E},\widetilde{\eta}), \tag{26}$$

where $\varpi = \min\{\mu\frac{\lambda_{\min}(P)}{\lambda_{\max}(P)}, 2\frac{\lambda_{\min}(\Theta\Theta^T)}{\lambda_{\max}(\Gamma^{-1})}\} > 0$ with $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ are minimum and maximum eigenvalues of a square matrix. Thus, based on the same discussion as in [8, 16], $V_T(\mathscr{E},\widetilde{\eta})$ tends to zero exponentially as $t\to\infty$, consequently $(\mathscr{E},\widetilde{\eta})$ exponentially tends to zero as $t\to\infty$. Since $V_T(\mathscr{E},\widetilde{\eta})$ is radially unbounded, the stability proof is global. □

4 The parameter values of the proposed adaptive control

To apply the proposed method, the values of the positive parameters μ, ς, δ, and r, and the positive-definite matrices Q and Γ should be appropriately chosen. In order to decrease burden of computation, Q and Γ are considered diagonal. Given (26) to have a desirable exponential convergence and at the same time to improve the performance of the parameter estimations, we need to increase $\mu\frac{\lambda_{\min}(P)}{\lambda_{\max}(P)}$ and $\frac{\lambda_{\min}(\Theta\Theta^T)}{\lambda_{\max}(\Gamma^{-1})}$. One may propose to increase μ and $\lambda_{\min}(\Gamma) = \frac{1}{\lambda_{\max}(\Gamma^{-1})}$, simultaneously without considering P and $\Theta\Theta^T$. However, increasing $\lambda_{\min}(\Gamma)$ means increasing adaptation gain which can stimulate unmodeled dynamics and in practice can make the closed-loop system unstable. Moreover, the differential equation of the adaptive law becomes stiff, and numerically unstable. On the other hand, increasing μ will increase the control effort which is also undesirable in practical use. Usually $\mu = 1$ and $\Gamma = I \in \mathbb{R}^{(m+1)\times(m+1)}$ are suitable choices. Based on the discussion in Subsection 3.1 for r, the number of recorded data, we should have $m+1 \leq r \leq r_{\max}$. $r_{\max}$ is limited due to memory usage and processing capability. In this paper, to make $\Theta\Theta^T$ highly informative, we choose $r = r_{\max}$. In order to choose the matrix Q which for decreasing the burden of computation is considered diagonal i.e., $Q = \mathrm{diag}[q_1,\ldots,q_n]$, the following optimization problem is proposed

$$\max_{q_1,\ldots,q_n} \frac{\lambda_{\min}(P)}{\lambda_{\max}(P)}. \\ \text{s.t. (13).} \tag{27}$$

Based on the discussion in Section 3, to have a desirable parameter estimation we need to choose the recording data such that $\lambda_{\min}(\Theta\Theta^T) = \sigma^2_{\min}(\Theta)$ is maximized where $\sigma_{\min}(\cdot)$ is the minimum singular value of a matrix. Algorithm 1 proposed in [7,8] and described in Remark 1, enlarges $\sigma_{\min}(\Theta)$, however, the values of the parameters ς and δ should be chosen in advance. In this paper, we optimize Algorithm 1 in [7, 8] by defining the following optimization problem

$$\max_{\varsigma,\delta} \sigma_{\min}(\Theta). \tag{28}$$

Solving the proposed optimization problems in (27) and (28) not only gives us the values of the control parameters but also dramatically enriches the performance. However, (27) and (28) are nonlinear optimization problems that cannot be solved using conventional methods. In order to find a suboptimal solution, we can utilize metaheuristic search algorithms [14].

There are several metaheuristic methods, the most effective ones are population-based algorithms. For instance, genetic algorithm (GA), ant colony optimization (ACO), and particle swarm optimization (PSO) are the three most commonly used population-based metaheuristic methods. In this manuscript, PSO is considered, which was introduced by Kennedy and Eberhart in [15]. The idea and formulation of the PSO algorithm were inspired by observing the social behavior of groups of birds and fish.

It has been observed that the primary PSO suffers from insufficient performance and sluggish convergence, especially for problems with multiple local optima [2]. In order to enrich the performance of the primary PSO several variants have been proposed. One of the simple yet effective variants is iteration PSO (IPSO), which by adding a new term enhances the performance of the PSO and decreases the convergence time [17, 21, 22]. Suppose that the solution space is n_d-dimensional, then in the beginning IPSO generates n_p number of random particles i.e., potential solutions. Each particle i at iteration k has a position vector and a velocity vector as $P_i^k = [p_{i1}^k, \dots, p_{in_d}^k]$ and $V_i^k = [v_{i1}^k, \dots, v_{in_d}^k]$, respectively. Consider the best position found by ith particle until the current iteration is denoted by $P_{best_i}^k = [p_{best_{i1}}^k, \dots, p_{best_{in_d}}^k]$ and the best position among all $P_{best_i}^k$ is denoted by $G_{best}^k = [G_{best_1}^k, \dots, G_{best_{n_d}}^k]$. Besides, $r_{best}^k = [r_{best_1}^k, \dots, r_{best_{n_d}}^k]$ is the best position achieved by any particle at iteration k. The update laws are as follows [17]

$$v_{ij}^{k+1} = \omega v_{ij}^k + c_1 r_1 (p_{best_{ij}}^k - p_{ij}^k) + c_2 r_2 (G_{best_j}^k - p_{ij}^k) + c_3 r_3 (r_{best_j}^k - p_{ij}^k),$$
$$p_{ij}^{k+1} = p_{ij}^k + v_{ij}^{k+1},$$

where ω is the inertia weight, c_1, c_2, and c_3 are acceleration coefficients, r_1, r_2, and r_3 are three random numbers generated uniformly from $[0,1]$.

However, the parameters of IPSO (c_1, c_2, ω, and c_3) should still be chosen in advance. In order to achieve both adaptive updating of the parameters and maintain the quality of the IPSO, in [21] a time-varying IPSO (TVIPSO) is proposed to derive the parameter values of IPSO in each iteration using the following equations

$$c_1 = c_{10} + \frac{c_{1f} - c_{10}}{k_{\max}} k, \quad c_2 = c_{20} + \frac{c_{2f} - c_{20}}{k_{\max}} k,$$
$$c_3 = c_1 (1 - e^{-c_2 k}), \qquad \omega = \omega_0 + \frac{\omega_f - \omega_0}{k_{\max}} k,$$

where c_{10}, c_{20}, ω_0 are the initial values and c_{1f}, c_{2f}, ω_f are the final values, furthermore, $k_{\max}$ is the maximum number of iterations. Also, in [21], the initial and final values $c_{10} = 1.75$, $c_{1f} = 0.5$, $c_{20} = 0.5$, $c_{2f} = 2$, $\omega_0 = 0.9$ and $\omega_f = 0.4$ are suggested.

5 Simulation example

To verify the efficiency and applicability of the proposed method, a third-order chaotic system is considered [18]

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = x_3, \quad \dot{x}_3 = a_1 x_1 + a_2 x_2 + a_3 x_3 + a_4 x_3^2 + a_5 x_1 x_2 + \psi(u), \tag{29}$$

where $a_1 = -2$, $a_2 = -1$, $a_3 = -1.1$, $a_4 = -0.3$ and $a_5 = 1$. Comparing (29) with (1), it can be seen that $n = 3$, $m = 5$, $a = [-2, -1, -1.1, -0.3, 1]^T$, $\varphi(X) = [x_1, x_2, x_3, x_3^2, x_1 x_2]^T$, $b = 1$, $\rho(X) = 1$ and

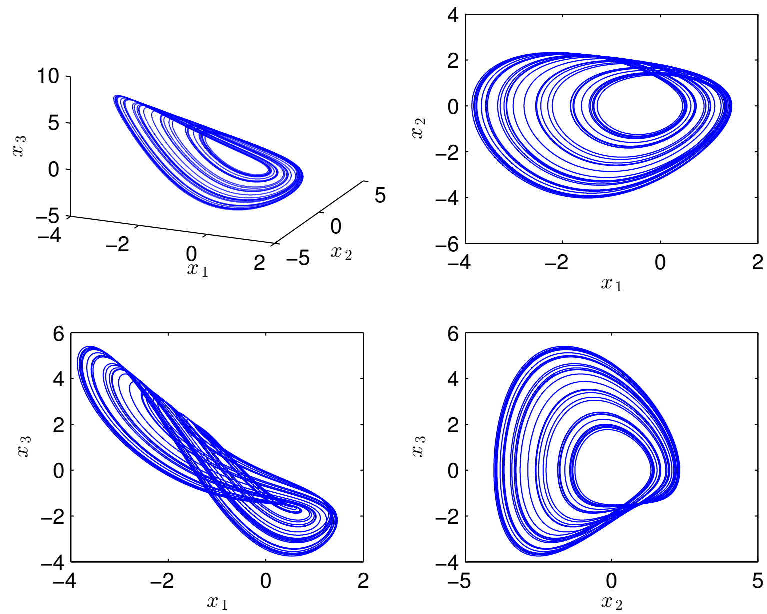


Figure 1: The chaotic attractor of system (29) without external force $\psi(u) = 0$ and $X(0) = [0.1, 0.1, 0.1]^T$.

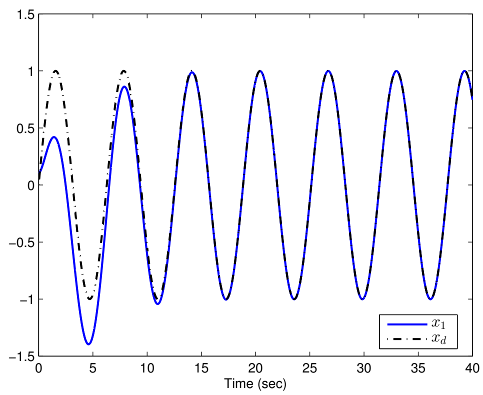
$X = [x_1, x_2, x_3]^T$. Furthermore, $\psi(u)$ represents the dead-zone input nonlinearity where u is the control input. Firstly, the external force $\psi(u) = 0$ is assumed. Figure 1 demonstrates chaotic attractor of the system without any external force for the initial condition $X(0) = [0.1, 0.1, 0.1]^T$. From this figure the chaotic behavior of the system can be easily seen.

The control objective is to design an adaptive control law to make x_1, x_2 and x_3 track $x_d = \sin(t)$, $\dot{x}_d = \cos(t)$ and $\ddot{x}_d = $ -$\sin(t)$, respectively, and to estimate the unknown parameters a_1, a_2, a_3, a_4, a_5 and b. Meanwhile, the control signal should be kept in $[-10, 10]$. To challenge the applicability of the proposed method, $\kappa = 1.2$, $\omega_r = 2$ and $\omega_l = 1$ are considered which shows a significant dead-zone. Without loss of generality $b_{\max} = 3$ is considered, besides, $\upsilon = 0.1$ is assumed. The proposed method is applied considering the control law in (6), the robust term in (9), the QP in (24) with $u_{\min} = u_{\max} = 10$ and $\vartheta = 30$ and the adaptive law in (23) with $\widehat{\eta}(0) = [2, 0, 0, 0, 0, 0]^T$. To avoid chattering phenomenon in practice, the sgn$(\cdot)$ function in (9) is approximated by $\tanh(\frac{\cdot}{\iota})$ with $\iota = 0.01$. It should be noted that the parameter ι is a trade off between accuracy and chattering. As stated in Section 4, $\mu = 1$ and $\Gamma = I_{6\times 6}$ are recommended, furthermore the number of recorded data $r = 100$ is considered. This number can be decreased or increased considering the limitation on memory and processing capability, but in any case it should be greater than 6. To choose the parameters q_i, $i = 1, 2, 3$ and ς and δ we apply the proposed optimization problems in (27) and (28). To this end, the intervals for the parameters $q_i \in [0.01, 100]$, $i = 1, 2, 3$, $\varsigma \in [1, 30]$ and $\delta \in [0.01, 1]$ are considered. To solve the optimization problem in (27) the TVIPSO with $k_{\max} = 500$ and $n_p = 10$ is used. Meanwhile, the optimization problem in (28) is solved by TVIPSO with $k_{\max} = 20$ and $n_p = 3$. The suboptimal parameters are obtained as in Table 1.

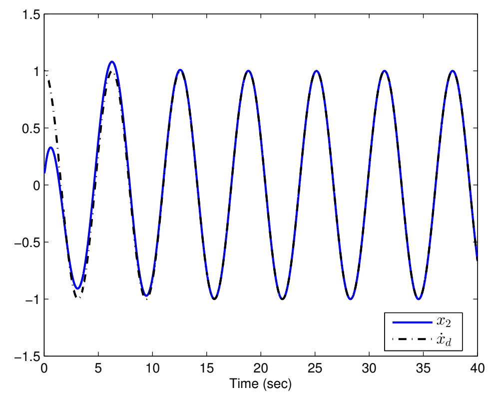
The simulation results using the suboptimal control parameters in Table 1 can be seen in Figures 2-5.

Table 1: Suboptimal parameter values obtained using the proposed method in Section 4.

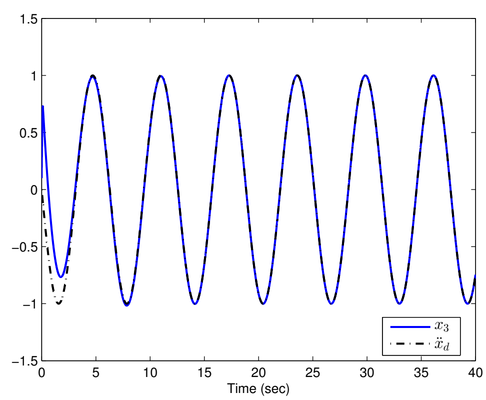
ς^*	δ^*	q_1^*	q_2^*	q_3^*
19.05923	0.02793	1	8.4186	8.4186



(a)

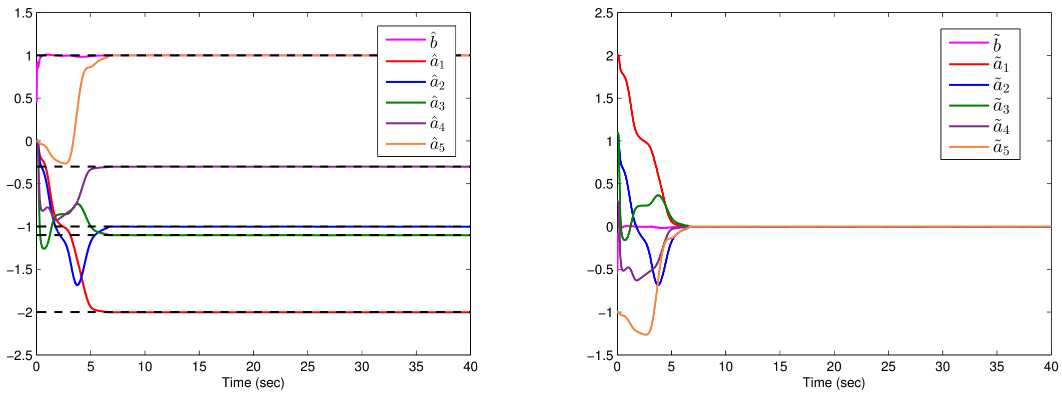


(b)



(c)

Figure 2: (a) x_1 and x_d (b) x_2 and $\dot{x}_d$ (c) x_3 and $\ddot{x}_d$.



(a) (b)

Figure 3: (a) The estimated parameters and the true values (b) The estimation errors.

Figure 2 shows the states and desired signals. It can be seen that satisfactory tracking is achieved for all three states. The estimation of the unknown parameters of the model is demonstrated in Figure 3. It can be understood that after about 5 seconds all estimations converge to the true values exponentially. Figures 4 (a)-(c), respectively show control signal u, the term z_{opt}, and the output of the dead-zone actuator $\psi(u)$. It can be seen that after the convergence of the states and parameters, u, z_{opt} and $\psi(u)$ are effectively reached their steady states. Figure 5 shows the evolution of the minimum singular value of the matrix Θ using the suboptimal values in Table 1.

6 Conclusion

A concurrent learning adaptive control for a class of nonlinear systems in the presence of dead-zone input nonlinearity is proposed. A feedback control law is proposed which contains an optimal term and a robust structure. The optimal term is derived by proposing a suitable quadratic programming (QP) based on control Lyapunov function (CLF). The proposed QP-CLF can also handle the prescribed control bounds. The robust term is designed to tackle the input nonlinearity caused by the dead-zone. The exponential convergence of the tracking and parameter estimation errors is analyzed using Lyapunov approach. The recent scholars usually tune the control parameters by trial and error, however, when the number of control parameters is large this becomes a tedious task. In this paper, to derive the values of the control parameters, with some theoretical analysis, two optimization problems have been proposed and solved using time-varying iterative particle swarm optimization. A jerk chaotic system is simulated to demonstrate the efficiency and applicability of the proposed method. In this work, the dead-zone as a non-smooth input nonlinearity with known parameters is considered. In future work, the unknown parameters case and other types of non-smooth nonlinearities such as backlash, saturation, and hysteresis can be studied.

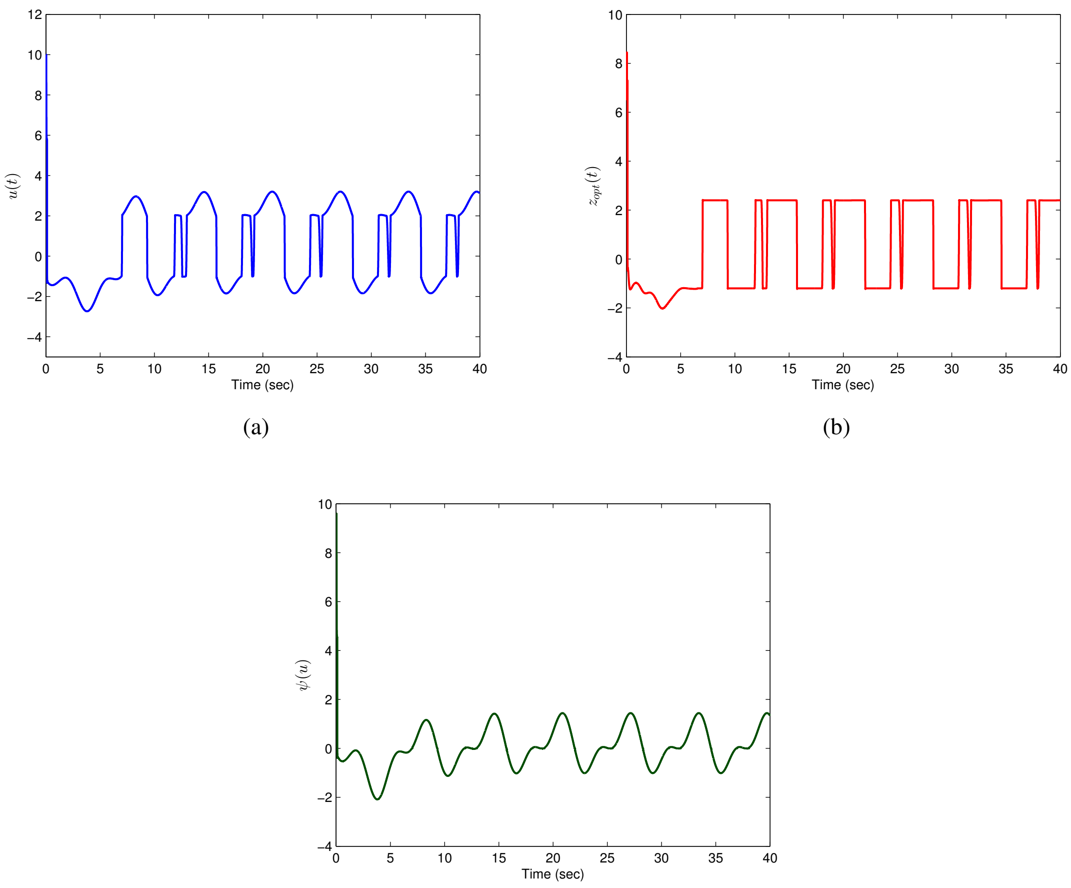


Figure 4: Control signals (a) u (b) z_{opt} (c) $\psi(u)$.

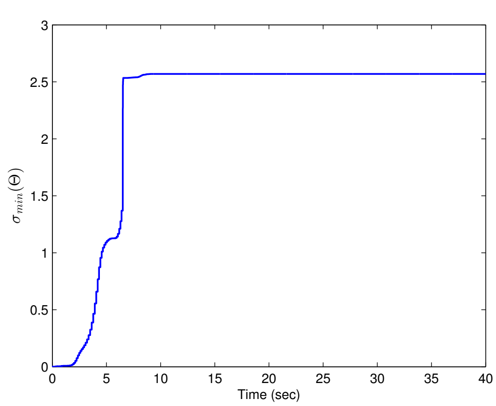


Figure 5: Evolution of $\sigma_{\min}(\Theta)$.

References

[1] A.D. Ames, J.W. Grizzle, P. Tabuada, *Control barrier function based quadratic programs with application to adaptive cruise control*, 53rd IEEE Conference on Decision and Control, IEEE, Dec 2014.

[2] H. Aminikhah, A. Jamalian, *A new algorithm for computing eigenpairs of matrices*, Math. Comput. Model. **54** (2011) 119–130.

[3] V. Azimi, S. Hutchinson, *Exponential control Lyapunov-barrier function using a filtering-based concurrent learning adaptive approach*, EEE Trans. Autom. Control. **67** (2022) 5376–5383.

[4] V. Azimi, P. Vela, *Performance reference adaptive control: A joint quadratic programming and adaptive control framework*, 2018 Annual American Control Conference (ACC), IEEE, Jun 2018.

[5] V. Azimi, P. Vela, *Robust adaptive quadratic programming and safety performance of nonlinear systems with unstructured uncertainties*, 2018 IEEE Conference on Decision and Control (CDC), IEEE, Dec 2018.

[6] G. Chowdhary, E. Johnson, *Concurrent learning for convergence in adaptive control without persistency of excitation*, 49th IEEE Conference on Decision and Control (CDC), IEEE, Dec 2010.

[7] G. Chowdhary, E. Johnson, *A singular value maximizing data recording algorithm for concurrent learning*, Proceedings of the 2011 American Control Conference, IEEE, Jun 2011.

[8] G. Chowdhary, M. Muhlegg, E. Johnson, *Exponential parameter and tracking error convergence guarantees for adaptive controllers without persistency of excitation*, Int. J. Control **87** (2014) 1583–1603.

[9] X. Fang, H. Fan, W. Wang, L.L., B. Wang, Z. Cheng, *Adaptive finite-time fault-tolerant control of uncertain systems with input saturation*, IEEE Trans. Syst. Man Cybern. Syst. **53** (2023) 165–177.

[10] R.A. Freeman, P. Kokotovic, *Robust Nonlinear Control Design*, Birkhäuser Boston, 1996.

[11] S.S. Ge, C. Hang, T. Zhang, *Nonlinear adaptive control using neural networks and its application to CSTR systems*, J. Process Control **9** (1999) 313–323.

[12] X. Guo, H. Yao, F. Gao, *Global prescribed-time stabilization of high-order nonlinear systems with asymmetric actuator dead-zone*, Mathematics **10** (2022) 2147.

[13] P.A. Ioannou, J. Sun, *Robust adaptive control*, PTR Prentice-Hall Upper Saddle River, NJ, 1996.

[14] A. Jamalian, H. Aminikhah, *A novel algorithm for computing interior eigenpairs of large nonsymmetric matrices*, Soft Comput. **25** (2021) 11865–11876.

[15] J. Kennedy, R. Eberhart, *Particle swarm optimization*, IEEE, In Proceedings of ICNN'95-International Conference on Neural Networks **4** (1995) 1942-1948.

[16] H.K. Khalil, *Nonlinear Control*, Pearson Education, 2013.

[17] T.Y. Lee, C.L. Chen, *Unit commitment with probabilistic reserve: An IPSO approach*, Energy Conv. Manag.**48** (2007) 486–493.

[18] M. Liu, B. Sang, N. Wang, I. Ahmad, *Chaotic dynamics by some quadratic jerk systems*, Axioms **10** (2021) 227.

[19] X. Luo, D. Mu, Z. Wang, P. Ning, C. Hua, *Adaptive full-state constrained tracking control for mobile robotic system with unknown dead-zone input*, Neurocomputing **524** (2023) 31–42.

[20] S.A. Malek, M. Shahrokhi, E. Vafa, A. Moradvandi, *Adaptive prescribed performance control of switched MIMO uncertain nonlinear systems subject to unmodeled dynamics and input nonlinearities*, Int. J. Robust Nonlinear Control.**28** (2018) 5981–5996.

[21] B. Mohammadi-ivatloo, A. Rabiee, M. Ehsan, *Time-varying acceleration coefficients IPSO for solving dynamic economic dispatch with non-smooth cost function*, Energy Convers. Manag.**56** (2012) 175–183.

[22] T.M. Shami, A.A. El-Saleh, M. Alswaitti, Q. Al-Tashi, M.A. Summakieh, S. Mirjalili, *Particle swarm optimization: A comprehensive survey*, IEEE Access **10** (2022) 10031–10061.

[23] W. Shi, R. Luo, B. Li, *Adaptive fuzzy prescribed performance control for MIMO nonlinear systems with unknown control direction and unknown dead-zone inputs*, ISA Transactions **66** (2017) 86–95.

[24] J.J.E. Slotine, J.A. Coestee, *Adaptive sliding controller synthesis for non-linear systems*, Int. J. Control **43** (1986) 1631–1651.

[25] J.J.E. Slotine, W. Li, *Composite adaptive control of robot manipulators*, Automatica **25** (1989) 509–519.

[26] X. Xu, P. Tabuada, J. Grizzle, A. Ames, *Robustness of control barrier functions for safety critical control*, IFAC-PapersOnLine **48** (2015) 54–61.

[27] P. Zhao, Y. Mao, C. Tao, N. Hovakimyan, X. Wang, *Adaptive robust quadratic programs using control Lyapunov and barrier functions*, 59th IEEE Conference on Decision and Control (CDC), Dec 2020.