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Analytical Approach Based on Tamimi-Ansari method for Solving Nonlinear Equations with Applications

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ABSTRACT

In this article, a new but powerful analytical method, the Tamimi-Ansari method (TAM), has been introduced to solve some nonlinear problems that have been used in physics. This method does not require any hypothesis to counter with the nonlinear term. These results are compared with the exact solution and two other analytical methods. A few examples have been presented to show that this method is effective and reliable.

1. Introduction

Nonlinear differential equations play an advanced role when modeling nonlinear real-life phenomena. These nonlinear phenomena may appear in a wide range of engineering, physical, mechanical and biological problems, and the need to provide accurate solutions to these problems is considered a challenge. This challenge comes from the fact that these models are nonlinear in their nature and the solutions for these models are hard to find. Many analytical methods that deal with providing a solution to these problems are employed either idealization or simplification to the original problem to overcome this issue. Perturbation methods for example entail a solution in a form of a power series expansion with certain physical conditions with some unknown coefficients that need to be evaluated to satisfy the problem [1]. These methods have proven over the last few years to be an effective and efficient tool for supplying accurate results to different types of nonlinear problems. It is worth mentioning that these methods do not provide exact solutions but they can provide an approximate solution that is acceptable to these models of nonlinear behavior.

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One of the promising methods that provide accurate results is the Temimi and Ansari method (TAM). This method was first introduced back in 2011 by Temimi and Ansari [2] and it was originated from the Homotopy perturbation method (HAM) [3]. It has been used ever since in solving different types of equations. For example, Al-Jawary et.al [4] have used the method for solving the well-known duffing equation with damping and undamping type equations. Tamimi et al. [5] used the self-invented novel method for solving different coupled nonlinear differential equations. Hussien et.al [6] adopted this method for solving the famous KdV type equations showing the capability of this proposed technique of providing accurate results. Some problems with application in chemistry have been solved the same technique by Al-Jawary et.al in [7]. In addition, Al-Jawary et.al solved the thin film problem in [8] using the same method. For more details about the methods and their application to physical and engineering problems, One may refer to [9-17] and the references therein.

In this paper, we are concerned with the application of the TAM method on some application problems with applications in heat transfer. These types of equations are nonlinear with different order and represent an important tool when modeling cooling using radiation or convection. In addition, the porous medium equation in the form of a nonlinear PDE is being solved along with the nonlinear heat equation with cubic nonlinearity. The results are compared with the exact solution for the problem to prove the efficiency and validity of this proposed method. The main contribution of the current paper is to implement this semi-analytic method for solving different types of nonlinear ODE and PDE type equations.

The organization of the paper is as follows: Section 2 reviews the basic idea and steps of the TAM method. In section 3 the results for solving different type of problem which provides accurate results. Section 4 is the closing stage that provides a conclusion for the study.

2. The fundamental of the Tamimi-Ansari method (TAM)

First, we present the general form for the nonlinear differential equation

$$L(y(x)) + g(x) + N(y(x)) = 0, \quad B(y, \frac{dy}{dx}) = 0. \tag{1}$$

where x represents the independent variable, $y(x)$ is the unknown function, $g(x)$ is a given known function, $L(.) = \frac{d^2}{dx^2}(.)$ is the linear operator, $N(.)$ is the nonlinear operator, and $B(.)$ is a boundary operator.

We first start by assuming that y_0 is an initial guess to solve the problem and that the solution starts via solving the following initial value problem [2]

$$L(y_0(x)) + g(x) = 0, \quad B(y_0, \frac{dy_0}{dx}) = 0. \tag{2}$$

The next estimated solutions are captured by solving the following problems

$$L(y_1(x)) + g(x) + N(y_0(x)) = 0, \quad B(y_1, \frac{dy_1}{dx}) = 0. \tag{3}$$

Therefore, we have a simple repetitive procedure that is the solution to a group of problems

$$L(y_{n+1}(x)) + g(x) + N(y_n(x)) = 0, \quad B(y_{n+1}, \frac{dy_{n+1}}{dx}) = 0. \tag{4}$$

Then, that the solution for Eq. (1) is given by

$$y = \lim_{n \to \infty} y_n. \tag{5}$$

The convergence of this method had been investigated in [12].

In the next section, we will apply the novel presented method to some application problems that arise in the modeling of heat transfer and porous medium equation.

3. Application

Example 1. (Cooling of a lumped system by mixed convection and radiation)

In this first example, we examine an application problem in heat transfer [18]. We first assume that the system has a surface area A, specific heat c, volume V, emissivity E, density ρ, and initial temperature T_0. The system is exposed to the surrounding with a convection heat shift with a coefficient h and a temperature T_a. The heat from the system is also reduced through radiation with sink temperature T_s. Thus, we can construct a Colling equation in the form

$$\rho V c \frac{dT}{dt} + hA(T - T_a) + E\sigma A(T^4 - T_S^4) = 0 \tag{6}$$

with conditions

$$t = 0, T = T_i. \tag{7}$$

For solving the equation, we do the following transforms of parameters

$$\theta = \frac{T}{T_i}, \theta_a = \frac{T_a}{T_i}, \theta_s = \frac{T_s}{T_i}, \tau = \frac{hAt}{\rho V c_a}, \varepsilon = \frac{E\sigma T_i^3}{h}. \tag{8}$$

After parameter transfer, the heat transfer equation will be like the following

$$\frac{d}{d\tau}\theta(\tau) + (\theta(\tau) - \theta_a) + \varepsilon(\theta(\tau)^4 - \theta_s^4) = 0, \tag{9}$$

with the condition $\theta(0) = 1$. For making it simple, we consider the case $\theta_a = \theta_s = 0$. Therefore, we have

$$\frac{d}{d\tau}\theta(\tau) + \theta(\tau) + \varepsilon\theta(\tau)^4 = 0, \ \theta(0) = 1. \tag{10}$$

To solve Eq. (10) we utilize TAM, we build a correcting practical, as pursues

$$L(\theta_{n+1}(\tau)) + N(\theta_n(\tau)) = 0, \ \theta_{n+1}(0) = 1, \ \theta_0(\tau) = e^{-\tau}. \tag{11}$$

Firstly, to get the zero estimation $\theta_0(\tau)$, the following initial issue must be determined. we consider the following

$$L(\theta_0) = -\theta \text{ with } \theta_0(0) = 1. \quad \textbf{(12)}$$

Then we get the following initial value

$$\theta_0(\tau) = e^{-\tau}. \quad \textbf{(13)}$$

Likewise, the rest of the other repetitions can be completed; the first repetition can be received by the computing

$$\frac{d}{d\tau}\theta_1(\tau) = -\theta_0(\tau) - \varepsilon\theta_0(\tau)^4, \quad \textbf{(14)}$$

with condition $\theta_1(0) = 1$. Then, the estimated answer for Eq. (14) will be

$$\theta_1(\tau) = \left(\frac{\varepsilon}{3}e^{-3\tau} + 1 - \frac{\varepsilon}{3}\right)e^{-\tau}. \quad \textbf{(15)}$$

The second repetition $\theta_2(\tau)$ can be gotten from determination the following

$$\frac{d}{d\tau}\theta_2(\tau) = -\theta_1(\tau) - \varepsilon\theta_1(\tau)^4, \quad \textbf{(16)}$$

with condition $\theta_2(0) = 1$. Then, by solving Eq. (16), we receive

$$\theta_2(\tau) = \Big(-\frac{\varepsilon}{81}\Big(\frac{2\varepsilon^4}{3(e^{\tau})^6} - \frac{6\varepsilon^3}{(e^{\tau})^6} - \frac{\varepsilon^4}{3(e^{\tau})^3} + \frac{18\varepsilon^2}{(e^{\tau})^6} + \frac{4\varepsilon^3}{(e^{\tau})^3} - \frac{18\varepsilon}{(e^{\tau})^6} - \frac{18\varepsilon^2}{(e^{\tau})^3} + \frac{36\varepsilon}{(e^{\tau})^3} - \frac{27}{(e^{\tau})^3}\Big)$$

$$+1 + \frac{\varepsilon(-\frac{1}{15}\varepsilon^4 + \varepsilon^3 - 6\varepsilon^2 + 18\varepsilon - 27)}{81}\Big)e^{-\tau} + \ldots.$$

Thus, we proceed to receive the exact solution $\theta(\tau) = \dfrac{1}{\sqrt[3]{-\varepsilon + (1+\varepsilon)e^{3\tau}}}$.

Example 2. (Porous media equation)

We consider the nonlinear heat equation called the porous media equation [19]

$$\frac{\partial}{\partial t}u(x,t) - \left(\frac{\partial}{\partial x}u(x,t)\right)^2 - u(x,t)\frac{\partial^2}{\partial x^2}u(x,t) = 0. \quad \textbf{(17)}$$

as to solve Eq. (17) utilizing TAM, we build a correcting practical, as pursues

$$L(u_{n+1}(x,t)) + N(u_n(x,t)) = 0,\ u_{n+1}(x,0) = x,\ u_0(x,t) = x. \quad \textbf{(18)}$$

Firstly, to get the zero estimation $\theta_0(\tau)$, the following initial issue must be determined

$$L(u_0(x,t)) = 0 \quad \textbf{(19)}$$

with the condition $u_0(x,0)=x$ then we get

$$u_0(x,t)=x. \quad (20)$$

Likewise, the rest of the other repetitions can be completed; the first repetition can be received by computing

$$\frac{\partial}{\partial t}u_1(x,t)-1=0, \quad (21)$$

where $u_1(x,0)=x$. Then, the estimated answer for Eq. (17) will be then

$$u_1(x,t)=x+t. \quad (22)$$

Thus, we proceed to receive the exact solution $u(x,t)=x+t$.

Example 3. (Nonlinear heat equation with cubic nonlinearity)

We consider the nonlinear heat transfer equation with cubic nonlinearity in the form of

$$\frac{\partial}{\partial t}u(x,t)+2(u(x,t))^3-\frac{\partial^2}{\partial x^2}u(x,t)=0. \quad (23)$$

as to solve Eq. (23) utilizing TAM, we build a correcting practical, as pursues:

$$L(u_{n+1}(x,t))+N(u_n(x,t))=0\ ,\ u_{n+1}(x,0)=\frac{1+2x}{x^2+x+1},\ \ u_0(x,t)=\frac{1+2x}{x^2+x+1}. \quad (24)$$

Firstly, to get the zero estimation $\theta_0(\tau)$, the following initial issue must be determined:

$L(u_0(x,t))=0$ with $u_0(x,0)=\dfrac{1+2x}{x^2+x+1}$. Then we get

$$u_0(x,t)=\frac{1+2x}{x^2+x+1}. \quad (25)$$

Likewise, the rest of the other repetitions can be completed; the first repetition can be received by computing

$$\frac{\partial}{\partial t}u_1(x,t)+6\frac{1+2x}{(x^2+x+1)^2}=0, \quad (26)$$

then $u_0(x,t)$ will have the following form

$$u_1(x,t)=\frac{1+2x}{x^2+x+1}. \quad (27)$$

Then, the estimated answer for Eq. (26) will be in the form

$$u_1(x,t)=-6\frac{(1+2x)t}{(x^2+x+1)^2}+\frac{1+2x}{x^2+x+1}. \quad (28)$$

The second repetition $\theta_2(\tau)$ can be gotten from determination the following

$$\frac{\partial}{\partial t}u_2(x,t)+2\left(-6\frac{(1+2x)t}{\left(x^2+x+1\right)^2}+\frac{1+2x}{x^2+x+1}\right)^3+36\frac{(1+2x)^3t}{\left(x^2+x+1\right)^4}$$
$$-72\frac{(1+2x)t}{\left(x^2+x+1\right)^3}+6\frac{1+2x}{\left(x^2+x+1\right)^2}-2\frac{(1+2x)^3}{\left(x^2+x+1\right)^3}=0, \quad \textbf{(29)}$$

and

$$u_2(x,0)=\frac{1+2x}{x^2+x+1} \quad \textbf{(30)}$$

Then, by solving Eq. (23), we find that

$$u_2(x,t)=\frac{72(1+2x)}{\left(x^2+x+1\right)^6}(\frac{1}{4}(24x^2+24x+6)t^4)+\frac{1}{3}(-12x^4-24x^3-27x^2-15x-3)t^3$$
$$+\frac{1}{2}(x^6+3x^5+6x^4+7x^3+6x^2+3x+1)t^2)+(-6\left(x^2+x+1\right)^{-2}$$
$$-12\frac{x}{\left(x^2+x+1\right)^2})t+\frac{1+2x}{x^2+x+1}, \quad \textbf{(31)}$$

then $u(x,t)=\lim_{n\to\infty}u_n(x,t)$.

In the next section, we will illustrate the performance of the proposed technique through the next results.

4. Results and discussion

In this section, the Tamimi-Ansari method (TAM) has been used to solve nonlinear equations that are applied in heat transfer. This method solves several nonlinear problems. Figs. (1-3), for $\varepsilon=0.4$, $\varepsilon=0.65$ and $\varepsilon=0.85$, in Example 1, gives the approximate solution that is noticed to be in good agreement with the exact solution. In Fig 4, the errors of TAM, VIM, and HPM are shown. That is in line with Table1. Tamimi-Ansari method (TAM) is better than other methods for solving these equations. As we can see, in fig.5. dimensionless temperature profile decreasing with increasing ε and as we know ε is increasing function of emissivity coefficient (E). At the same time when the emissivity coefficient of the body increases, the body loses more heat energy, as a result, body temperature decreases, which compatible with Fig. 5. This method does not require linearization and physically unrealistic assumptions. In addition, Fig. 6 and Fig. 7 show the approximate solution for Example 2 and Example 3, respectively.

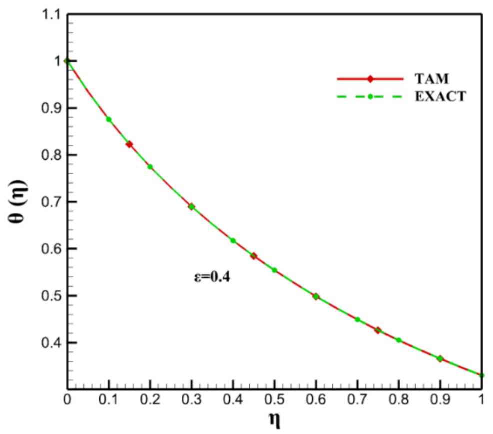

Figure 1. The comparison of the result of TAM for Example 1, at $\varepsilon = 0.4$.

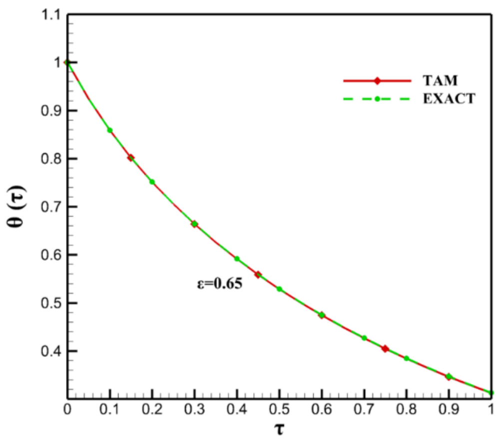

Figure 2. The comparison of the result of TAM for Example 1, at $\varepsilon = 0.65$.

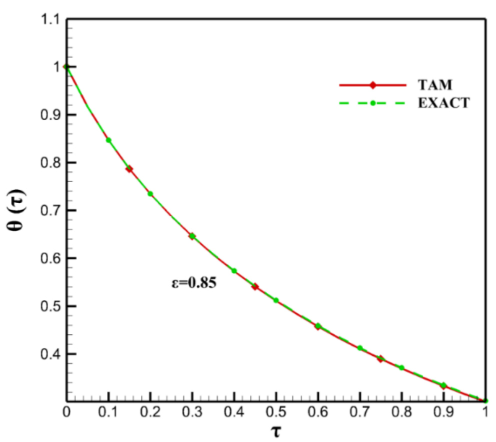

Figure 3. The comparison of the result of TAM for Example 1, at $\varepsilon = 0.85$.

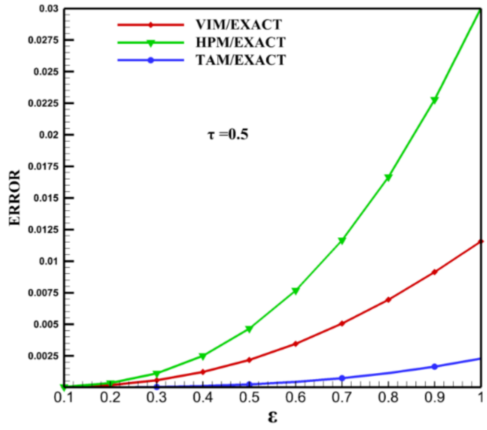

Figure 4. The comparison of the errors in answers resulted from VIM, TAM, and HPM for Example 1, at $\tau = 0.5$.

Table 1. The results of VIM, TAM, and HPM methods and their errors at $\tau = 0.5$.

ε	VIM	HPM	TAM	Exact	Error of VIM	Error of HPM	Error of TAM
0	0.606531	0.606531	0.606531	0.606531	1.65E-10	1.65E-10	1.65E-10
0.1	0.591616	0.591637	0.5915908	0.5915914	4.29E-05	7.80E-05	6.31E-07
0.2	0.578207	0.578371	0.5780143	0.578023	3.19E-04	6.03E-04	8.84E-06
0.3	0.566185	0.566732	0.5655803	0.56562	1.00E-03	1.97E-03	3.94E-05
0.4	0.55544	0.55672	0.5541066	0.554217	2.21E-03	4.52E-03	1.10E-04
0.5	0.545868	0.548335	0.5434418	0.543681	4.02E-03	8.56E-03	2.39E-04
0.6	0.537369	0.541576	0.5334597	0.533903	6.49E-03	1.44E-02	4.44E-04
0.7	0.52985	0.536445	0.5240549	0.524793	9.64E-03	2.22E-02	7.38E-04
0.8	0.523226	0.53294	0.5151385	0.531062	1.35E-02	3.23E-02	1.14E-03
0.9	0.517412	0.531062	0.5066358	0.508284	1.80E-02	4.48E-02	1.65E-03
1	0.512333	0.530812	0.4984832	0.500765	2.31E-02	6.00E-02	2.28E-03

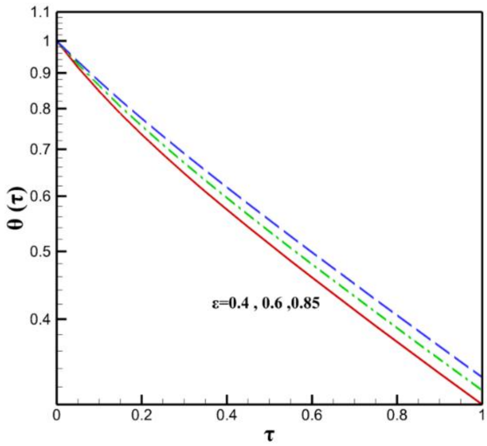

Figure 5. influence of different epsilon on the dimensionless temperature profile.

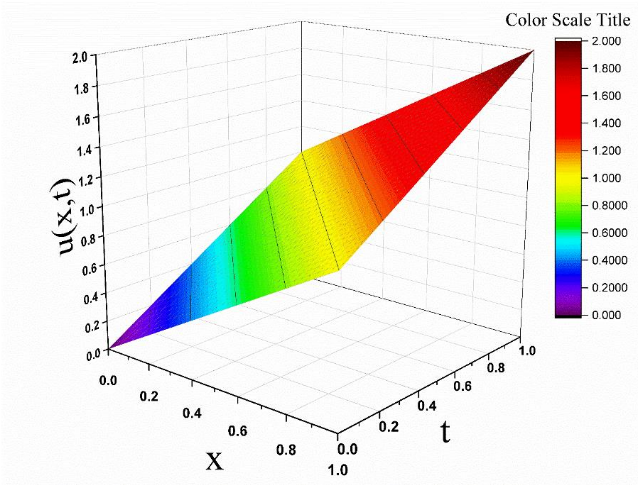

Figure 6. the solution of Example2 via TAM.

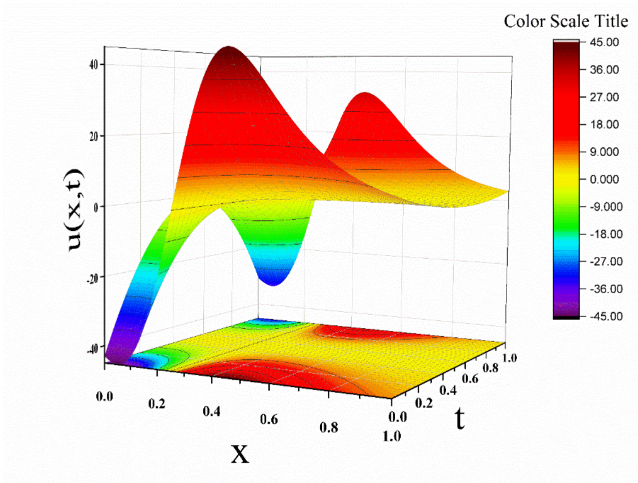

Figure 7. the solution Example3 via TAM.

5. Conclusion

In this article, Tamimi and Ansari method (TAM) has been introduced to solve different types of nonlinear equations with applications in heat transfer. The method has proven to be accurate, simple, effective, and straightforward for solving nonlinear equations. In addition, the method has been tested to several problems and is compared to the exact solution. Thus, the method can be extended to similar problems in the future.

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Conflict of Interest

All of the authors declare no conflict of interest

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