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Multiple complex and real soliton solutions to the new integrable (2+1) dimensional Hirota–Satsuma–Ito equation

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ABSTRACT

A new version of the integrable (2+1)-dimensional Hirota–Satsuma–Ito (2D-HSI) equation is studied in the present paper. The analysis is conducted systematically by considering the bilinear form of the new integrable 2D-HSI equation and utilizing different approaches. As a consequence, a number of multiple complex and real soliton solutions to the model are formally constructed. The findings can be useful to deeply understand the dynamical features of multiple-soliton solutions in mathematical physics.

1. Introduction

During the last decades, considerable attention has been paid to integrable equations particularly their multiple-soliton solutions. Multiple-soliton solutions are a class of explicit exact solutions that play a significant role in mathematical physics. Different systematic schemes have been used to deal with nonlinear integrable equations. For example, Wazwaz [1] obtained multiple real and complex soliton solutions of the integrable VP and MVP equations through the simplified Hirota's method. Zhou and Manukure [2] utilized the linear superposition principle to obtain complexiton solutions of the integrable 2D-HSI equation, and Liu et al. [3] derived multiple real soliton solutions of the integrable 2D-HSI equation using the Hirota's bilinear method. For more studies, see [4-20].

Our aim in the current paper is to look for multiple complex and real soliton solutions of the following new integrable (2+1)-dimensional Hirota–Satsuma–Ito equation [21]

$$\begin{aligned} &v_t + u_{xxt} + 3(uw)_x + \alpha u_x = 0, \\ &u_y - v_x = 0, \end{aligned} \quad (1)$$

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$u_t - w_x = 0,$

through Hirota's bilinear method, linear superposition principle, and ansatz technique. The above nonlinear integrable equation is a new version of the integrable 2D-HSI equation [2, 3, 14]

$$w_t - u_{xxt} - 3uu_t + 3u_x v_t - \alpha u_x = 0,$$
$$w_x + u_y = 0,$$
$$v_x + u = 0.$$

Recently, N-soliton solutions and interaction solutions of the new integrable 2D-HSI equation have been extracted by Liu et al. in [21].

The organization of this work is as follows: In Section 2, the bilinear form of the new integrable 2D-HSI equation is presented. In Section 3, multiple complex and real soliton solutions of the new integrable 2D-HSI equation are obtained using Hirota's bilinear method, linear superposition principle, and ansatz technique. Section 4 is devoted to presenting some remarks.

2. Bilinear form of the new integrable 2D-HSI equation

A direct computation illustrates that the nonlinear model (1) can be written in its bilinear form as [21]

$$B_{\mathrm{HSI}}(f) := ff_{xxxt} - f_t f_{xxx} - 3f_{xxt}f_x + 3f_{xt}f_{xx} + ff_{yt} - f_y f_t + \alpha(ff_{xx} - f_x^2) = 0, \tag{2}$$

under the transformations

$$u = 2(\ln f)_{xx} = 2\frac{f_{xx}f - f_x^2}{f^2}, \quad v = 2(\ln f)_{xy} = 2\frac{f_{xy}f - f_x f_y}{f^2}, \quad w = 2(\ln f)_{xt} = 2\frac{f_{xt}f - f_x f_t}{f^2}.$$

3. Multiple complex and real soliton solutions of the new integrable 2D-HSI equation

In this section, Hirota's bilinear method, linear superposition principle, and ansatz technique are used to procure multiple complex and real soliton solutions of the new integrable 2D-HSI equation.

3.1. Double complex soliton solutions

For double complex soliton solution, an auxiliary complex function is considered as

$$f(x,y,t) = I + e^{k_1 x + r_1 y + c_1 t} + e^{k_2 x + r_2 y + c_2 t} - I a_{12} e^{k_1 x + r_1 y + c_1 t} e^{k_2 x + r_2 y + c_2 t}, \quad I = \sqrt{-1},$$

where k_1, r_1, c_1, k_2, r_2, and c_2 are unknowns to be calculated. Setting the above solution in the bilinear form (2) results in the dispersion relations and the phase shift as

$$c_i = -\frac{\alpha k_i^2}{k_i^3 + r_i}, \quad i = 1,2,$$

$$a_{12} = \big(k_1^6 k_2^2 - 3k_1^5 k_2^3 + 4k_1^4 k_2^4 - 3k_1^3 k_2^5 + k_1^2 k_2^6 + k_1^4 k_2 r_2 + 2k_1^3 k_2^2 r_1 - 3k_1^3 k_2^2 r_2 - 3k_1^2 k_2^3 r_1 + 2k_1^2 k_2^3 r_2 + k_1 k_2^4 r_1 + k_1^2 r_2^2 - 2k_1 k_2 r_1 r_2 + k_2^2 r_1^2\big)/\big(k_1^6 k_2^2 + 3k_1^5 k_2^3 + 4k_1^4 k_2^4 + 3k_1^3 k_2^5 + k_1^2 k_2^6 + k_1^4 k_2 r_2 + 2k_1^3 k_2^2 r_1 + 3k_1^3 k_2^2 r_2 + 3k_1^2 k_2^3 r_1 + 2k_1^2 k_2^3 r_2 + k_1 k_2^4 r_1 + k_1^2 r_2^2 - 2k_1 k_2 r_1 r_2 + k_2^2 r_1^2\big).$$

Now, a double complex soliton solution to the new integrable 2D-HSI equation is gained as

$$u = 2(\ln f)_{xx} = 2\frac{f_{xx}f - f_x^2}{f^2}, \quad v = 2(\ln f)_{xy} = 2\frac{f_{xy}f - f_x f_y}{f^2}, \quad w = 2(\ln f)_{xt} = 2\frac{f_{xt}f - f_x f_t}{f^2}.$$

The physical structures and characteristics of the double complex soliton solution have been shown in Figure 1.

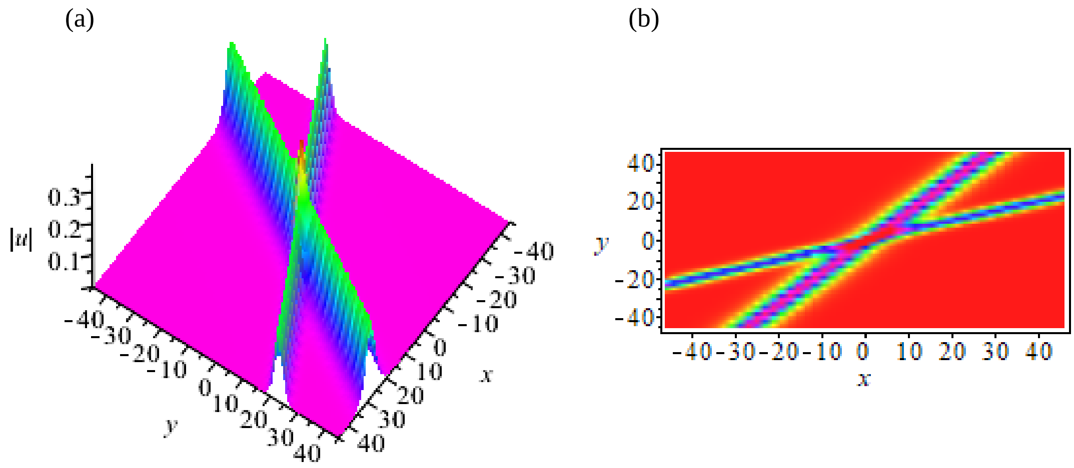

Figure 1. *(a) and (b) Three dimensional and density plots of* $|u(x,y,t)|$ *when* $k_1 = 0.5, r_1 = -0.3, k_2 = -0.5, r_2 = 1, \alpha = -1,$ *and* $t = 0$.

3.2. Multiple real soliton solutions

According to the linear superposition principle, we look for $a_i, i = 1,2,3$ and $n_i, i = 1,2,3$ such that

$$a_3(x^{n_3} - y^{n_3})a_1^3(x^{n_1} - y^{n_1})^3 + \alpha a_1^2(x^{n_1} - y^{n_1})^2 + a_3(x^{n_3} - y^{n_3})a_2(x^{n_2} - y^{n_2}) = 0. \quad (3)$$

From (3), $a_i, i = 1,2,3$ can be gained if (n_1, n_2, n_3) is selected as $(1,3,-1)$. Consequently, by setting the coefficients of (3) to zero, one derives

$$-a_1^3 a_3 - a_2 a_3 = 0,$$

$$4a_1^3 a_3 + \alpha a_1^2 + a_2 a_3 = 0,$$

$$-6a_1^3 a_3 - 2\alpha a_1^2 = 0.$$

By solving the above algebraic set, we get $a_2 = -a_1^3$ and $a_3 = -\alpha/3a_1$ where $a_1 \neq 0$. Now, by considering $a_1 = 1$, one can acquire the following multiple real soliton solutions to the new integrable 2D-HSI equation

$$u = 2(\ln f)_{xx} = 2\frac{f_{xx}f - f_x^2}{f^2}, \quad v = 2(\ln f)_{xy} = 2\frac{f_{xy}f - f_x f_y}{f^2}, \quad w = 2(\ln f)_{xt} = 2\frac{f_{xt}f - f_x f_t}{f^2},$$

in which

$$f(x,y,t) = \sum_{i=1}^{N} e^{k_i x - k_i^3 y - \frac{1\,\alpha}{3k_i}t}.$$

The physical structures and characteristics of the triple real soliton solution have been shown in Figure 2.

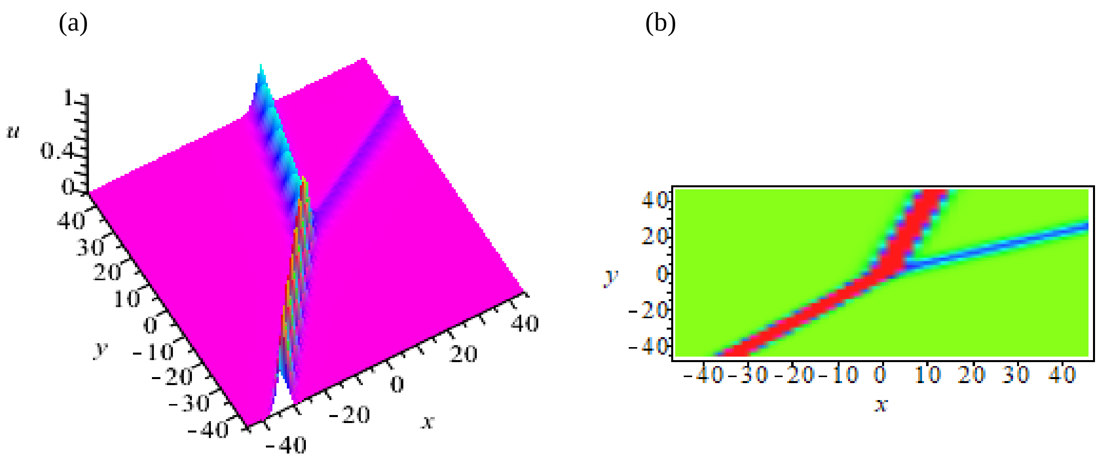

Figure 2. *(a) and (b) Three dimensional and density plots of* $u(x,y,t)$ *when* $N=3$, $k_1=-0.5$, $k_2=1$, $k_3=0.5$, $\alpha=-1$, *and* $t=0$.

3.3. Other soliton solutions

In order to obtain other soliton solutions of the new integrable 2D-HSI equation, the following test functions are considered

$$i.\quad \begin{cases} u(x,y,t)=a_0+a_1\tanh^2(k_1x+r_1y+c_1t),\\ v(x,y,t)=(r_1/k_1)\big(a_0+a_1\tanh^2(k_1x+r_1y+c_1t)\big),\\ w(x,y,t)=(c_1/k_1)\big(a_0+a_1\tanh^2(k_1x+r_1y+c_1t)\big). \end{cases} \tag{4}$$

$$ii.\quad \begin{cases} u(x,y,t)=a_0+a_1\coth^2(k_1x+r_1y+c_1t),\\ v(x,y,t)=(r_1/k_1)\big(a_0+a_1\coth^2(k_1x+r_1y+c_1t)\big),\\ w(x,y,t)=(c_1/k_1)\big(a_0+a_1\coth^2(k_1x+r_1y+c_1t)\big). \end{cases}$$

$$iii.\quad \begin{cases} u(x,y,t)=a_0+a_1\operatorname{sech}^2(k_1x+r_1y+c_1t),\\ v(x,y,t)=(r_1/k_1)\big(a_0+a_1\operatorname{sech}^2(k_1x+r_1y+c_1t)\big),\\ w(x,y,t)=(c_1/k_1)\big(a_0+a_1\operatorname{sech}^2(k_1x+r_1y+c_1t)\big). \end{cases} \tag{5}$$

$$iv.\quad \begin{cases} u(x,y,t)=a_0+a_1\operatorname{csch}^2(k_1x+r_1y+c_1t),\\ v(x,y,t)=(r_1/k_1)\big(a_0+a_1\operatorname{csch}^2(k_1x+r_1y+c_1t)\big),\\ w(x,y,t)=(c_1/k_1)\big(a_0+a_1\operatorname{csch}^2(k_1x+r_1y+c_1t)\big). \end{cases}$$

In the first case, inserting the solution (4) into equation (1) leads to

$$a_1=-2k_1^2,\quad r_1=-\frac{k_1(-8c_1k_1^2+\alpha k_1+6a_0c_1)}{c_1}.$$

Now, the following single soliton solution to the new integrable 2D-HSI equation can be derived

$$u(x,y,t)=a_0-2k_1^2\tanh^2\left(k_1x-\frac{k_1(-8c_1k_1^2+\alpha k_1+6a_0c_1)}{c_1}y+c_1t\right),$$

$$v(x,y,t)=-\frac{(-8c_1k_1^2+\alpha k_1+6a_0c_1)}{c_1}\left(a_0-2k_1^2\tanh^2\left(k_1x-\frac{k_1(-8c_1k_1^2+\alpha k_1+6a_0c_1)}{c_1}y+c_1t\right)\right),$$

$$w(x,y,t)=\frac{c_1}{k_1}\left(a_0-2k_1^2\tanh^2\left(k_1x-\frac{k_1(-8c_1k_1^2+\alpha k_1+6a_0c_1)}{c_1}y+c_1t\right)\right).$$

In the second case, one can get the following single soliton solution to the new integrable 2D-HSI equation

$$u(x,y,t)=a_0-2k_1^2\coth^2\left(k_1x-\frac{k_1(-8c_1k_1^2+\alpha k_1+6a_0c_1)}{c_1}y+c_1t\right),$$

$$v(x,y,t)=-\frac{(-8c_1k_1^2+\alpha k_1+6a_0c_1)}{c_1}\left(a_0-2k_1^2\coth^2\left(k_1x-\frac{k_1(-8c_1k_1^2+\alpha k_1+6a_0c_1)}{c_1}y+c_1t\right)\right),$$

$$w(x,y,t)=\frac{c_1}{k_1}\left(a_0-2k_1^2\coth^2\left(k_1x-\frac{k_1(-8c_1k_1^2+\alpha k_1+6a_0c_1)}{c_1}y+c_1t\right)\right).$$

In the third case, by setting the solution (5) in equation (1), we obtain

$$a_1=2k_1^2,\quad r_1=-\frac{k_1(4c_1k_1^2+\alpha k_1+6a_0c_1)}{c_1}.$$

Now, the following single soliton solution to the new integrable 2D-HSI equation can be derived

$$u(x,y,t)=a_0+2k_1^2\,\mathrm{sech}^2\left(k_1x-\frac{k_1(4c_1k_1^2+\alpha k_1+6a_0c_1)}{c_1}y+c_1t\right),$$

$$v(x,y,t)=-\frac{(4c_1k_1^2+\alpha k_1+6a_0c_1)}{c_1}\left(a_0+2k_1^2\,\mathrm{sech}^2\left(k_1x-\frac{k_1(4c_1k_1^2+\alpha k_1+6a_0c_1)}{c_1}y+c_1t\right)\right),$$

$$w(x,y,t)=\frac{c_1}{k_1}\left(a_0+2k_1^2\,\mathrm{sech}^2\left(k_1x-\frac{k_1(4c_1k_1^2+\alpha k_1+6a_0c_1)}{c_1}y+c_1t\right)\right).$$

Finally, from the last case, one derives

$$u(x,y,t)=a_0-2k_1^2\,\mathrm{csch}^2\left(k_1x-\frac{k_1(4c_1k_1^2+\alpha k_1+6a_0c_1)}{c_1}y+c_1t\right),$$

$$v(x,y,t)=-\frac{(4c_1k_1^2+\alpha k_1+6a_0c_1)}{c_1}\left(a_0-2k_1^2\,\mathrm{csch}^2\left(k_1x-\frac{k_1(4c_1k_1^2+\alpha k_1+6a_0c_1)}{c_1}y+c_1t\right)\right),$$

$$w(x,y,t)=\frac{c_1}{k_1}\left(a_0-2k_1^2\,\mathrm{csch}^2\left(k_1x-\frac{k_1(4c_1k_1^2+\alpha k_1+6a_0c_1)}{c_1}y+c_1t\right)\right).$$

The physical structures and characteristics of the second and fourth soliton solutions have been demonstrated in Figures 3 and 4.

4. Concluding remarks

A new integrable (2+1)-dimensional Hirota–Satsuma–Ito equation was successfully explored in the present paper. For this purpose, based on the bilinear form of the new integrable 2D-HSI equation, Hirota's bilinear method, linear superposition principle, and ansatz technique, a number of multiple complex and real soliton solutions to the model were formally extracted. The results highlight the effectiveness of the schemes particularly the linear superposition principle to handle the integrable models.

(a) (b)

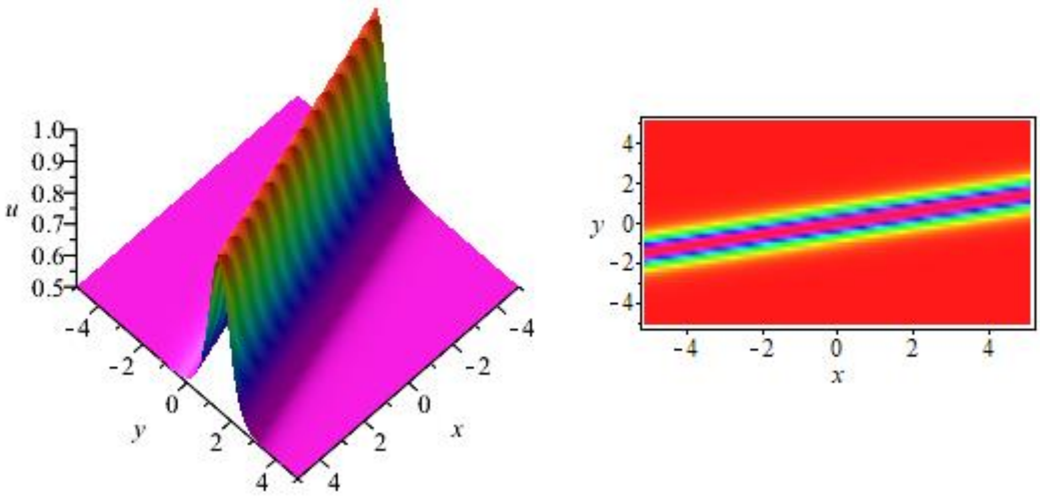

Figure 3. *(a) and (b) Three dimensional and density plots of* $u(x, y, t)$ *in the first case when* $a_0 = 1$, $k_1 = 0.5$, $c_1 = 1$, $\alpha = -1$, *and* $t = 0$.

(a) (b)

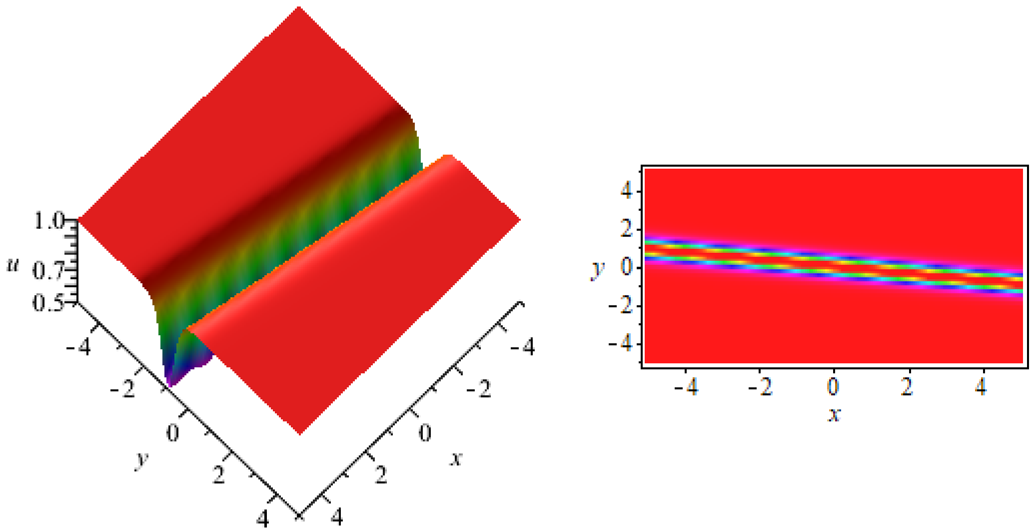

Figure 4. *(a) and (b) Three dimensional and density plots of* $u(x, y, t)$ *in the third case when* $a_0 = -1$, $k_1 = 0.5$, $c_1 = 1$, $\alpha = -1$, *and* $t = 0$.

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