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THE UNIVERSAL $\mathcal{AIR}$- COMPACTIFICATION OF A SEMIGROUP

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Abstract. In this paper we establish a characterization of abelian compact Hausdorff semigroups with unique idempotent and ideal retraction property. We also introduce a function algebra on a semitopological semigroup whose associated semigroup compactification is universal with respect to these properties.

1. Introduction and preliminaries

Throughout this paper S is a semitopological semigroup. A semigroup compactification of S is a pair (ψ, X), where X is a compact, Hausdorff, right topological semigroup and $\psi : S \longrightarrow X$ is a continuous homomorphism such that $\overline{\psi(S)} = X$ and $\psi(S) \subset \Lambda(X) := \{t \in X :$ the function $s \longmapsto ts : X \longrightarrow X$ is continuous$\}$.

The C^*-algebra of all continuous bounded complex-valued functions on S is denoted by $\mathcal{C}(S)$. For $\mathcal{C}(S)$, left and right translations L_s and R_t are defined for all $s, t \in S$ and $f \in \mathcal{C}(S)$ by $(L_s f)(t) = f(st) = (R_t f)(s)$. A translation invariant C^*-subalgebra $\mathcal{F}$ of $\mathcal{C}(S)$ (i.e. $L_s f \in \mathcal{F}$ and $R_s f \in \mathcal{F}$ for all $s \in S$ and $f \in \mathcal{F}$) containing the constant functions is called m-admissible if the function $s \mapsto (T_\mu f)(s) = \mu(L_s f)$ is in $\mathcal{F}$ for all $f \in \mathcal{F}$ and $\mu \in S^{\mathcal{F}}$ (=the spectrum of $\mathcal{F}$); then the product of $\mu, \nu \in S^{\mathcal{F}}$ can be defined by $\mu\nu = \mu \circ T_\nu$ and the Gelfand topology on $S^{\mathcal{F}}$ makes $(\epsilon, S^{\mathcal{F}})$ a semigroup compactification (called the

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$\mathcal{F}$-compactification) of S, where $\epsilon : S \longrightarrow S^{\mathcal{F}}$ is the evaluation mapping. Conversely, if (ψ, X) is a compactification of S, then $\psi^*(\mathcal{C}(X))$ is an m-admissible subalgebra of $\mathcal{C}(S)$, where ψ^* is the dual mapping of ψ, and this correspondence between compactifications of S and m-admissible subalgebras of $\mathcal{C}(S)$ is one-to-one up to isomorphism(see [3, Theorem 3.1.7]). Some of the usual m-admissible subalgebras of $\mathcal{C}(S)$, that we need in the rest of this paper, are the following:
$\mathcal{LMC}(S) = \{f \in \mathcal{C}(S) : R_s f$ is relatively compact in $\mathcal{C}(S)$ in the topology of pointwise convergence on $S\}$,
$\mathcal{WAP}(S) = \{f \in \mathcal{C}(S) : f$ is weakly almost periodic function on $S\}$,
$\mathcal{D}(S) = \{f \in \mathcal{LMC}(S) : \mu\eta\nu(f) = \mu\nu(f)$ for $\mu, \nu, \eta \in S^{\mathcal{LMC}}$ with $\eta^2 = \eta\}$,
$\mathcal{MD}(S) = \{f \in \mathcal{D}(S) : \eta\mu(f) = \mu(f)$ for $\mu, \eta \in S^{\mathcal{LMC}}$ with $\eta^2 = \eta\}$,
$\mathcal{SD}(S) = \{f \in \mathcal{D}(S) : \mu\eta(f) = \mu(f)$ for $\mu, \eta \in S^{\mathcal{LMC}}$ with $\eta^2 = \eta\}$,
$\mathcal{GP}(S) = \mathcal{MD}(S) \cap \mathcal{SD}(S)$ [3].
We also write $\mathcal{AB}$ for $\{f \in \mathcal{WAP}(S) : f(st) = f(ts)$, and $f(stu) = f(sut)$ for all $s, t, u \in S\}$ [4].

A P-compactification (ψ, X) of S, that is an extension of other P-compactification of S is called a universal P-compactification of S. That is for any other compactification (φ, Y), having the property P, there exists a homomorphism $\pi : (\psi, X) \longrightarrow (\varphi, Y)$, where π is a continuous mapping from X onto Y with $\pi o \psi = \varphi$, or equivalently, $\varphi^*(\mathcal{C}(Y)) \subseteq \psi^*(\mathcal{C}(X))$ (see [3, Theorem 3.1.9]). The $\mathcal{LMC}$,$\mathcal{WAP}$ and $\mathcal{D}$-compactifications are universal with respect to the properties of being a (right topological) semigroup, a semitoplogical semigroup and an inflation of a rectangular group, respectively [3], [6]. The reader is referred to [3] for more information about compactifications of S ,m-admissible subalgebras of $\mathcal{C}(S)$ and universal P-compactifications.

A left ideal (respectively, right ideal, ideal) of a semigroup S is said to be minimal if it properly contains no left ideal (respectively, right ideal , ideal) of S. An idempotent e in a semigroup S is said to be minimal if it satisfies the following equivalent conditions :
(i) Se is a minimal left ideal.
(ii) eS is a minimal right ideal.
(iii) $eSe(= eS \cap Se)$ is a group.
Let S be a compact, Hausdorff, right topological semigroup. Then S has a minimal idempotent [3, Theorem 1.3.11].The intersection of all ideals of a semigroup S is denoted by $K(S)$. For a semigroup S with a minimal idempotent, S has a unique minimal ideal $K := K(S)$ and $K = \cup\{Se : e \in E(K)\} = \cup\{eS : e \in E(K)\} = \cup\{eSe : e \in E(K)\}$ [3, Theorem 1.2.12].

A semigroups has the ideal retraction property if each of its ideals is the image of some idempotent endomorphism. Aucoin et al. [1], [2] initiated the study of semigroup with ideal retraction property.

2. Ideal retraction property on compact abelian semitopological semigroup

In this section we characterize the equivalent condition with the ideal retraction property for a compact abelian semitopological semigroup. It is easy to see that every group has the ideal retraction property. We notice that groups have a unique idempotent.

The following example presents a semigroup S with a unique idempotent enjoying the ideal retraction property, while S is not a group.

Example 2.1. Let $S = \{a, b, c\}$ be the semigroup with the following multiplication table:

	a	b	c
a	a	a	a
b	a	b	a
c	a	a	c

Then S is an abelian semigroup with unique minimal idempotent a. Moreover S has ideal retraction property, while S is not group.

In the following example we introduce an abelian semigroup with unique idempotent that has not ideal retraction property.

Example 2.2. Let $S = \{a, b, c\}$ be the semigroup with the following multiplication table:

	a	b	c
a	a	a	a
b	a	a	a
c	a	a	b

Finally the following semigroup is a compact abelian semigroup with a unique idempotent while it is not simple.

Example 2.3. Let $S = \{a, b, c\}$ be the semigroup with the following multiplication table:

Then a is unique minimal idempotent of S. Moreover S has ideal retraction property and $K(S) = \{a, c\} = S^2$. we notice that S is not group.

The previous examples lead us to the following theorem.

	a	b	c
a	a	c	c
b	c	a	a
c	c	a	a

Theorem 2.4. *Let S be an abelian compact Hausdorff semigroup with unique idempotent e. Then the following statements are equivalent:*
(i) $K(S) = S^2$;
(ii) S has the ideal retraction property.

Proof. First we notice that since S is compact, e is a minimal idempotent and $K(S) = Se = eS = eSe$. So $K(S) \subseteq S^2$.
$(i) \Longrightarrow (ii)$: Suppose that S has ideal retraction property, and $s, t \in S$. Then there exists a homomorphic retraction $\phi : S \longrightarrow (st)^2 S^1$. Then $s^2t^2 = (st)^2 = \phi((st)^2) = \phi(st)\phi(st)$, so there exists $u \in S^1$ such that $s^2t^2 = ((st)^2u)((st)^2u) = s^4t^4u^2$. So $(s^2t^2u^2)^2 = s^4t^4u^4 = (s^4t^4u^2)u^2 = (s^2t^2)u^2 = s^2t^2u^2$. Hence $s^2t^2u^2$ is an idempotent and we conclude that $s^2t^2u^2 = e$ and $s^2t^2 = s^4t^4u^2 = (s^2t^2)(s^2t^2u^2) = s^2t^2e$. So $s^2t^2 \in K(S)$. Now for ideal stS^1 there exists a homomorphic retraction $\varphi : S \longrightarrow stS^1$. Thus $st = \varphi(st) = \varphi(s)\varphi(t)$, so there exists $v, w \in S^1$ such that $st = (stv)(stw) = (s^2t^2)(vw) \in K(S)S \subseteq K(S)$ and this implies that $S^2 \subseteq K(S)$.
$(ii) \Longrightarrow (i)$: Let $K(S) = S^2$ and I be an ideal of S. Define $\phi : S \longrightarrow I$ by

$$\phi(s) = \begin{cases} s & s \in I \\ se & s \notin I. \end{cases}$$

It is easy to see that $\phi \mid_I = 1_I$. To see that ϕ is homomorphism let $s, t \in S$. Then $st \in S^2 = K(S) \subseteq I$, so $\phi(st) = st$. Moreover since $st \in K(S) = Se$, there exists some $u \in S$ such that $st = ue$, thus $ste = uee = ue = st$. So if $s, t \in I$ or $s \in I, t \notin I$ or $s, t \notin I$ we conclude that $\phi(st) = \phi(s)\phi(t)$ and this completes the proof. □

The compactification (ψ, X) of S is called the $\mathcal{AIR}$- compactification of S, if X is an abelian semigroup with unique idempotent and ideal retraction property.

In the following lemma, we provide essential condition for existness of universal $\mathcal{AIR}$- compactification in the next theorem.

Lemma 2.5. *The $\mathcal{AIR}$- property of compactifications is invariant under subdirect products.*

Proof. Let $(\psi, X) = \vee\{(\psi_i, X_i) : i \in I\}$, where, for each $i \in I$, (ψ_i, X_i) is a compactification of S with $\mathcal{AIR}$- property. If (ϕ, Y) denote the universal compactification of S, then there exist onto homomorphisms

$\pi : (\phi, Y) \longrightarrow (\psi, X)$ and $\pi_i : (\phi, Y) \longrightarrow (\psi_i, X_i)$, $i \in I$, such that $\pi(y) = (\pi_i(y))$, $y \in Y$. Now by Theorem 2.2 and [3, Theorem 1.3.16] we have $K(X) = \pi(K(Y)) = \pi(Y^2) = \pi(Y)^2 = X^2$. Therefore (ψ, X) has $\mathcal{AIR}$- property, as claimed. □

Theorem 2.6. *Let S be a semitopological semigroup, then S has a universal $\mathcal{AIR}$- compactification.*

Proof. In the previous lemma, we proved that the $\mathcal{AIR}$- property of compactifications of a semitopological semigroup S is invariant under subdirect products. Moreover in the similar way $\mathcal{AIR}$- property is invariant under isomorphisms, so by [3, Theorem 3.3.4] S has a universal $\mathcal{AIR}$- compactification. □

Let (ψ, X) be the universal $\mathcal{AIR}$- compactification of S. Then $\psi^*(\mathcal{C}(X))$ is an m- admissible subalgebra of $\mathcal{C}(S)$. We denote $\psi^*(\mathcal{C}(X))$ by $\mathcal{AIR}(S)$ and the canonical $\mathcal{AIR}(S)$- compactification of S by $(\epsilon, S^{\mathcal{AIR}})$.

3. Algebra of functions on a semitopological semigroup

In this section we study the relation between $\mathcal{AIR}$- compactification of a semitopoligical semigroup S and its corresponding algebra of functions. The next lemma characterizes $\mathcal{AB}$ and $\mathcal{D}$ in terms of the elements of $S^{\mathcal{WAP}}$ and $S^{\mathcal{LMC}}$, respectively.

Lemma 3.1. *Let S be a semitopological semigroup.*

(i) A function $f \in \mathcal{WAP}(S)$ is in $\mathcal{AB}(S)$ if and only if $\mu\nu(f) = \nu\mu(f)$ and $T_{\mu\nu}f = T_{\nu\mu}f$ for all $\mu, \nu \in S^{\mathcal{WAP}}$ [4, Lemma 3.1].

(ii) A function $f \in \mathcal{LMC}(S)$ is in $\mathcal{D}(S)$ if and only if $\mu\eta\nu(f) = \mu\nu(f)$ for all $\mu, \nu \in S^{\mathcal{LMC}}$ and $\eta \in E(S^{\mathcal{LMC}})$ [3, Lemma 4.6.2].

Lemma 3.2. *Let S be a semitopological semigroup.*

(i) $(\epsilon, S^{\mathcal{D}})$ is the semigroup compactification of S which is universal among compactifications (ψ, X) with the property that $xey = xy (x, y \in X, e \in E(X))$ [3, Theorem 4.6.5].

(ii) $(\epsilon, S^{\mathcal{AB}})$ is the semigroup compactification of S which is universal among compactifications (ψ, X) with the property that $xy = yx (x, y \in X)$[4, Theorem 3.2].

For a semitopological semigroup S, we write $\mathcal{ABD}$ for $\mathcal{AB} \cap \mathcal{D}$. By the above lemmas, a function $f \in \mathcal{WAP}(S)$ is in $\mathcal{ABD}(S)$ if and only if $\mu\nu(f) = \nu\mu(f)$, $T_{\mu\nu}f = T_{\nu\mu}f$ for all $\mu, \nu \in S^{\mathcal{WAP}}$ and for all idempotent $\eta \in S^{\mathcal{LMC}}$ such that $\mu\eta\nu(f) = \mu\nu(f)$, moreover $\mathcal{ABD}$ is an m-admissible subalgebra of $\mathcal{C}(S)$.

Lemma 3.3. *Let S be a semitopological semigroup, then $S^{\mathcal{ABD}}$ has the $\mathcal{AIR}$- property.*

Proof. Let $X = S^{\mathcal{ABD}}$ and $f \in \mathcal{ABD}(S)$, Then X is a compact Hausdorff right topological semigroup. Moreover since $\mu\nu(f) = \nu\mu(f)$ for all $\mu, \nu \in X$, X is abelian. Now we should prove that X has a unique idempotent and $K(X) = X^2$. Let $\eta \in E(S^{\mathcal{LMC}})$. So $\mu\nu(f) = \mu\eta\nu(f) = \mu\nu\eta(f)$, for all $\mu, \nu \in X$. Therefore $\mu\nu = \mu\nu\eta \in X\eta$. So we have $X^2 \subseteq \cup\{X\eta : \eta \in E(X)\} = K(X)$ and this implies that $X^2 = K(X)$. Moreover since X is abelian semigroup and $K(X) \neq \emptyset$, by [3, Corollary 1.2.14] $K(X)$ is a group and X has an unique minimal idempotent. Thus X has the $\mathcal{AIR}$- property, as required. □

The previous lemma implies that $\mathcal{ABD}(S) \subseteq \mathcal{AIR}(S)$. In the following theorem we prove the converse of this relation.

Theorem 3.4. *Let S be a semitopological semigroup. Then $\mathcal{ABD}(S) = \mathcal{AIR}(S)$.*

Proof. By Lemma 3.3, $\mathcal{ABD}(S) \subseteq \mathcal{AIR}(S)$. Suppose that $(\psi, X) = (\epsilon, S^{\mathcal{AIR}})$ and $f \in AIR(S)$. So there exists $g \in \mathcal{C}(X)$ such that $f = \psi^*(g) \in \mathcal{WAP}$ and

$$\mu\nu(f) = \mu\nu(\psi^*(g)) = g(\pi(\mu\nu)) = g(\pi(\nu\mu)) = \nu\mu(f),$$

$$\mu\eta\nu(f) = \mu\eta\nu(\psi^*(g)) = g(\pi(\mu\eta\nu)) = g(\pi(\mu\nu)) = \mu\nu(f),$$

where $\pi : (\epsilon, S^{\mathcal{WAP}}) \longrightarrow (\psi, X)$ is the canonical homomorphism whose existence is guaranteed by the universal property of $(\epsilon, S^{\mathcal{WAP}})$. Hence $f = \psi^*(g) \in \mathcal{ABD}(S)$, as claimed. □

Corollary 3.5. *Let S be a semitopological semigroup. Then $(\epsilon, S^{\mathcal{AIR}})$ is the semigroup compactification of S which is universal among compactifications (ψ, X) with the property that $xey = xy$ and $xy = yx$, $(x, y \in X, e \in E(X))$.*

4. Reductive compactifications and E-algebras

An action of a semigroup S on a topological space X is a mapping $\sigma : S \times X \longrightarrow X$ such that

(i) $\sigma(S, .) : X \longrightarrow X$ is continuous for each $s \in S$, and

(ii) $\sigma(st, x) = \sigma(s, \sigma(t, x)$ for all $s, t \in S$ and $x \in X$.

A flow is a triple (S, X, σ), where S is a semigroup, X is a nonempty, compact, Hausdorff, topological space, and σ is an action of S on X. Let (ψ, X) be a compactification of S, then the mapping $\sigma : S \times X \longrightarrow X$, defined by $\sigma(s, x) = \psi(s)x$, is separately continuous and so (S, X, σ) is a flow. If Σ_X denotes the enveloping semigroup of the flow (S, X, σ)

(i.e., the pointwise closure of semigroup $\{\sigma(s,.) : s \in S\}$ in X^X) and the mapping $\sigma_X : S \longrightarrow \Sigma_X$ is defined by $\sigma_X(s) = \sigma(s,.)$ for all $s \in S$, then (σ_X, Σ_X) is a compactification of S (see [3, Proposition 1.6.5]).

A semigroup S is called right reductive if for each $u, v \in S$, $ut = vt$ for all $t \in S$ implies $u = v$. One can easily verify that $\Sigma_X = \{\lambda_x : x \in X\}$, where $\lambda_x(y) = xy$ for each $y \in X$. If we define the mapping $\theta : X \longrightarrow \Sigma_X$ by $\theta(x) = \lambda_x$, then θ is a continuous homomorphism with the property that $\theta o \psi = \sigma_X$. So (σ_X, Σ_X) is a factor of (ψ, X), that is $(\psi, X) \geq (\sigma_X, \Sigma_X)$. By definition, θ is one-to-one if and only if X is right reductive.

Proposition 4.1. *Let S be a semitopological semigroup and Let $X = S^{\mathcal{AIR}}$. Then Σ_X is right reductive.*

Proof. Let $x_1, x_2 \in X$, $\lambda_{x_1}\lambda_y = \lambda_{x_2}\lambda_y$ for all $y \in X$. Since $X = S^{\mathcal{AIR}}$, X is is an abelian semigroup with unique idempotent and ideal retraction property. Let e be a unique minimal idempotent of X, So e is identity for X^2. Thus for $z \in X$ and $y = e$ we have

$$\lambda_{x_1}(z) = x_1 z e = x_1 e z = \lambda_{x_1}\lambda_e(z) = \lambda_{x_2}\lambda_e(z) = x_2 e z = x_2 z e = \lambda_{x_2}(z).$$

Hence $\lambda_{x_1} = \lambda_{x_2}$, as claimed. □

Proposition 4.2. [5, Proposition 2.1] *Let (ψ, X) be a compactification of S. Then $(\sigma_X, \Sigma_X) \cong (\psi, X)$ if and only if X is right reductive.*

Corollary 4.3. *Let S be a semitopological semigroup and Let $X = S^{AIR}$. Then $(\sigma_{\Sigma_X}, \Sigma_{\Sigma_X}) \cong (\sigma_X, \Sigma_X)$.*

Proof. By Proposition 4.1 and Proposition 4.2, the proof is clear. □

Proposition 4.4. *Let S be a semitopological semigroup and Let $X = S^{\mathcal{AIR}}$. Then Σ_X is an abelian group.*

Proof. Let e be a unique minimal idempotent of X, so e is identity for X^2. First we notice that λ_e is identity for Σ_X, because for all $x, y \in x$ we have

$$\lambda_x\lambda_e(y) = \lambda_{xe}(y) = xey = yxe = yx = xy = \lambda_x(y).$$

So $\lambda_x\lambda_e = \lambda_x$. similarly, we have $\lambda_e\lambda_x = \lambda_x$. Also, since $xe \in X^2 = K(X)$ and e is identity for the group X^2, there exists $y \in X$ such that $(xe)(ye) = e$. Now for $z \in X$ we have

$$\lambda_x\lambda_y = \lambda_{xe}\lambda_{ye} = \lambda_{xeye} = \lambda_e.$$

Thus every element of Σ_X is invertible. Finally

$$\lambda_x\lambda_y(z) = \lambda_{xy}(z) = xyz = yxz = \lambda_{yx}(z) = \lambda_y\lambda_x(z).$$

Hence Σ_X is an abelian and this completes the proof. □

Example 4.5. Let S be the presented semigroup in Example 2.3. Then a is unique minimal idempotent of S. Moreover S has ideal retraction property and $K(S) = \{a, c\} = S^2$. So by Proposition 4.4, since Σ_S is an abelian group, Σ_S is right reductive. We notice that for all $s \in S$ we have $bs = cs$, while $b \neq c$. Thus S is not right reductive.

An m-admissible subalgebra F of $\mathcal{C}(S)$ is called an E-algebra if there is a compactification (ψ, X) such that $(\sigma_X, \Sigma_X) \cong (\epsilon, S^{\mathcal{F}})$. In this setting (ψ, X) is called an EF-compactification of S [7, Definition 1.6.5].

Theorem 4.6. *Let S be a semitopological semigroup. Then $(\epsilon, S^{\mathcal{AIR}})$ is an $E\mathcal{AB} \cap \mathcal{GP}$-compactification of S.*

Proof. Let $X = S^{\mathcal{AIR}}$ and e be the unique minimal idempotent of X. For $(\sigma_X, \Sigma_X) \cong (\epsilon, S^{\mathcal{AB}\cap\mathcal{GP}})$, we prove that ${\sigma_X}^*(\mathcal{C}(\Sigma_X)) = \mathcal{AB} \cap \mathcal{GP}$. Since $(\epsilon, S^{\mathcal{AB}\cap\mathcal{GP}})$ is the universal abeleian group compactification of S, ${\sigma_X}^*(\mathcal{C}(\Sigma_X)) \subseteq \mathcal{AB} \cap \mathcal{GP}$. For the convese, let $f \in \mathcal{AB} \cap \mathcal{GP}$. We define the mapping $g : \Sigma_X \longrightarrow \mathbb{C}$ by $g(\lambda_x) = x(f)$. We show that g is well-define, bounded and continuos. for this goal, let $\lambda_x = \lambda_y$. So for all $z \in X$ we have $zx = zy$. By setting $z = e$, $ex = ey$. Thus for $f \in \mathcal{GP}$, $ex(f) = ey(f)$, hence $x(f) = y(f)$. Simillarly g is continuous. We notice that $go\sigma_X = f$ as claimed. ☐

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