

[Research]

## Economically optimal cutting cycle in a beech forest, Iranian Caspian Forests

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### ABSTRACT

The aim of this study was to determine the optimal cutting cycle in an uneven-aged beech forest in the North of Iran. First of all, a logistic growth model was determined for an uneven aged forest. Then, the stumpage price was predicted via an autoregressive model. The average stumpage price of beech was derived from actual timber, round wood, fire and pulpwood prices at road side minus the variable harvesting costs. Price and growth models were used in order to determine the optimal cutting cycle under different rates of interest and setup costs. The Faustmann's model was used for optimal cutting cycle. The results indicated that the optimal cutting cycle will decrease if the rate of interest increased. The results also indicated that if the setup costs increase, the optimal cutting cycle will also increase.

**Keywords:** Optimal cutting cycle, Caspian Forests, Faustmann's model, Stumpage price

### INTRODUCTION

The aim of economically optimal harvest interval in an uneven-aged forest is to maximize the expected present value over the infinite horizon. The interval between harvests in an uneven-aged stand is called the cutting cycle. A cycle starts with a harvest that leaves a specified volume of reserve growing stock. This volume grows for the number of years in the cycle. A harvest cut removes the merchantable portion of this growth, plus or minus whatever adjustments are desired in the reserve growing stock to initiate the next cycle. Uneven-aged stands generally lack a definite beginning or end in time. Management of this type of stand is basically a periodic cycle of partial harvests which influence the species composition and size structure of the residual stand. The optimal cutting cycle or harvest interval is calculated as a function of the entering stock, the price state, the harvesting cost, and the rate of interest in the capital market (Mohammadi Limaiei, 2006).

The optimal growing stock and cutting cycle were determined for an uneven-aged stand (Chang, 1981). He used a generalized Faustmann model for determining the

optimal cutting cycle. His results indicated that the rate of interest and stumpage price effect on optimal cutting cycle.

Optimal residual growing stock and cutting cycle in mixed uneven-aged maritime pine stands in Northwestern Spain was calculated (Orois et al, 2004). They used Faustmann's formula to calculate the land expectation value for the economic evaluation of the different management option. Their results indicated that in low sites the uneven-aged management is superior to the even-aged management.

A theoretical model was developed to determine the best cutting policy (cutting cycle and residual stock) in a regulated selection forest. (Buongiorno and Lu, 1990) The model was obtained by a simple modification of the objective function in a linear-programming model of a non-regulated stand. Their results showed that regulating an uneven-aged forest to get a yearly production had a cost, compared to cutting the entire forest at intervals equal to the cutting cycle. For management of uneven-aged stands, a generalized Faustmann's model was developed by Chang and Gadow (2010) to allow the number of years and the level of residual

growing stock to vary from one cutting cycle to the next. Sensitivity analyses reveal that for the uneven-aged loblolly-shortleaf pine stand both the length of the cutting cycle and the level of the residual growing stock are very sensitive to changes in land value in the future, in the stumpage prices of trees selected for harvest, in the stumpage prices of the residual growing stock, and in the interest rate.

Haight (1990) studied feedback of thinning policies for uneven-aged stand management with stochastic prices. Buongiorno (2001) developed a generalization of Faustmann's formula for stochastic forest growth and prices with a Markov decision process model. Stochastic dynamic programming technique was used to determine the optimal extraction level in the Iranian Caspian forest (Mohammadi Limaei, 2011). The results indicated that the expected present value will increase under the stochastic price model.

The aim of this research is to determine the optimal cutting cycle in a beech forest at Shafarod watershed, North of Iran. The Faustmann's model (Faustmann, 1849) was used to determine the optimal cutting cycle in the study area, so that price and growth models under different rates of interest and setup costs are practiced.

### Material and Methods:

#### Study area:

The study area is located in district No 2 in the Shafarod forested watershed, Caspian Forests in the North of Iran. This district is located between  $48^{\circ} 30'$  and  $48^{\circ} 27'$  longitude, and  $37^{\circ} 22'$  and  $37^{\circ} 25'$  latitude. Total area of the district is 2402 hectares. The altitude ranges from 850 to 2000 meters. Almost 95% of the species in this forest belonged to beech and the harvest focuses on this species. The numerical growth and stumpage price data is to be obtained from collected data from this area.

Required data for the model:

The forest state (growth) in each period can be estimated by making an inventory, or based on previous inventory data. The

forest growth in the future periods is determined by the current forest growth.

The interest rate, stumpage price and harvesting cost are based on the current period. In analysis of forest management problems, it is usually assumed that:

- The capital market is perfect (the interest rate for lending is equal to that for borrowing, and any amount can be lent or borrowed at the prevailing interest rate).
- The interest rate is constant over time.
- The timber market as well as the production factors are perfectly competitive.

#### Price model prediction:

In this research, the possible assumption is that the price is a stationary autoregressive (AR) process; in the sense that changes in one period are generally not assumed to affect expected prices very much in other periods. The best forecast of the future price is given by the mean of the process (when the time distance to the future period of interest approaches infinity). The price in this assumption can be estimated as  $P_{t+1} = \alpha + \beta P_t$ ,  $0 < \beta < 1$ . So, a regression analysis was used to estimate the stumpage price process.

The first order autoregressive (AR) model is:

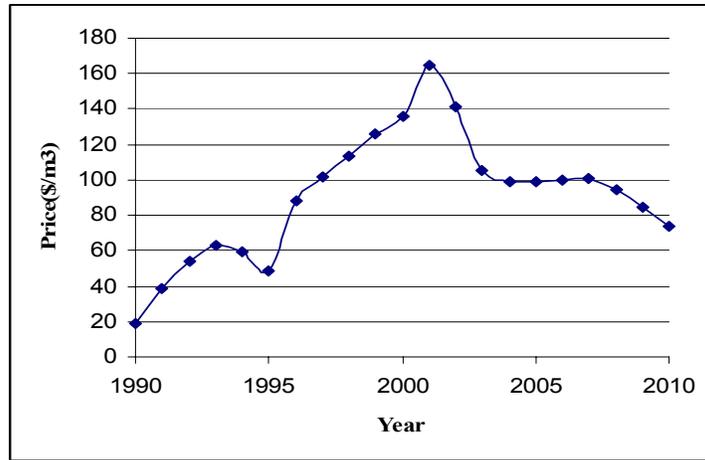
$$P_{t+1} = \delta + \varphi P_t + \varepsilon_t \quad (1)$$

It was assumed that  $\varepsilon_t$  is a series of normally distributed errors with mean zero and autocorrelation zero, and  $0 < \varphi < 1$ .

The mean of the price process was calculated based on the first order AR model parameters:

$$P_{eq} = \frac{\delta}{1 - \varphi} \quad (2)$$

In the numerical calculation in this paper, the data of the period 1990-2010 was collected from the Shafarod Forest Company in north of Iran. The average stumpage price of beech was derived from actual timber, roundwood, fire and pulpwood prices at road side minus the variable harvesting costs. Then it was deflated or adjusted to consumer price index (CPI) of Iran for the base year of 2004 (Fig. 1), (Central Bank of the Islamic Republic of Iran, 2011).



**Fig 1.** Stumpage prices of beech, adjusted to the price level to year 2004 in Iran during the period 1990-2010 (\$/m<sup>3</sup>).

**Growth model prediction:**

Predicting the future growth and yield of forest stand(s) is an essential part of the planning process for forest land.

The following logistic growth model was used for the growth estimation:

$$G = \gamma + \alpha V + \beta V^2 + \varepsilon \tag{3}$$

$G$  = growth (m<sup>3</sup>/ha/year),  $\gamma$ ,  $\alpha$  and  $\beta$  are estimated parameters,  $V$  = stock level (m<sup>3</sup>/ha).

It was assumed that  $\varepsilon$  is a series normally distributed errors with mean zero and autocorrelation zero.

Regression analysis was used to estimate the growth function.

**Faustmann Model:**

The Faustmann Model or land rent theory is used as a benchmark model for determining optimal timber rotation age. Faustmann (1849) showed that the value of a forest can be expressed as a sum of discounted net cash flow over an infinite time period. The goal of a forest owner is to choose a rotation period that the value of a forest is maximized. Therefore, the mathematical model is:

$$Max V_F = \frac{R - C(1+r)^T}{(1+r)^T - 1} \tag{4}$$

Where,  $V_F$  is Soil Expectation value,  $R$  is cutting value at harvest age,  $C$  is plantation costs,  $r$  is the discount rate and  $T$  is harvest age.

The basic logic of this Faustmann formula (or land expectation value, LEV or soil

expectation value, SEV) is as follows: for even-aged timber production, the net present value is basically formed by a perpetual periodic series of clear-cutting revenues at the end of every rotation of  $T$  years. By compounding each rotation's regeneration and other possible costs (as well as possible revenues from thinning to the end of rotation), all (compounded) cash flows can be added to the end of rotation and applied to a general present value formula for a perpetual periodic series, In this research to determine the optimal harvesting period in the presence of different setup costs, the expected present value was calculated. The first harvest will begin  $t_1$  years from now, the expected present value is:

$$\pi = \frac{\bar{P}gT - C}{(1+r)^T - 1} \tag{5}$$

Where,  $\bar{P}$  is the mean of the net price process,  $g$  is the annual growth (m<sup>3</sup>/ha).  $C$  is the setup cost per hectare and occasion.  $T$  is the harvest time interval and  $r$  is the rate of interest in the capital market

**Results:**

**Results of growth model prediction:**

In the numerical calculation of this paper, the growth data (Fig. 2) was collected from a research carried out in Shafarod uneven-aged beech forest (Bonyad, 2005). The data was volume (m<sup>3</sup>/ha) and annual growth for different areas. Therefore, for different standing timber or volume, the actual

growth was determined (Fig. 2). Almost 95% of the species at this forest belonged to beech and the harvest focus on this species.

Hence, the beech volume and growth data was used for prediction the growth model.

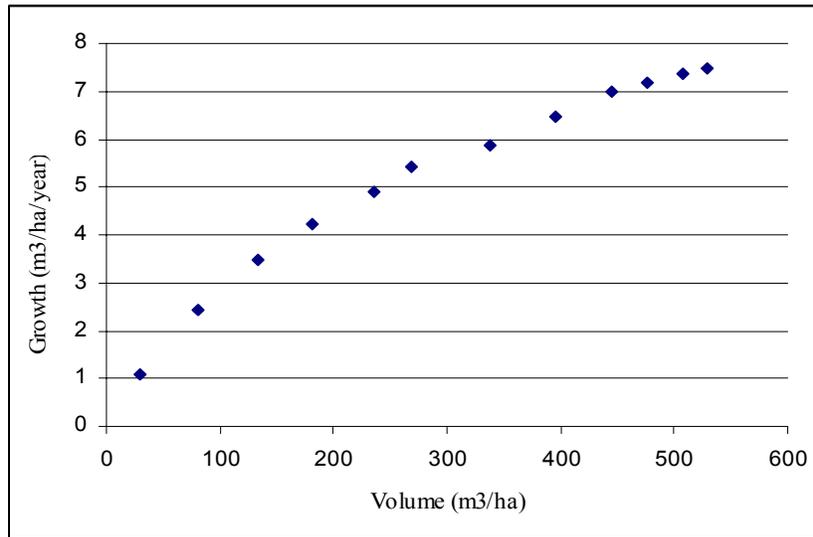


Fig 2. Relation between growth and volume in an un-even aged beech forest in the study area

Regression analysis was used to estimate the growth function. The results show that there is a significant relation between  $G$ ,  $V$  and  $V^2$  with confidence interval of 95%

(Table 1). The growth model based on equation (3) could be calculated as:

$$G = 0.756737 + 0.020917V - 0.000015V^2 \pm 0.1638 \quad (6)$$

Table 1. Estimated growth function parameters based on the growth data from an un-even aged beech forest.

	$\gamma$	$\alpha$	$\beta$	R <sup>2</sup>	R	$\delta$ (standard deviation of $\varepsilon$ )
Parameter value	0.756737	0.020917	-0.000015	0.995	0.997	0.1638
Standard deviation	0.1544	0.001233	2.02240			
t-statistics	4.9011	16.96	-7.63			

**Results of the price model prediction:**

Regression analysis was used to determine the price process parameters based on equation (1). The results show that there is a significant relation between  $P_{t+1}$  and  $P_t$  with confidence interval of 95% (Table 2). The price model based on

equation (1) could be calculated as:

$$p_{t+1} = 22.787 + 9.947 p_t \pm 16.98 \quad (7)$$

If we use the estimates of  $\delta$  and  $\varphi$  from Table 2 and using equation (2), the mean of the net price process is 104.5 \$/m<sup>3</sup>. This means that the expected price in a long term period would be 104.5 \$/m<sup>3</sup>.

Table 2. Estimated price parameters based on the stumpage price data for the first order AR process from period 1980-2003.

	$\delta$	$\varphi$	R <sup>2</sup>	R	$\sigma$ (standard deviation of $\varepsilon_t$ )
Parameter value	22.787	0.782	0.76	0.87	16.98
Standard deviation	9.947	0.1010			
t-statistics	2.290	7.740			

**Determination of Optimal cutting cycle:**

Using growth and price models, the expected present value was calculated under different setup costs and rate of interests.

**Optimal cutting cycle under different setup cost:**

The Optimal cutting cycle was calculated according to equation (5). Different setup costs were assumed in order to determine

the optimal cutting cycle. The required values to solve the problem is  $\bar{P}, g, T, C$  and  $r$ .

$\bar{P}$  is 104.5 \$/m<sup>3</sup> which is the mean of the net price process and it was determined before. When the initial volume is 100 m<sup>3</sup>/ha, and

using the parameters values of  $\gamma, \alpha$  and  $\beta$  into equation (3), the growth ( $g$ ) would be 2.8 m<sup>3</sup>/ha/year.  $T$  is the harvesting cutting cycle that is assumed to vary from 1 to 20 years. Set up costs or  $\alpha$  is assumed to

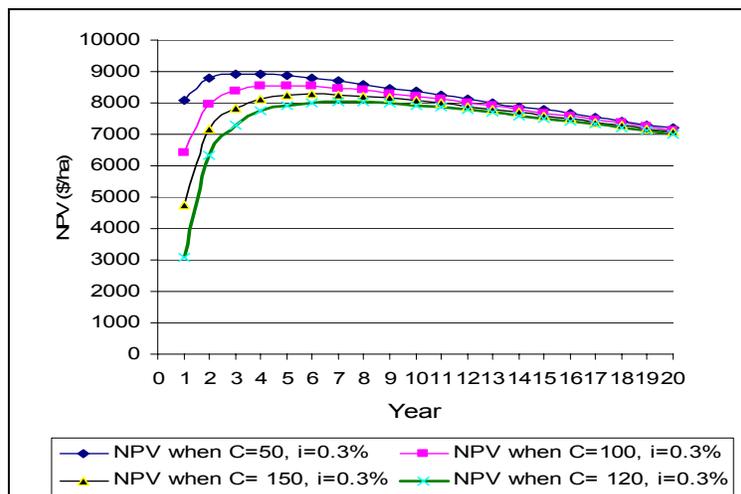
varies from 50\$/ha, 100\$/ha, 150\$/ha and 200\$/ha.

The results indicated that if the set up cost increases, the optimal cutting cycle also increases and the peak of expected net present value shifts to the right side under different setup costs (Table 3 and Fig. 3).

The optimal cutting cycle is 3,5, 6 and 7 years, if the setup cost is 50, 100, 150 and 200 \$/ha, and the present value is 8927.281, 8557.576, 8274.058 and 8040.061 \$/ha (Table 3 and Fig. 3).

**Table 3.** The expected net present value (NPV) under different set up costs (C).

Year	NPV (\$/ha) when C=50	NPV (\$/ha) when C=100	NPV(\$/ha) when C=150	NPV(\$/ha) when C=200
1	8086.667	6420	4753.333	3086.667
2	8788.177	7967.159	7146.141	6325.123
3	8927.281	8388.064	7848.847	7309.629
4	8926.863	8528.485	8130.107	7731.728
5	8871.5	8557.576	8243.652	7929.727
6	8789.383	8531.721	8274.058	8016.396
7	8692.593	8475.083	8257.572	8040.061
8	8587.17	8399.743	8212.315	8024.888
9	8476.468	8312.411	8148.355	7984.298
10	8362.498	8217.114	8071.73	7926.345
11	8246.54	8116.411	7986.282	7856.153
12	8129.446	8012.009	7894.572	7777.135
13	8011.803	7905.087	7798.372	7691.656
14	7894.033	7796.489	7698.945	7601.401
15	7776.44	7686.829	7597.218	7507.607
16	7659.254	7576.569	7493.884	7411.199
17	7542.648	7466.061	7389.473	7312.886
18	7426.757	7355.576	7284.395	7213.214
19	7311.686	7245.33	7178.974	7112.617
20	7197.518	7135.492	7073.465	7011.439



**Fig 3.** The optimal cutting cycle under different setup costs.

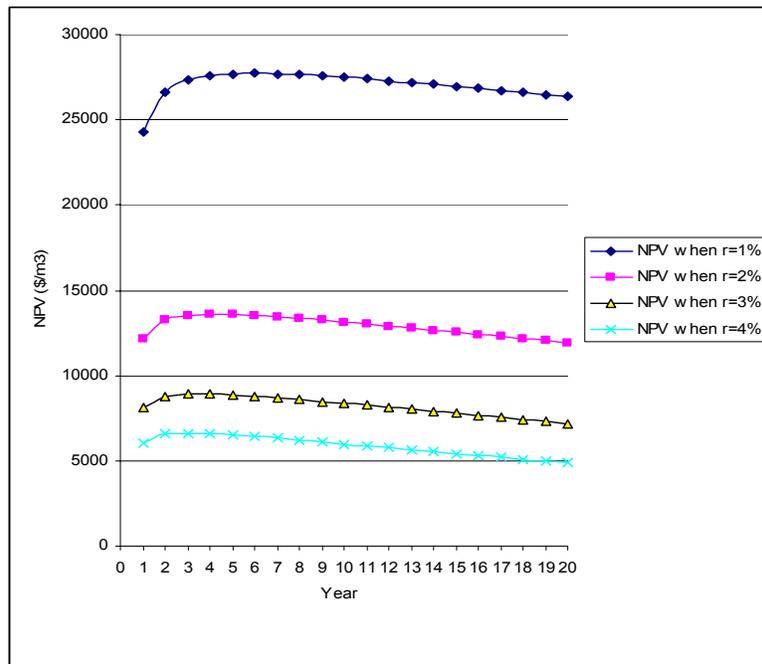
**Optimal cutting cycle under different interest rate:**

The optimal cutting cycle was determined when setup cost is 50\$/ha and different rate of interest. The results indicated that the expected net present value decreased when the rate of interest increased from 1% to 4%. The results also indicated that if

the rate of interest increased, the optimal cutting cycle decreased. The optimal cutting cycle is 6,5, 4 and 3 years, if the interest rate is 1%, 2%, 3% and 4%, respectively. Then the expected net present value is 27724.25, 13575.99, 8926.863 and 6629.613\$/ha (Table 4 and Fig. 4).

**Table 4.** NPV under different rates of interest.

Year	NPV when r=1%	NPV when r=2%	NPV when r=3%	NPV when r=4%
1	24260	12130	8086.667	6065
2	26626.87	13247.52	8788.177	6558.824
3	27319.23	13524.38	8927.281	6629.613
4	27593.33	13591.78	8926.863	6596.076
5	27700.42	13575.99	8871.5	6521.953
6	27724.25	13519.08	8789.383	6428.488
7	27700.7	13439.09	8692.593	6324.783
8	27647.73	13345.03	8587.17	6215.389
9	27575.31	13241.92	8476.468	6102.83
10	27489.41	13132.75	8362.498	5988.619



**Fig 4.** Expected present value under different rates of interest.

**DISCUSSION:**

In this paper we first estimated a growth model of an uneven- aged forest in the Iranian Caspian region. The logistic growth was used to predict the function (Clark , 1976): Many papers have been written about growth functions in uneven-aged forests such as: Hann and Bare (1979),

Buongiorno *et al.* (1995), Virgilietti and Buongiorno (2001), Peng (2000).

The stumpage price of beech was predicted via an autoregressive model. In this study, it is shown that the stumpages price during the period 1990-2010 fluctuates over time. The mean of the stumpage price process according to the first order AR

process estimated was 104.5 \$/m<sup>3</sup>.

Lohmander (1987) investigated the time series of stumpage prices in Sweden, Norway and Finland. He used an AR model to predict the price process. Gong (1990) used AR models for Swedish saw timber prices and price predictions. Howard (1995) estimated price trends for stumpage and selected agricultural products in Costa Rica. Mohammadi Limaiei (2006) investigated the time series of stumpage prices via AR model in Iran. He showed that the stumpages price in Iran during the period 1980-2003 fluctuates over time may be regarded as a stationary stochastic process. The mean of the stumpage price process according to the first order AR process was estimated 47.57\$/m<sup>3</sup>.

Consequently, the optimal cutting cycle was calculated based on price and growth in a beech forest in north of Iran. The optimal cutting cycle or harvest interval is calculated as a function of the entering stock, the price state, the harvesting cost, and the rate of interest in the capital market.

The optimal growing stock and cutting cycle was determined for an uneven-aged stand (Chang, 1981). His results indicated that the rate of interest and stumpage price effect on optimal cutting cycle. A generalized Faustmann model is developed for uneven-aged management to allow the number of years and the level of residual growing stock to vary from one cutting cycle to the next (Chang, and Gadow , 2010). Their results showed that the length of the cutting cycle and the level of the residual growing stock are very sensitive to changes in land value in the future, in the stumpage prices of trees selected for harvest, in the stumpage prices of the residual growing stock, and in the interest rate. The optimal cutting cycle for an un-even aged forest was calculated in Khirodkenar, north of Iran (Lohmander and Mohammadi Limaiei, 2008). Their results showed that the optimal harvest interval is 6 (9) years, if the set up cost is 50 € (100 €) per hectare, rate of interest is 3% and the present value is 2757.56 € (2551.62) per hectare. There are similarity between their results and the results of this research whereas if the setup cost increases, the optimal harvest interval increases. The

optimal cutting cycle in this research is 3,5, 6 and 7 years, if the setup cost is 50, 100, 150 and 200 \$/ha, and the present value is 8927.281 , 8557.576, 8274.058 and 8040.061 \$/ha

Finally, the primal cutting cycle under different interest rates were also calculated. The results indicated that the expected net present value decreased when the rate of interest increased from 1% to 4%. The results also indicated that if the rate of interest increased, the optimal cutting cycle decreased. The optimal cutting cycle is 6,5, 4 and 3 years, if the interest rate is 1%, 2%, 3% and 4%, respectively. The results of this research could be a guideline for foresters to make economically optimal harvest decisions.

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## چرخش برداشت بهینه اقتصادی در یک جنگل راش، جنگل های خزری ایران

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### چکیده

هدف از این مطالعه تعیین چرخش برداشت بهینه اقتصادی در یک جنگل ناهمسال در شمال ایران بود. ابتدا یک مدل لجستیک رویش تعیین شد. سپس قیمت چوب سرپا با استفاده از مدل خود کاهشی تعیین گردید. میانگین قیمت چوب سرپا با کسر هزینه های متغیر بهره برداری از قیمت واقعی چوب آلات مختلف مثل گرده بینه، چوب های هیزمی در کنار جاده جنگلی بدست آمد. مدل های قیمت و رویش جهت تعیین چرخش بهینه اقتصادی به ازای نرخ های مختلف سود بانکی و هزینه های ثابت مختلف محاسبه شد. نتایج نشان داد که طول چرخش بهینه با افزایش نرخ سود بانکی کاهش پیدا می کند. نتایج همچنین نشان داد که اگر هزینه ثابت بهره برداری افزایش یابد، طول چرخش بهینه افزایش پیدا می کند.