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On pseudo 2-prime hyperideals

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Abstract. Let R be a multiplicative hyperring. In this paper, we introduce and investigate the notion of pseudo 2-prime hyperideals. Several fundamental properties of pseudo 2-prime hyperideals in multiplicative hyperrings are established. In particular, we examine the relationships between pseudo 2-prime hyperideals and other important classes of hyperideals, including prime hyperideals, 2-prime hyperideals, quasi-primary hyperideals, and 2-absorbing primary hyperideals. Moreover, we prove that a hyperideal J is pseudo 2-prime if and only if J/γ^* is a pseudo 2-prime ideal of R/γ^*. Finally, we establish a Pseudo 2-Prime Avoidance Theorem for a pseudo 2-prime hyperideal of R.

Keywords: 2-prime hyperideal, Pseudo 2-prime hyperideal, Multiplicative hyperring.
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1 Introduction

The notion of hypergroups was first introduced in 1934 by the French mathematician Marty [12] at the 8th Congress of Scandinavian Mathematicians. Multiplicative hyperrings constitute an important class of algebraic hyperstructures that generalize classical rings. The systematic study of these structures was initiated by Rota in 1982, where multiplication is defined as a hyperoperation while addition remains a standard operation [15]. Fundamental concepts in algebraic hyperstructure theory can be found in [6, 8]. Procesi and Rota briefly introduced and studied prime hyperideals of multiplicative hyperrings [13, 14], and this notion was further generalized by U. Dasgupta [7]. Ameri et al. [1] described multiplicative hyperrings of fractions and coprime hyperideals. Subsequent research has explored various generalizations of prime hyperideals in multiplicative hyperrings. In particular, P. Ghiasvand [9] introduced and studied 2-absorbing hyperideals as a generalization of prime hyperideals, while Anbarloei investigated 2-absorbing and 2-absorbing primary hyperideals in [3, 4]. In [10], the notion of S-prime hyperideals in multiplicative hyperrings was introduced and systematically studied as a generalization of prime

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hyperideals. In [5], Anbarloei introduced and investigated the concept of 2-prime hyperideals of multiplicative hyperrings, which The notion of pseudo 2-prime hyperideals is introduced as a generalization of classical prime and 2-prime ideals to the hyperring framework, where the absence of single-valued multiplication prevents a direct extension of standard primeness conditions. Pseudo 2-prime hyperideals provide a weaker yet mathematically significant form of primeness that is compatible with hyperoperations and better reflects the intrinsic structure of hyperrings. This concept offers a flexible framework for investigating zero-divisor phenomena, factorization behavior, and ideal-theoretic decompositions in hyperrings, enabling a natural extension of several fundamental results from classical ring theory The main objective of the present study is to extend the notion of pseudo 2-prime ideals, originally introduced by S. Koç [11] in commutative rings, to multiplicative hyperrings as a generalized problem in hyperstructure theory.

In this paper, we introduce and study pseudo 2-prime hyperideals as a new generalization of prime hyperideals and derive several fundamental and novel properties. Through explicit examples, we demonstrate that 2-prime hyperideals, primary hyperideals, and quasi-primary hyperideals are distinct from pseudo 2-prime hyperideals. In particular, pseudo 2-prime hyperideals constitute a strictly broader class that neither coincides with nor reduces to any of the aforementioned notions, capturing structural behaviors not detectable by existing generalizations. Moreover, we investigate conditions under which pseudo 2-prime hyperideals, 2-prime hyperideals, and quasi-primary hyperideals coincide. We also prove that a hyperideal J is pseudo 2-prime if and only if J/γ^* is a pseudo 2-prime ideal of R/γ^*. Moreover, we investigate the conditions under which pseudo 2-prime hyperideals, 2-prime hyperideals, and quasi-primary hyperideals coincide. We also show that a hyperideal J is pseudo 2-prime if and only if J/γ^* is a pseudo 2-prime ideal of R/γ^*. Finally, we formulate the Pseudo 2-Prime Avoidance Theorem for pseudo 2-prime hyperideals of R.

2 Preliminaries

In this section, we present some definitions and results on hyperstructures that are needed for the development of this paper. For basic properties and further information on hyperstructures, we refer the reader to [7, 8].

Definition 1 ([12]). *Let H be a nonempty set. By $\mathcal{P}^*(H)$, we mean the set of all nonempty subsets of H. A hyperoperation on H is a map $\circ : H \times H \to \mathcal{P}^*(H)$. $(H, \circ)$ is called a hypergroupoid. A hypergroup is a hypergroupoid $(H, \circ)$ which satisfies the associative and the reproductive law, i.e.,*

(1) $x \circ (y \circ z) = (x \circ y) \circ z, \ \forall x, y, z \in H,$

(2) $x \circ H = H \circ x = H, \ \forall x \in H.$

A hypergroupoid $(H, \circ)$ is called a semihypergroup if only the associative law is hold. Here, we mean a semihypergroup by a nonempty set H with an associative hyperoperation $\circ$, i.e.,

$$a \circ (b \circ c) = \bigcup_{t \in (b \circ c)} a \circ t = \bigcup_{s \in (a \circ b)} s \circ c = (a \circ b) \circ c$$

for all $a, b, c \in H$.

For any two nonempty subsets A and B of R and $x \in R$, we define

$$A \circ B = \bigcup_{a \in A, b \in B} a \circ b, \quad A \circ x = A \circ \{x\}.$$

Definition 2 ([15]). *A triple $(R, +, \circ)$ is called a multiplicative hyperring, if it has the following properties:*

(i) $(R, +)$ *is an abelian group;*

(ii) $(R, \circ)$ *is a semihypergroup;*

(iii) For all $a, b, c \in R$, $a \circ (b + c) \subseteq a \circ b + a \circ c$ and $(b + c) \circ a \subseteq b \circ a + c \circ a$;

(iv) $a \circ (-b) = (-a) \circ b = -(a \circ b)$.

If in *(iii)* we have equalities instead of inclusions, then we say that the multiplicative hyperring is strongly distributive.

Definition 3 ([15]). *(a) If R is a multiplicative hyperring with $a \circ b = b \circ a$ for all $a, b \in R$, then R is called a commutative multiplicative hyperring.*

(b) A nonzero element a of a multiplicative hyperring R is said to be unit, if $1 \in a \circ x$ and $1 \in x \circ a$ for some $x \in R$. The set of all unit elements of R is denoted by $U(R)$.

Example 1. Let $(K, +, \cdot)$ be a field and V be a vectorial space over K. If for all $a, b \in K$ we denote by (a, b) the subspace generated by the subset $\{a, b\}$ of V, then we define the following hyperoperation on V: For all $a, b \in V$, $a \circ b = (a, b)$. It follows that $(V, +, \circ)$ is a multiplicative hyperring, which is not strongly distributive.

Definition 4 ([8]). *(a) Let $(R, +, \circ)$ be a multiplicative hyperring and S be a non-empty subset of R. Then S is said to be a subhyperring of R if $(S, +, \circ)$ is itself a multiplicative hyperring.*

(b) A non-empty subset I of a multiplicative hyperring R is a hyperideal of R if

(i) $a, b \in I$, then $a - b \in I$,

(ii) $a \in I$ and $r \in R$, then $r \circ a \subseteq I$.

Definition 5 ([7]). *A proper hyperideal P of a multiplicative hyperring R is said to be a prime hyperideal of R, if for any $a, b \in R$, $a \circ b \subseteq P$, then $a \in P$ or $b \in P$.*

Definition 6 ([7]). *(a) Let C be the class of all finite products of elements of a multiplicative hyperring R i.e., $C = \{r_1 \circ r_2 \circ \cdots \circ r_n : r_i \in R, n \in \mathbb{N}\} \subseteq P^*(R)$. A hyperideal I of R is said to be a C-ideal of R if for any $A \in C$, $A \cap I \neq \emptyset$, then $A \subseteq I$.*

(b) Let I be a hyperideal of a multiplicative hyperring $(R, +, \circ)$. The intersection of all prime hyperideals of R containing I is called the radical of I, being denoted by $rad(I)$. If the multiplicative hyperring R does not have any prime hyperideal containing I, we define $rad(I) = R$. Also, the hyperideal $\{r \in R : r^n \subseteq I \text{ for some } n \in \mathbb{N}\}$ will be designated by $D(I)$ and note that the inclusion $D(I) \subseteq rad(I)$ always holds. In addition, if I is a C-ideal of R, other inclusion holds.

Definition 7. (*a*) *A proper hyperideal Q of a multiplicative hyperring R is said to be a primary hyperideal of R, if for any $a, b \in R$, $a \circ b \subseteq Q$, then $a \in Q$ or $b^n \subseteq Q$ for some $n \in \mathbb{N}$ [7].*

(*b*) *A proper hyperideal I of a multiplicative hyperring R is called a 2-absorbing primary hyperideal of R, if for any $a, b, c \in R$, $a \circ b \circ c \subseteq I$, then $a \circ b \subseteq I$ or $b \circ c \subseteq rad(I)$ or $a \circ c \subseteq rad(I)$ [9].*

(*c*) *Let Q be a proper hyperideal of a multiplicative hyperring R. We say that Q is a quasi primary hyperideal if for all $a, b \in R$ such that $a \circ b \subseteq Q$, then either $a \in rad(Q)$ or $b \in rad(Q)$.*

(*d*) *A proper hyperideal I of R is called a 2-prime hyperideal if for all $a, b \in R, a \circ b \subseteq I$ implies $a^2 \subseteq I$ or $b^2 \subseteq I$ [5].*

Definition 8 ([1])**.** *Let I be a hyperideal of a multiplicative hyperring R and let $R/I = \{r+I \mid r \in R\}$. Define the operation $+$ and the hyperoperation $\circ$ on R/I by $(a + I) + (b + I) = a + b + I$ and $(a + I) \circ (b + I) = \bigcup\{c + I \mid c \in a \circ b\}$. Then $(R/I, +, \circ)$ is called a quotient hyperring.*

Definition 9 ([1])**.** *Let R and R' be multiplicative hyperrings. A mapping $\phi : R \to R'$ is said to be a hyperring good homomorphism, if for all $a, b \in R$;*

(1) $\phi(a + b) = \phi(a) + \phi(b)$,

(2) $\phi(a \circ b) = \phi(a) \circ \phi(b)$.

Throughout this paper R is a commutative multiplicative hyperring with scalar identity 1.

3 Main results

First, we present the definition of pseudo 2-prime hyperideal of a commutative multiplicative hyperring $(R, +, \circ)$ as a new generalization of prime hyperideals and by providing examples, we show that definitions of 2-prime hyperideals, primary hyperideals and quasi-primary hyperideals are different from definition of pseudo 2-prime hyperideals.

Definition 10. *A proper hyperideal J of a commutative multiplicative hyperring R is called a pseudo 2-prime hyperideal if, for any $a, b \in R$ with $a \circ b \subseteq J$, there exists a positive integer $n \in \mathbb{N}$ such that either $a^{2n} \subseteq J^n$ or $b^{2n} \subseteq J^n$.*

Example 2. (*a*) Let $R = (k[X, Y], +, \cdot)$ be the polynomial ring over a field k. Define $(R, +, \circ) = R_A$ to be a commutative multiplicative hyperring with $A = \{1, 3, 5, 6\}$, where for any $f, g \in R_A$, the hyperoperation $\circ$ is defined by $f \circ g = \{f \cdot a \cdot g : a \in A\}$. Let $Q = (X^2, XY, Y^2)$. It is straightforward to verify that Q is a pseudo 2-prime hyperideal of R.

(*b*) Let $R = (k[X, Y]/J, +, \cdot)$ be the polynomial ring over a field k and $J = (X^6, XY, Y^6)$. Define $(R, +, \circ) = R_A$ to be a commutative multiplicative hyperring with $A = \{1, 2, 4, 7\}$, where $R_A = k[X, Y]/J$ and, for any $f, g \in R_A$, the hyperoperation $\circ$ is defined by $f \circ g = \{f \cdot a \cdot g : a \in A\}$. Consider the hyperideal $P = (X^3, XY, Y^3)/J$. We claim that P is a pseudo 2-prime hyperideal of R. To prove this, assume that $f \circ g \subseteq P \subseteq rad(P)$ for some $f, g \in R_A$. Since $rad(P) = (X, Y)/J$ is a prime hyperideal of R_A, it follows that $f \in rad(P)$ or $g \in rad(P)$. Moreover, since $rad(P)^6 = (0 + J)$, we obtain $f^{12} = \underbrace{f \circ f \circ \cdots \circ f}_{12} \subseteq P^6 = (0 + J)$ or $g^{12} = \underbrace{g \circ g \circ \cdots \circ g}_{12} \subseteq P^6 = (0 + J)$. Hence, P is a pseudo 2-prime hyperideal of R_A.

Example 3. Let $R = (\mathbb{Z}, +, \cdot)$ be the ring of integers. Then $(R, +, \circ) = R_A$ with $A = \{1, 2, 3, 6\}$ is a commutative multiplicative hyperring, where $R_A = \mathbb{Z}$ and for any $x, y \in R_A$, the hyperoperation $\circ$ is defined by $x \circ y = \{x \cdot a \cdot y : a \in A\}$. Let $Q = (6)$ be a hyperideal of R_A. We show that Q is not a pseudo 2-prime hyperideal of R_A. Indeed, we have $2 \circ 3 = \{6, 12, 18, 36\} \subseteq Q$ but neither $2^{2n} = \underbrace{2 \circ 2 \circ \cdots \circ 2}_{2n} \subseteq Q^n$ nor $3^{2n} = \underbrace{3 \circ 3 \circ \cdots \circ 3}_{2n} \subseteq Q^n$ for any $n \in \mathbb{N}$. Hence, Q is not a pseudo 2-prime hyperideal of R_A.

Remark 1. *Note that for any hyperideal J, $rad(J^n) = rad(J)$ for every $n \in \mathbb{N}$. Since $J^n = \underbrace{J \circ J \circ \cdots \circ J}_{n} \subseteq J$, then $rad(J^n) \subseteq rad(J)$. In the other hand, $x \in rad(J)$, then $x^k \subseteq J$ for some $k \in \mathbb{N}$. Thus $x^{nk} \subseteq J^n$, and $x \in rad(J^n)$. Therefore, $rad(J) \subseteq rad(J^n)$. Consequently, $rad(J^n) = rad(J)$.*

In the following theorem, we establish the relationship between these definitions.

Theorem 1. *Let R be a commutative multiplicative hyperring and J be a proper C-hyperideal of R. The following statements are satisfied.*

(i) Every pseudo 2-prime hyperideal J of R is a quasi primary hyperideal of R, that is, $rad(J)$ is a prime hyperideal of R.

(ii) Every 2-prime hyperideal J of R is a pseudo 2-prime hyperideal of R. In particular, every prime hyperideal is pseudo 2-prime.

(iii) Every pseudo 2-prime hyperideal J of R is a 2-absorbing primary hyperideal of R.

Proof. (i) Let $a \circ b \subseteq rad(J)$ for some $a, b \in R$. Then $(a \circ b)^t \subseteq J$ for some $t \in \mathbb{N}$. Hence $a^t \circ b^t \subseteq J$, so there exists $n \in \mathbb{N}$ such that $a^{2nt} \subseteq J^n$ or $b^{2nt} \subseteq J^n$. It implies that $a \in rad(J^n) = rad(J)$ or $b \in rad(J^n) = rad(J)$. Therefore, J is a quasi primary hyperideal of R.

(ii) Let $a, b, c \in R$ be such that $a \circ b \circ c \subseteq J$ and $a \circ b \nsubseteq J$. Since J is a 2-prime hyperideal of R, we have $a^2 \subseteq J$ or $b^2 \subseteq J$. Consequently, $a^{2n} \subseteq J^n$ or $b^{2n} \subseteq J^n$. Therefore, J is a pseudo 2-prime hyperideal of R.

(iii) Let $a \circ b \circ c \subseteq J$ and $a \circ b \nsubseteq J$ for some $a, b, c \in R$. By the associativity and commutativity of the hyperoperation, we have $(a \circ b \circ c) \circ c = (a \circ c) \circ (b \circ c) \subseteq J \subseteq \mathrm{rad}(J)$. By part (i), $\mathrm{rad}(J)$ is a prime hyperideal of R. Therefore, $a \circ c \subseteq \mathrm{rad}(J)$ or $b \circ c \subseteq \mathrm{rad}(J)$. Thus, since $a \circ b \nsubseteq J$, the defining condition of a 2-absorbing primary hyperideal is satisfied. Consequently, J is a 2-absorbing primary hyperideal of R. □

Let $a, b, c \in R$ be such that $a \circ b \circ c \subseteq J$ and $a \circ b \nsubseteq J$. Since J is a hyperideal, multiplying the inclusion $a \circ b \circ c \subseteq J$ by c yields $(a \circ b \circ c) \circ c \subseteq J$. By the associativity and commutativity of the hyperoperation, we have $(a \circ b \circ c) \circ c = (a \circ c) \circ (b \circ c)$. Hence, $(a \circ c) \circ (b \circ c) \subseteq J \subseteq \mathrm{rad}(J)$. By part (i), $\mathrm{rad}(J)$ is a prime hyperideal of R. Therefore, $a \circ c \subseteq \mathrm{rad}(J)$ or $b \circ c \subseteq \mathrm{rad}(J)$. Thus, since $a \circ b \nsubseteq J$, the defining condition of a 2-absorbing primary hyperideal is satisfied. Consequently, J is a 2-absorbing primary hyperideal of R.

Example 4. (a) Suppose $Q = (X^2, XY, Y^2)$ is the hyperideal defined in Example 2(a). It is straightforward to see that Q is a pseudo 2-prime hyperideal of R_A, but is not a prime hyperideal of R_A, since $X \circ Y = \{XY, 3XY, 5XY, 6XY\} \subseteq Q$, while $X \notin Q$ and $Y \notin Q$.

(b) Suppose $P = (X^3, XY, Y^3)/J$ is the hyperideal defined in Example 2(b). Then P is a pseudo 2-prime hyperideal of R, but it is not a 2-prime hyperideal of R_A. Indeed, we have $(x + J) \circ (y + J) = x \circ y + J = \{xy + J, 2xy + J, 4xy + J, 7xy + J\} \subseteq P$ while $(x + J)^2 = (x + J) \circ (x + J) = x \circ x + J = \{x^2 + J, 2x^2 + J, 4x^2 + J, 7x^2 + J\} \nsubseteq P$ and $(y + J)^2 = (y + J) \circ (y + J) = y \circ y + J = \{y^2 + J, 2y^2 + J, 4y^2 + J, 7y^2 + J\} \nsubseteq P$.

Example 5. Let be the polynomial ring over a field k. Define $(R, +, \circ) = R_A$ with $A = \{1, 4, 7, 8\}$ as a commutative multiplicative hyperring, where $R_A = k[X, Y]$ and, for any $f, g \in R_A$, the hyperoperation $\circ$ is defined by $f \circ g = \{f \cdot a \cdot g : a \in A\}$.

(a) Let $T = (X^3, XY, Y^3)$. It is straightforward to see that $rad(T) = (X, Y)$ is a primary hyperideal of R_A; hence, T is a quasi-primary hyperideal of R_A. However,although $X \circ Y = \{XY, 4XY, 7XY, 8XY\} \subseteq T$ we have $X^{2n} = \underbrace{X \circ X \circ \cdots \circ X}_{2n} \nsubseteq T^n$ and $Y^{2n} = \underbrace{Y \circ Y \circ \cdots \circ Y}_{2n} \nsubseteq T^n$. Therefore, we conclude that T is not a pseudo 2-prime hyperideal of R_A.

(b) Let $J = (X^2, XY)$. Then $rad(J) = (X)$ is a prime hyperideal of R_A. However, J is not a primary hyperideal of R_A, since

$$X \circ Y = \{XY, 4XY, 7XY, 8XY\} \subseteq J,$$

while $X \notin J$ and $Y \notin rad(J)$. Nevertheless, J is a pseudo 2-prime hyperideal of R_A.

Now, we examine the conditions under which the notions of prime hyperideals, primary hyperideals, pseudo 2-prime hyperideals, 2-prime hyperideals, and quasi-primary hyperideals of a commutative multiplicative hyperring coincide.

Proposition 1. *Let R be a commutative multiplicative hyperring, and let J be a proper C-hyperideal of R such that $rad(J)^2 \subseteq J$. Then the following statements are equivalent:*

(i) J is a 2-prime hyperideal of R.

(ii) J is a pseudo 2-prime hyperideal of R.

(iii) J is a quasi primary hyperideal of R.

Proof. The implications, $(i) \Rightarrow (ii) \Rightarrow (iii)$, follow from Theorem 1. Now, we prove that $(iii) \Rightarrow (i)$. Let $a \circ b \subseteq J \subseteq rad(J)$ for some $a, b \in R$. Then $a \in rad(J)$ or $b \in rad(J)$, since $rad(J)$ is a prime hyperideal. Because $rad(J)^2 \subseteq J$, it follows that $a^2 \subseteq rad(J)^2 \subseteq J$ or $b^2 \subseteq rad(J)^2 \subseteq J$. Therefore, J is a 2-prime hyperideal of R. □

Proposition 2. *Let R be a commutative multiplicative hyperring, and let J be a 2-absorbing primary C-hyperideal of R. Then J is a 2-prime hyperideal of R if and only if J is a pseudo 2-prime hyperideal of R.*

Proof. It is clear that if J is a 2-prime hyperideal of R, then J is a pseudo 2-prime hyperideal of R. Now, suppose that J is a pseudo 2-prime hyperideal of R. Since J is a 2-absorbing primary hyperideal of R, it follows that $rad(J)^2 \subseteq J$. The remaining statement then follows directly from Proposition 1. □

Proposition 3. *Let R be a commutative multiplicative hyperring. Then the zero hyperideal of R is a quasi primary hyperideal of R if and only if it is a pseudo 2-prime hyperideal.*

Proof. Let the zero hyperideal be a quasi primary hyperideal. Then by Theorem 1, it is a pseudo 2-prime hyperideal of R. Conversely, suppose that the zero hyperideal is a pseudo 2-prime hyperideal. Let $a \circ b \subseteq \{0\}$ where $a, b \in R$. Then $a \in rad(\{0\})$ or $b \in rad(\{0\})$, since $\{0\}$ is a quasi primary hyperideal. Therefore, $a^{2n} \subseteq \{0\}^n$ or $b^{2n} \subseteq \{0\}^n$ which shows that the zero hyperideal is indeed a pseudo 2-prime hyperideal of R. □

Remark 2 ([2])**.** *A hyperring R is called a regular hyperring if, for each $a \in R$, there exists $b \in R$ such that $a = a \circ b \circ a$. It is well known that R is regular if and only if for every hyperideal I and $n \in \mathbb{N}$, $I = I^n$ if and only if $I = rad(I)$ for every hyperideal I of R.*

Theorem 2. *Let R be a regular hyperring, and let J be a proper hyperideal of R. The following statements are equivalent:*

(i) J is a prime hyperideal.

(ii) J is a primary hyperideal.

(iii) J is a 2-prime hyperideal.

(iv) J is a pseudo 2-prime hyperideal.

(v) J is a quasi primary hyperideal.

Proof. $(i) \Leftrightarrow (ii) \Leftrightarrow (iii)$: The proofs follow from the fact that $I = rad(I)$ for every hyperideal I of R of a regular hyperrings.

$(iii) \Leftrightarrow (iv) \Leftrightarrow (v)$: The proofs follow from the fact that $rad(J)^2 = J^2 \subseteq J$ of a regular hyperrings and Proposition 1. □

Theorem 3. *Let $\varphi : R_1 \to R_2$ be a good homomorphism between two hyperrings, and let J be a proper hyperideal of R. The following statements hold.*

(i) If φ is onto and J is a pseudo 2-prime hyperideal of R_1 containing $Ker(\varphi)$, then $\varphi(J)$ is a pseudo 2-prime hyperideal of R_2.

(ii) If $\varphi(J)$ is a pseudo 2-prime hyperideal of R_2 such that $Ker(\varphi) \subseteq J^n$ for each $n \in \mathbb{N}$, then J is a pseudo 2-prime hyperideal of R_1.

Proof. (i) Let $c \circ d \subseteq \varphi(J)$ for some $c, d \in R_2$. Since φ is onto, there exist $a, b \in R_1$ such that $c = \varphi(a)$ and $d = \varphi(b)$ and $c \circ d = \varphi(a \circ b) \subseteq \varphi(J)$. Thus $a \circ b \subseteq J$ because, if $x \in a \circ b$, then $\varphi(x) \in \varphi(a \circ b) \subseteq \varphi(J)$. Hence $\varphi(x) = \varphi(y)$ for some $y \in J$, so $x - y \in Ker(\varphi) \subseteq J$ and $x \in J$. Since J is a pseudo 2-prime hyperideal of R_1, we conclude $a^{2n} \subseteq J^n$ or $b^{2n} \subseteq J^n$ for some $n \in \mathbb{N}$. Therefore, $c^{2n} = \varphi(a^{2n}) \subseteq \varphi(J)^n$ or $d^{2n} = \varphi(c^{2n}) \subseteq \varphi(J)^n$. Hence $\varphi(J)$ is a pseudo 2-prime hyperideal of R_2.

(ii) Let $a \circ b \subseteq J$ for some $a, b \in R_1$. Then $\varphi(a) \circ \varphi(b) = \varphi(a \circ b) \subseteq \varphi(J)$. As $\varphi(J)$ is a pseudo 2-prime hyperideal of R_2, there exists $n \in \mathbb{N}$ such that $\varphi(a)^{2n} = \varphi(a^{2n}) \subseteq \varphi(J^n)$ or $\varphi(b)^{2n} = \varphi(b^{2n}) \subseteq \varphi(J^n)$. Hence $a^{2n} \subseteq J^n$ or $b^{2n} \subseteq J^n$, since $Ker(\varphi) \subseteq J^n$. Therefore, J is a pseudo 2-prime hyperideal of R_1. □

Corollary 1. *Let R be a commutative multiplicative hyperring and J and I be hyperideals of R. Then the following statements hold:*

(i) Let J be a pseudo 2-prime hyperideal of R and $I \subseteq J$. Then J/I is a pseudo 2-prime hyperideal of R/I.

(ii) Let J/I be a pseudo 2-prime hyperideal of R/I and $I \subseteq J^n$ for each $n \in \mathbb{N}$. Then J is a pseudo 2-prime hyperideal of R.

Proof. It follows directly from Theorem 3. □

Let $(R_1, +, \circ)$ and $(R_2, +, \circ)$ be two multiplicative hyperrings with identity 1. Then $(R = R_1 \times R_2, +, \circ)$ is a multiplicative hyperring with operation $+$ and the hyperoperation $\circ$ are defined respectively as $(x, y) + (z, t) = (x + z, y + t)$ and $(x, y) \circ (z, t) = \{(a, b) \in R \mid a \in x \circ z, b \in y \circ t\}$ for all $(x, y), (z, t) \in R$.

Theorem 4. *Let $R = R_1 \times R_2$ be a decomposable hyperring, where R_1 and R_2 are multiplicative hyperrings with identity 1. Suppose that $J = J_1 \times J_2$ is a hyperideal of R. Then the following statements are equivalent:*

(i) J is a pseudo 2-prime ideal of R.

(ii) Either $J_1 = R_1$ and J_2 is a pseudo 2-prime hyperideal of R_2, or $J_2 = R_2$ and J_1 is a pseudo 2-prime hyperideal of R_1.

Proof. $(i) \Rightarrow (ii)$ Let J be a pseudo 2-prime hyperideal of R. Since $(1, 0) \circ (0, 1) = (0, 0) \in J$, there exists $n \in \mathbb{N}$ such that $(1, 0)^{2n} = (1, 0) \in J^n \subseteq J$ or $(0, 1)^{2n} = (0, 1) \in J^n \subseteq J$. Hence, we conclude that $J_1 = R_1$ or $J_2 = R_2$. Assume that $J_1 = R_1$. We show that J_2 is a pseudo 2-prime hyperideal of R_2. Let $a \circ b \subseteq J_2$ for some $a, b \in R_2$. Then $(0, a) \circ (0, b) \subseteq J$. Since J is a pseudo 2-prime hyperideal of R, there exists $n \in \mathbb{N}$ such that $(0, a)^{2n} \subseteq J^n$ or $(0, b)^{2n} \subseteq J^n$. Consequently, $a^{2n} \subseteq J_2^n$ or $b^{2n} \subseteq J_2^n$. Thus J_2 is a pseudo 2-prime hyperideal of R_2. In the other case, can be proved similarly to show that J_1 is a pseudo 2-prime hyperideal of R_1.

$(ii) \Rightarrow (i)$ Let $J_1 = R_1$ and J_2 be a pseudo 2-prime hyperideal of R_2. We show that J is a pseudo 2-prime ideal of R. Let $(a, b) \circ (c, d) \subseteq J$ for some $(a, b), (c, d) \in R$. Then $b \circ d \subseteq J_2$. Since J_2 is a pseudo 2-prime hyperideal of R_2, there exists $n \in \mathbb{N}$ such that $b^{2n} \subseteq J_2^n$ or $d^{2n} \subseteq J_2^n$. Hence, we conclude that $(a, b)^{2n} \subseteq J^n$ or $(c, d)^{2n} \subseteq J^n$, which shows that J is a pseudo 2-prime ideal of R. In the other case, one can prove that J is a pseudo 2-prime ideal of R. □

Corollary 2. *Let $R = R_1 \times R_2 \times \cdots \times R_2$ be a decomposable hyperring where $R_1, R_2, \ldots, R_n$ are multiplicative hyperrings with identity 1 and let $J = J_1 \times J_2 \times \cdots \times J_n$ be a hyperideal of R. Then the following statements are equivalent:*

(i) J is a pseudo 2-prime hyperideal of R.

(ii) J_i is a pseudo 2-prime hyperideal of R_i for some $i \in \{1, 2, \ldots, n\}$ and $J_j = R_j$ for each $i \neq j$.

Let $(R, +, \circ)$ be a hyperring with identity 1. We define the relation γ as follows: $a\gamma b$ if and only if $a, b \subseteq U$ where U is a finite sum of finite products of elements of R, i.e.,
$a\gamma b \Leftrightarrow \exists z_1, z_2, \ldots, z_n \in R$ such that $\{a, b\} \subseteq \sum_{j\in J} \prod_{i\in I_j} z_i$; $I_j, J \subseteq \{1, 2, \ldots, n\}$.
We denote the transitive closure of γ by γ^*. The relation γ^* is the smallest equivalence relation on a multiplicative hyperring $(R, +, \circ)$ such that the quotient R/γ^*, the set of all equivalence classes, is a fundamental ring. Let U be the set of all finite sums of products of elements of R. We can rewrite the definition of γ^* on R as follows:
$a\gamma b \Leftrightarrow \exists z_1, z_2, \ldots, z_n \in R$ with $z_1 = a$ and $z_{n+1} = b$ and $u_1, u_2, \ldots, u_n \in U$ such that $\{z_i, z_{i+1}\} \subseteq u_i$ for $i \in \{1, 2, \ldots, n\}$.

Suppose that $\gamma^*(a)$ is the equivalence class containing $a \in R$. Then, both the sum $\oplus$ and the product $\odot$ in R/γ^* are defined as follows: $\gamma^*(a) \oplus \gamma^*(b) = \gamma^*(c)$ for all $c \in \gamma^*(a) + \gamma^*(b)$ and $\gamma^*(a) \odot \gamma^*(b) = \gamma^*(d)$ for all $d \in \gamma^*(a) \circ \gamma^*(b)$. Then R/γ^* is a ring, which is called a fundamental ring of R (see [16]).

Proposition 4. *Let R be a multiplicative hyperring with identity 1. Then the hyperideal J is pseudo 2-prime if and only if J/γ^* is a pseudo 2-prime ideal of R/γ^*.*

Proof. Let J be a pseudo 2-prime hyperideal of R and let $xy \in J/\gamma^*$ with $x, y \in R/\gamma^*$. Then there exist $a, b \in R$ such that $x = \gamma^*(a)$ and $y = \gamma^*(b)$. Hence $xy = \gamma^*(a \circ b) \in J/\gamma^*$, which implies $a \circ b \subseteq J$. Since J is a pseudo 2-prime hyperideal of R, there exists $n \in \mathbb{N}$ such that $a^{2n} \subseteq J^n$ or $b^{2n} \subseteq J^n$. Therefore $\gamma^*(a)^{2n} = \gamma^*(a^{2n}) \in (J/\gamma^*)^n$ or $\gamma^*(b)^{2n} = \gamma^*(b^{2n}) \in (J/\gamma^*)^n$ since γ^* is a strongly regular relation of R. Conversely, assume that J/γ^* is a pseudo 2-prime ideal of R/γ^*. Let $a \circ b \subseteq J$ for some $a, b \in R$. Then $\gamma^*(a \circ b) = \gamma^*(a)\gamma^*(b) \in J/\gamma^*$. Since J/γ^* is a pseudo 2-prime ideal, there exists $n \in \mathbb{N}$ such that $\gamma^*(a)^{2n} \in (J/\gamma^*)^n$ or $\gamma^*(b)^{2n} \in (J/\gamma^*)^n$. Hence, $a^{2n} \subseteq J^n$ or $b^{2n} \subseteq J^n$, as required. □

Let R be a multiplicative hyperring. Then $M_n(R)$ denotes the set of all hypermatrices of R. Moreover, for any $A = (A_{ij})_{nn}$, $B = (B_{ij})_{nn} \in P^*(M_n(R))$, we have $A \subseteq B$ if and only if $A_{ij} \subseteq B_{ij}$.

Example 6. Let R be a multiplicative hyperring with identity 1, and let J be a hyperideal of R. If $M_n(J)$ is a pseudo 2-prime hyperideal of $M_n(R)$, then J is a pseudo 2-prime hyperideal of R.

Proof. Let $x \circ y \subseteq J$ where $x, y \in R$. Then

$$\begin{pmatrix} x\circ y & 0 & \ldots & 0 \\ 0 & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & 0 \end{pmatrix} \subseteq M_n(J).$$

We have

$$\begin{pmatrix} x\circ y & 0 & \ldots & 0 \\ 0 & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & 0 \end{pmatrix} = \begin{pmatrix} x & 0 & \ldots & 0 \\ 0 & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & 0 \end{pmatrix} \begin{pmatrix} y & 0 & \ldots & 0 \\ 0 & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & 0 \end{pmatrix}.$$

Since $M_n(J)$ is a pseudo 2-prime hyperideal of $M_n(R)$, then

$$\begin{pmatrix} x & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{pmatrix}^{2n} \subseteq M_n(J)^n \quad \text{or} \quad \begin{pmatrix} y & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{pmatrix}^{2n} \subseteq M_n(J)^n.$$

Hence,

$$\begin{pmatrix} x^{2n} & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{pmatrix} \subseteq M_n(J)^n \quad \text{or} \quad \begin{pmatrix} y^{2n} & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{pmatrix} \subseteq M_n(J)^n.$$

Therefore, $x^{2n} \subseteq J^n$ or $y^{2n} \subseteq J^n$. Hence J is a pseudo 2-prime hyperideal of R. □

Let $I, I_1, \dots, I_n$ be hyperideals of R. A covering $I \subseteq I_1 \cup \dots \cup I_n$ is said to be efficient precisely when I is not contained in the union of any $n-1$ of the hyperideals $I_1, \dots, I_n$. We shall say that $I \subseteq I_1 \cup \dots \cup I_n$ is an efficient union if none of the I_k, $1 \leq k \leq n$, may be excluded.

Lemma 1. *Let R be a commutative multiplicative hyperring and $J \subseteq J_1 \cup \dots \cup J_n$ be an efficient covering of hyperideals of R. Assume that $J \cap rad(J_k) \not\subseteq J \cap rad(J_m)$ for each $k \neq m$. Then no J_i is a pseudo 2-prime hyperideal for each $i \in \{1, 2, \dots, n\}$.*

Proof. Let $J \subseteq J_1 \cup \dots \cup J_n$ be an efficient covering of hyperideals of R and $J \cap rad(J_k) \not\subseteq J \cap rad(J_m)$ for each $k \neq m$. Suppose that J_1 is a pseudo 2-prime hyperideal of R. Then by Theorem 1, $rad(J_1)$ is a prime hyperideal of R. As the covering is efficient, we have $J \cap (\cap_1^n J_i) \subseteq J \cap J_1$. By assumption, there exists $a_j \in J \cap rad(J_j) - rad(J_1)$ for each $j = 2, 3, \dots, n$. Thus there exists $s_j \in \mathbb{N}$ such that $a_j^{s_j} \subseteq J \cap J_j$. Put $t = max\{s_1, s_2, \dots, s_n\}$, hence $a_j^t \subseteq J \cap J_j$. Set $x = \{a_2\}$ and $y = a_3 \circ a_4 \circ \dots \circ a_n$. Since $rad(J_1)$ is a prime hyperideal and $a_3, a_4, \dots, a_n \notin rad(J_1)$, we conclude $y^{2nt} \not\subseteq J_1^n$ for all $n \in \mathbb{N}$. Otherwise, $y = a_3 \circ a_4 \circ \dots \circ a_n \subseteq rad(J_1^n) = rad(J_1)$ and $a_I \in rad(J_1)$ for some $i \in \{3, 4, \dots, n\}$, which is a contradiction. Now, since J_1 is a pseudo 2-prime hyperideal and $x^t b^t \subseteq J_1$, we have $x^{2nt} = a_2^{2nt} \subseteq J_1^n$ for some $n \in \mathbb{N}$. Thus $a_2 \in rad(J_1^n) = rad(J_1)$, a contradiction. Therefore, no J_i is a pseudo 2-prime hyperideal of R. □

Now, we state the Pseudo 2-Prime Avoidance Theorem for pseudo 2-prime hyperideals of R.

Theorem 5. *(Pseudo 2-Prime Avoidance Theorem) Let R be a commutative multiplicative hyperring and $J \subseteq J_1 \cup \dots \cup J_n$ where $J, J_1, \dots, J_n$ are hyperideals of R such that at most two J_i are not pseudo 2-prime hyperideals of R. Assume that $J \cap rad(J_k) \not\subseteq J \cap rad(J_m)$ for each $k \neq m$. Then $J \subseteq J_i$ for some $i \in \{1, 2, \dots, n\}$.*

Proof. Let $J \subseteq J_1 \cup \dots \cup J_n$ be a covering such that at least $n-2$ of the hyperideals $I_1, I_2, \cdots, I_n$ are 1-absorbing prime. Without loss of generality, one may reduce the covering to an efficient covering. If $n = 2$, then it is obvious. Suppose that $n > 2$. Then we get a contradiction by using Lemma 1. Therefore, we have $n \leq 2$. The rest is clear. □

4 Conclusion

The notion of pseudo 2-prime hyperideals was proposed and basic properties based on their formations were introduced. We showed by providing examples that the concepts of 2-prime hyperideals, primary hyperideals and quasi-primary hyperideals are different from pseudo 2-prime hyperideals and many results concerning of them do not hold for pseudo 2-prime hyperideals. Also, we investigated the conditions under which prime hyperideals, 2-prime hyperideals, quasi primary hyperideals, 2-absorbing primary hyperideals and pseudo 2-prime hyperideals are coincide. We also explored the behaviour of pseudo 2-prime hyperideals under homomorphism hyperrings, in factor hyperrings, Cartesian products of hyperrings and the fundamental relation in the context of multiplicative hyperrings with some related results. We hope that these results are helpful for further studies in the theory of hyperrings. Also, we will work on graded such hyperstructures.

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